Induced Compton Scattering in Fast Radio Bursts

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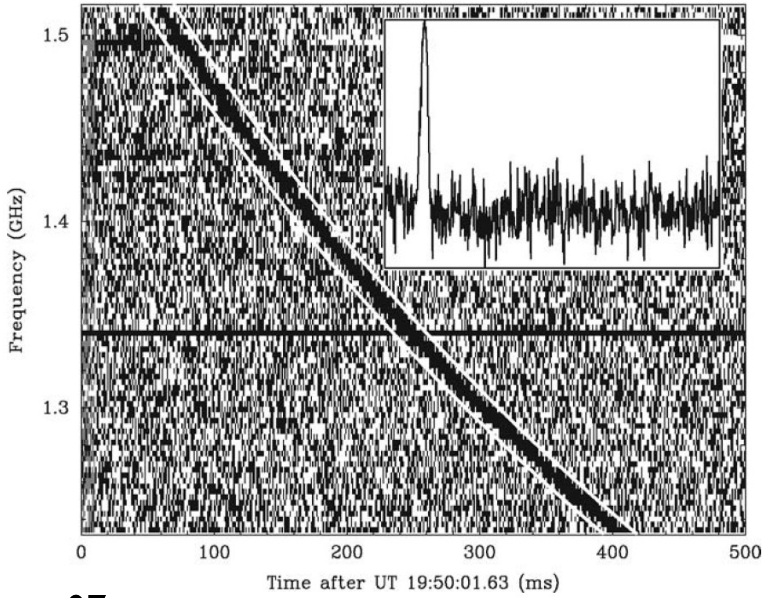
Fast Radio Bursts (FRB)



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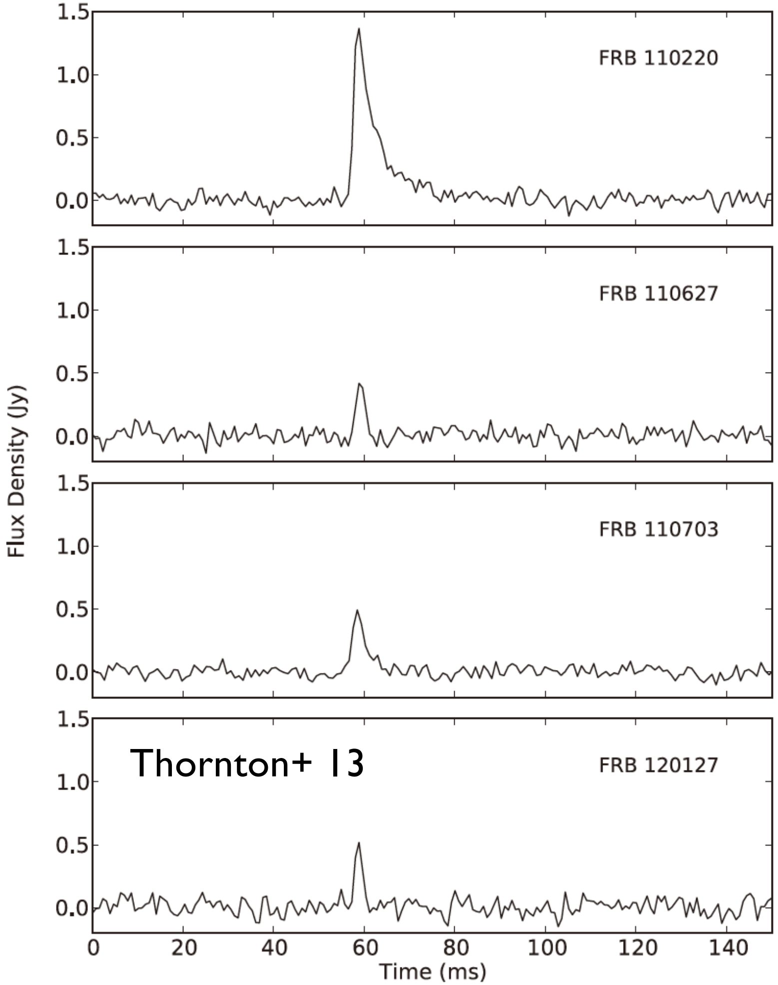
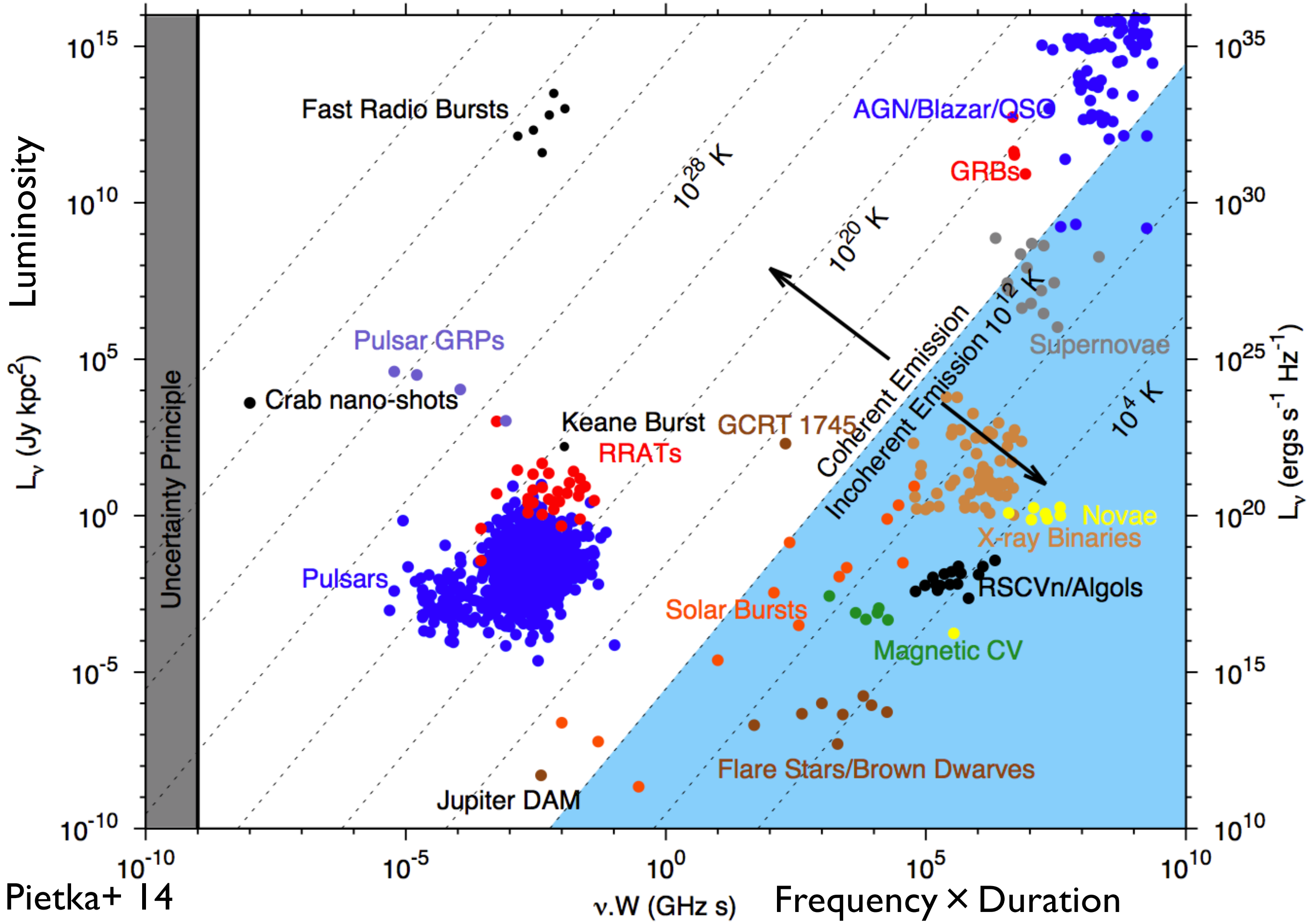
Most luminous radio transients discovered in 2007



Brightness Temperature



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Brightness temperature

$$T = \frac{c^2 I_\nu}{2k\nu^2} > 10^{35}\mathrm{K}\,\frac{F_{\nu,\mathrm{Jy}} d_{\mathrm{Gpc}}^2}{\Delta t_{\mathrm{ms}}^2 \nu_{\mathrm{GHz}}^2}$$

$$F_\nu \simeq I_\nu \Delta\Omega \simeq I_\nu \frac{\pi \ell^2}{d^2}$$

$$\ell < c\Delta t$$

$$\rightarrow \nu = \Gamma\nu', I_\nu/\nu^3 = I'_{\nu'}/\nu'^3, \ell < c\Gamma\Delta t$$

$$T' > 10^{35}\mathrm{K}\,\frac{F_{\nu,\mathrm{Jy}} d_{\mathrm{Gpc}}^2}{\Delta t_{\mathrm{ms}}^2 \nu_{\mathrm{GHz}}^2 \Gamma^3}$$

Coherent number

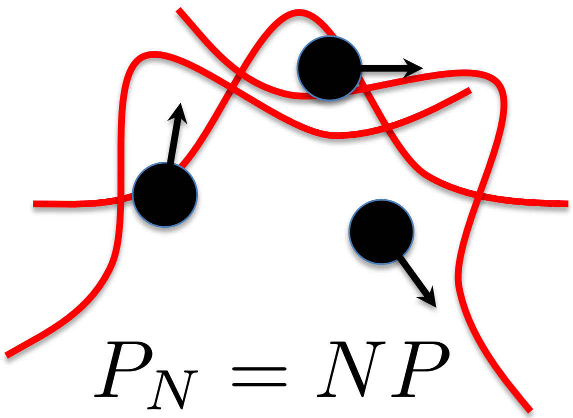
$$\mathcal{N} \sim \frac{kT'}{\gamma m_e c^2} \sim 10^{25}\,\frac{F_{\nu,\mathrm{Jy}} d_{\mathrm{Gpc}}^2}{\Delta t_{\mathrm{ms}}^2 \nu_{\mathrm{GHz}}^2 \gamma \Gamma^3}$$

Coherent Emission

$$P_N = \left| \sum_{k=1}^{N} E_k e^{i\phi_k} \right|^2$$

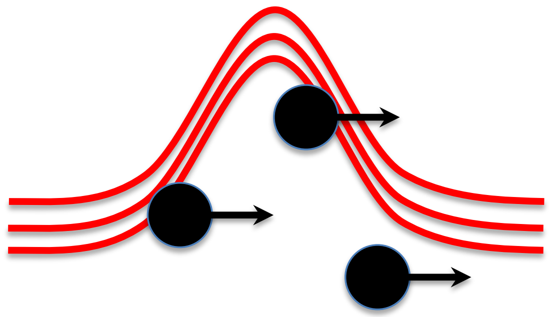
$$= N\,|E|^2 + |E|^2 \sum_{k \neq j} e^{i(\phi_k - \phi_j)}$$

Incoherent



$$P_N = NP$$

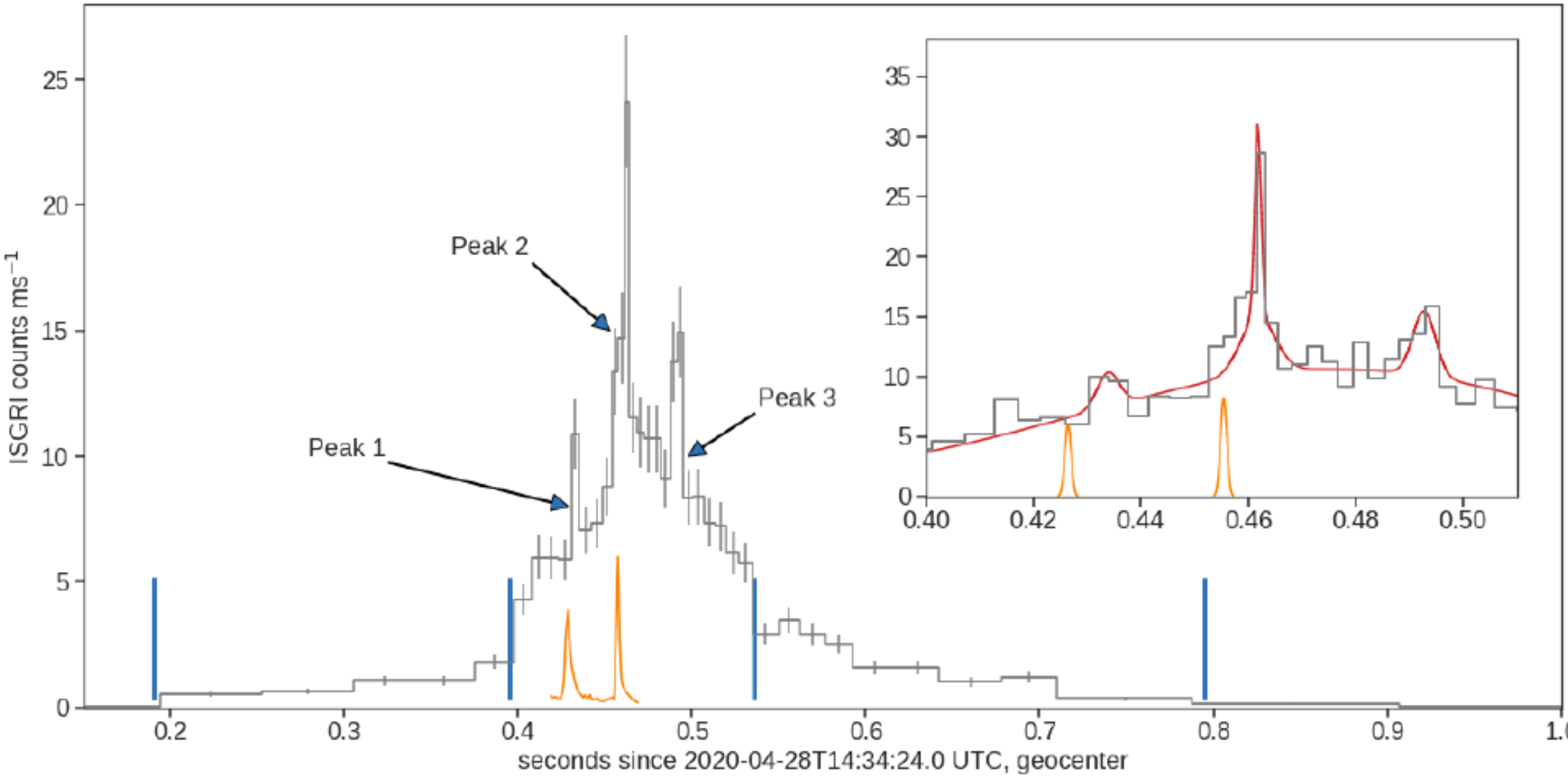
Coherent



$$P_N = N^2 P$$

Galactic FRB from Magnetar Bursts

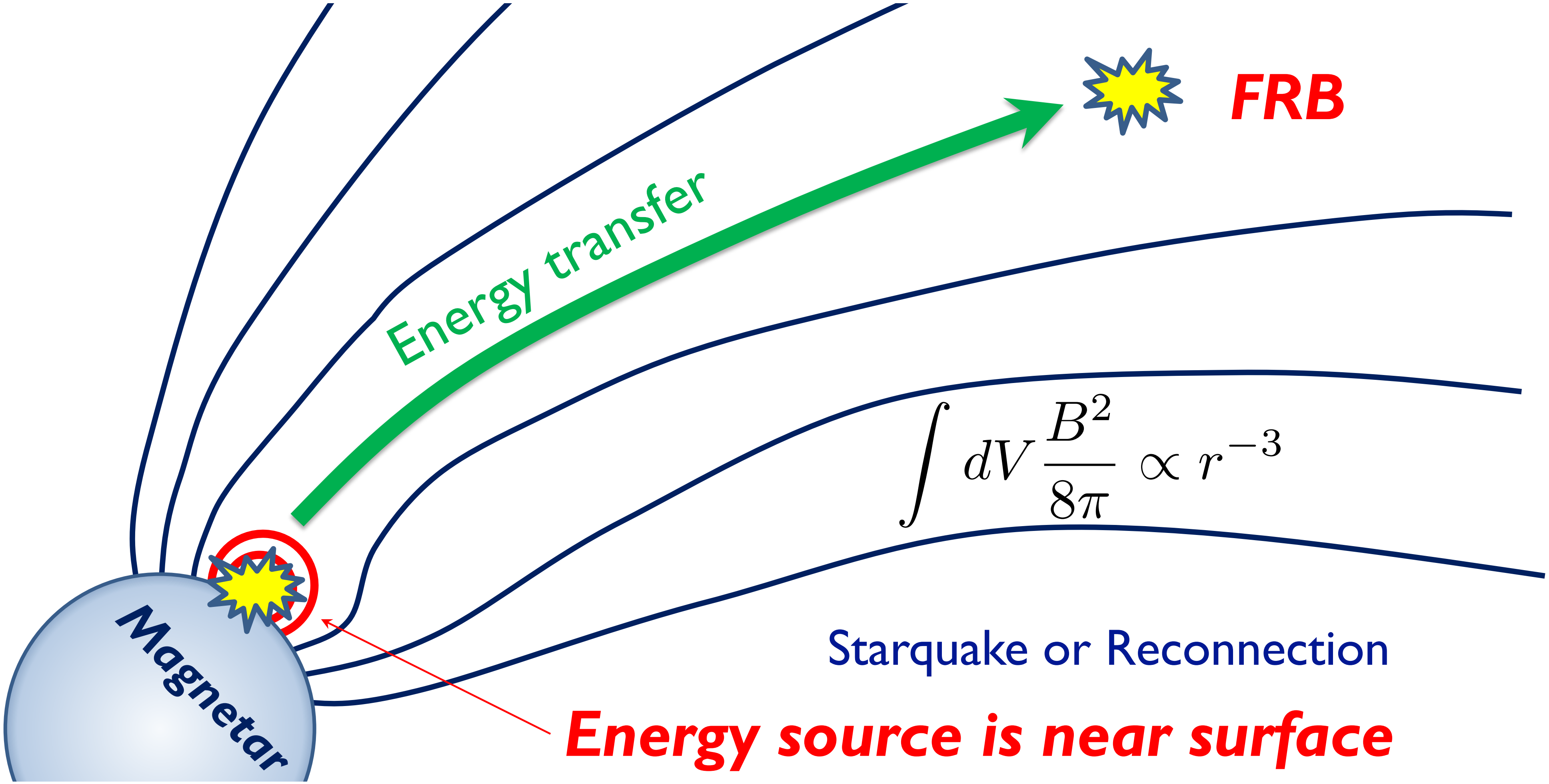
A smoking gun! Magnetar: One of the origins



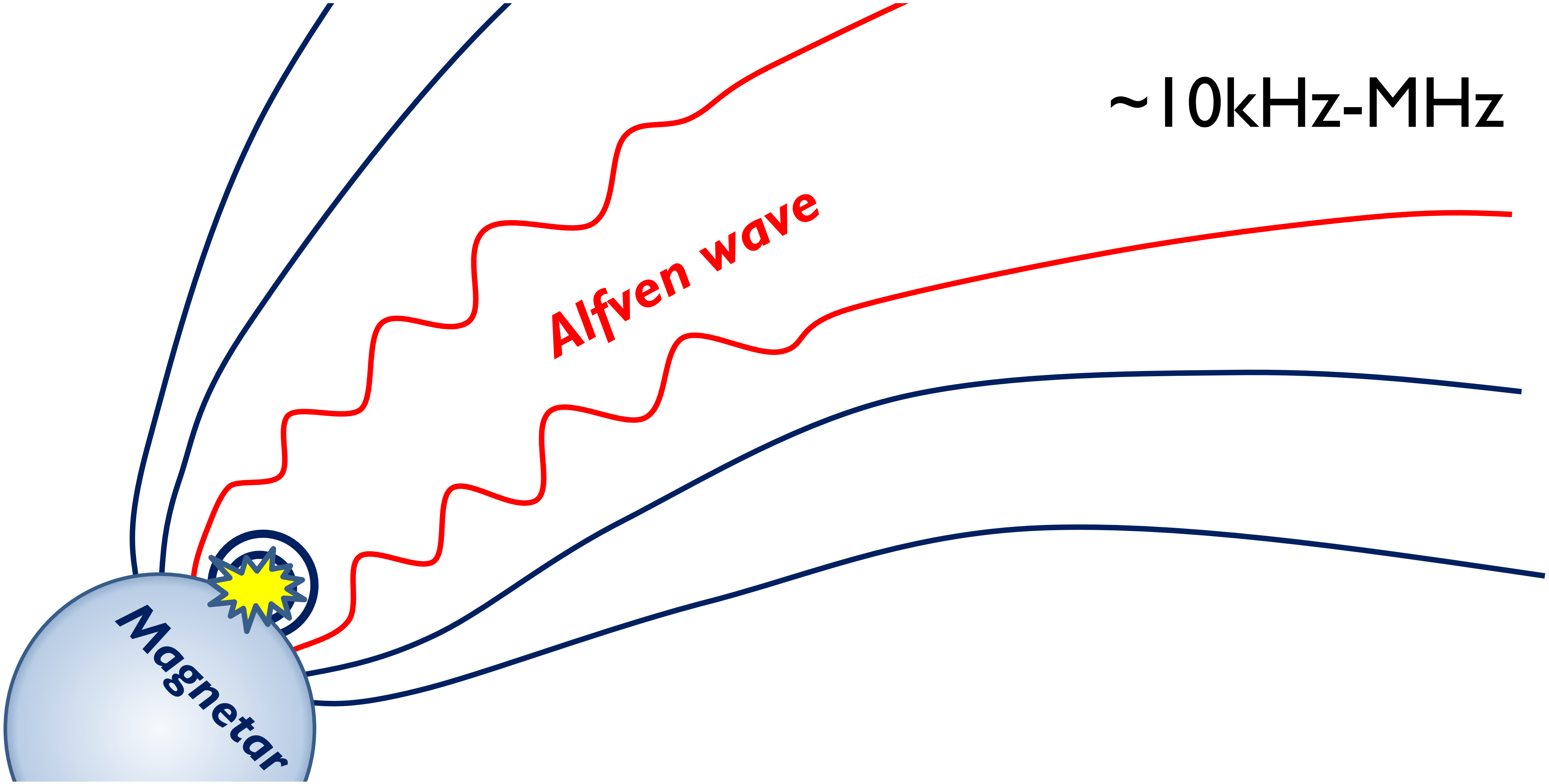
$L_X \sim 10^{41}$ erg/s >> $L_{FRB} \sim 10^{38}$ erg/s

Mereghetti+ 20,
Bochenek+ 20,
CHIME/FRB+ 20,
Li+ 20,
Ridnaia+ 20,
Tavani+ 20

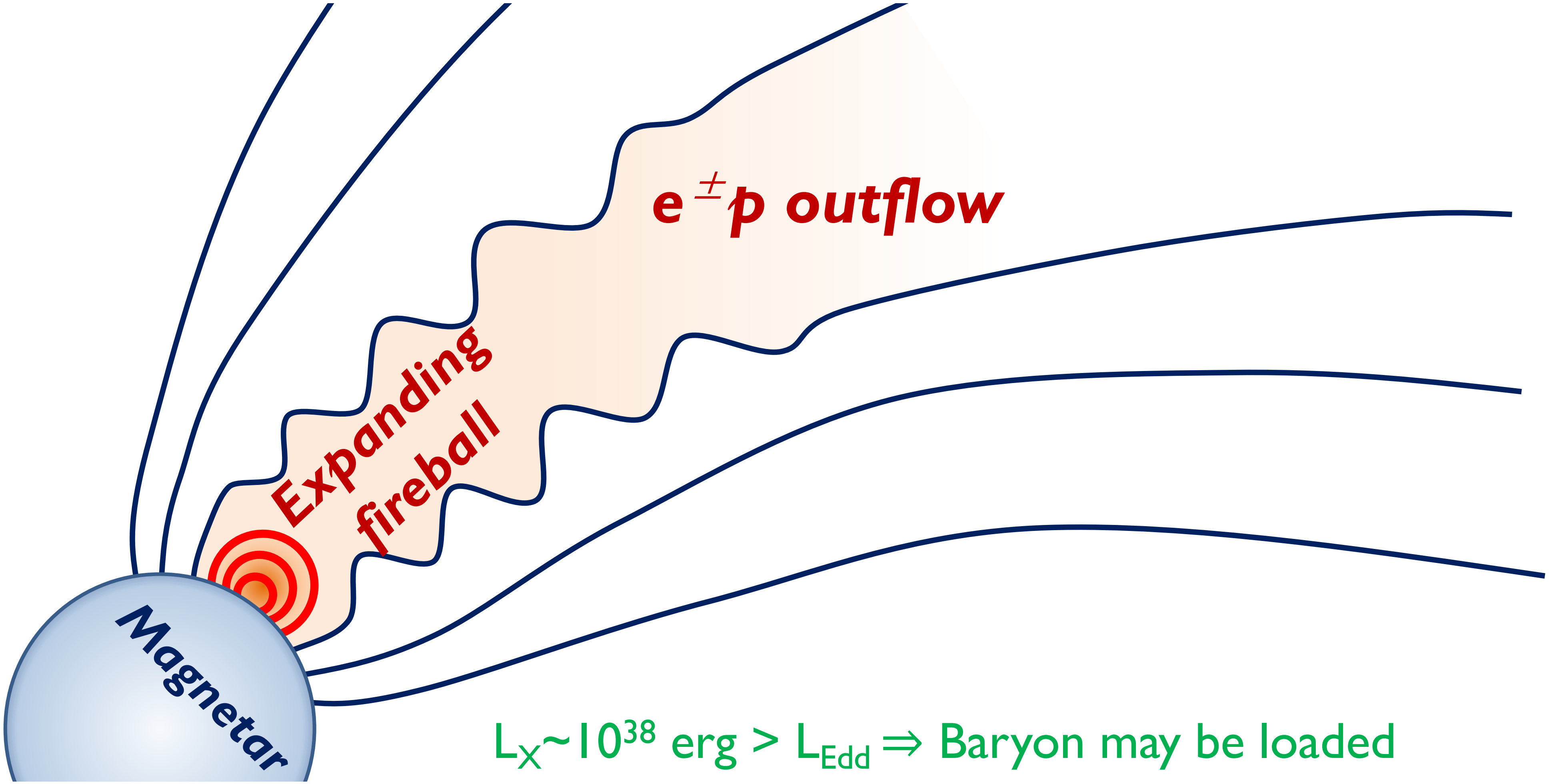
Kinetic? Magnetic?



Magnetic Pulse: Poynting Flux

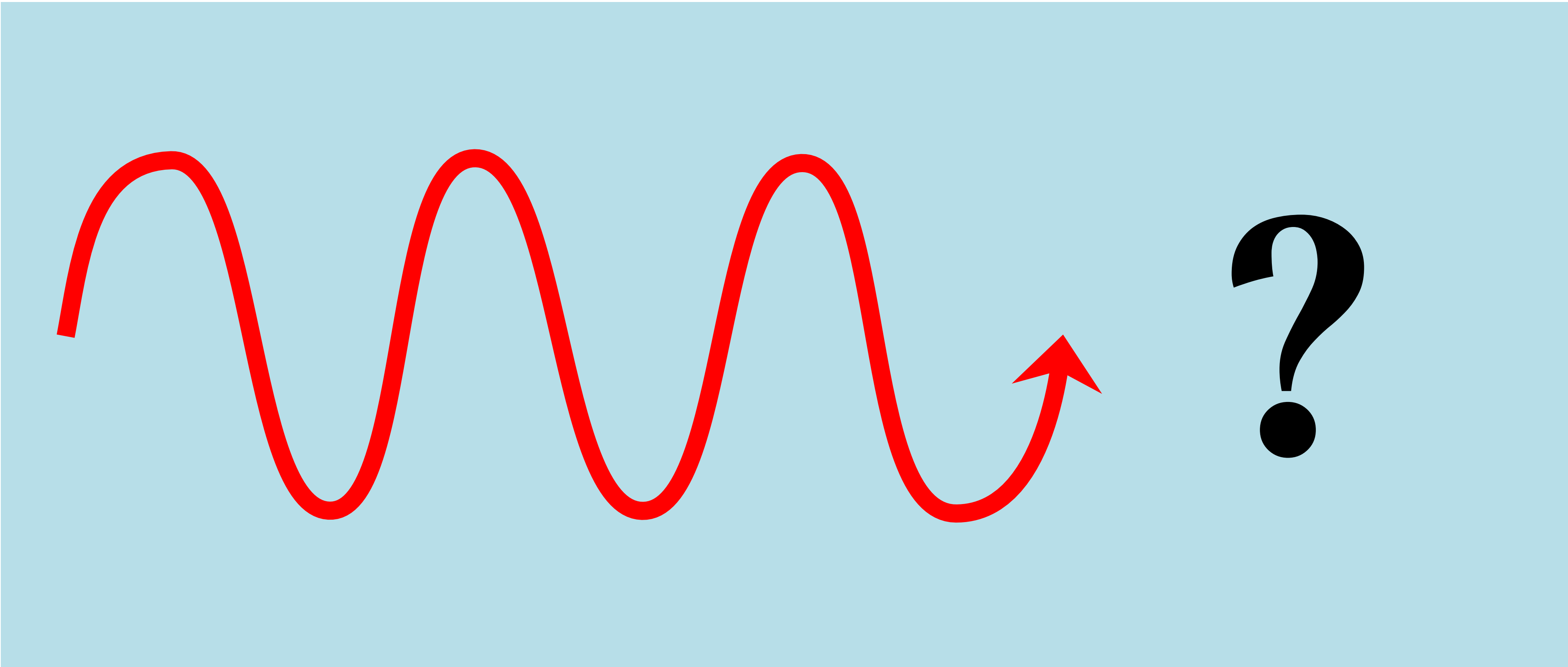


Fireball: Kinetic Flux



$L_X \sim 10^{38}$ erg > L_{Edd} ⇒ Baryon may be loaded

Wave in Plasma

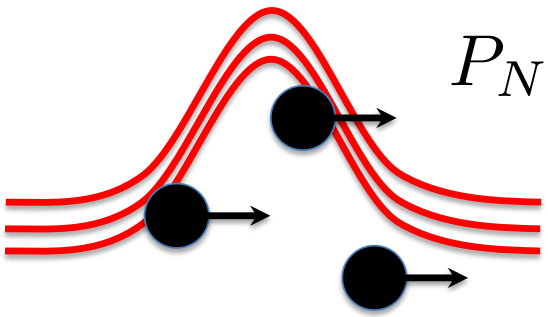


$\omega > \omega_p$ plasma frequency

Antenna vs. Maser

Antenna mechanism

$$f(\boldsymbol{x}, \boldsymbol{p}, t) \propto \delta(\boldsymbol{x})\delta(\boldsymbol{p})$$



$$P_N = N^2 P$$

N particles

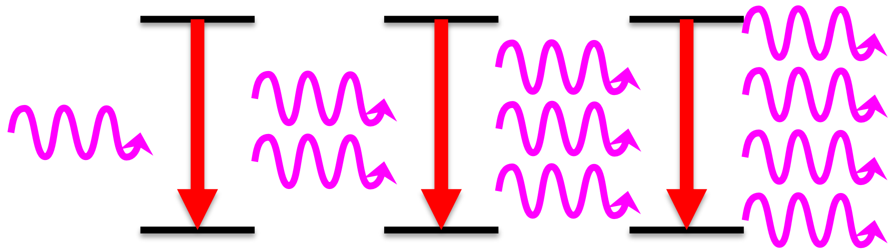
wavelength λ

Spontaneous

Maser mechanism ✔

$$f(\boldsymbol{x}, \boldsymbol{p}, t) \propto \delta(\boldsymbol{p})$$

or even
w/o bunch

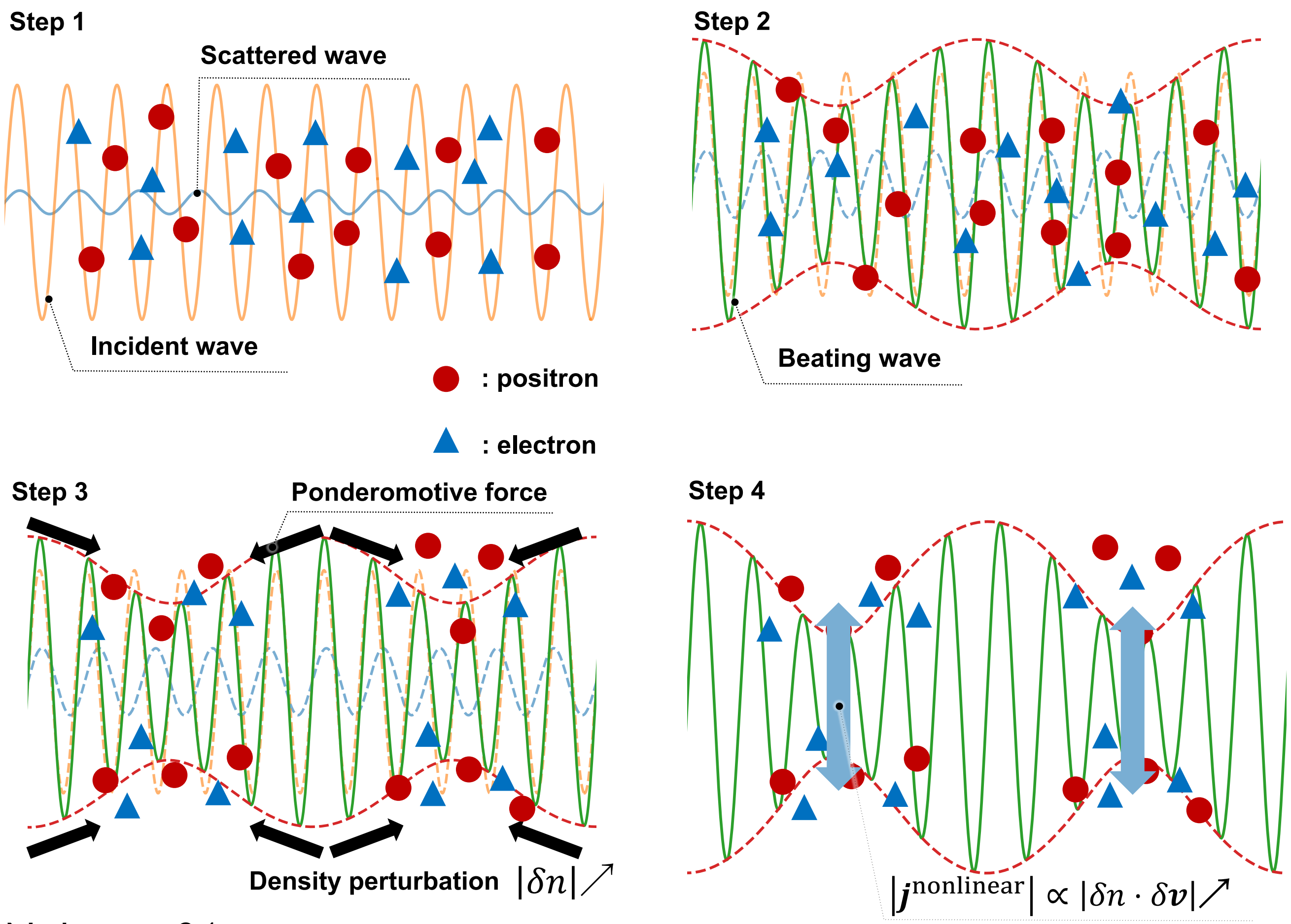


N particles

$>\lambda$

Induced (Stimulated)

Induced Scattering



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Maser (induced emission)

Classical plasma process

Parametric instability

- ***Induced Compton***
- Induced Brillouin
- Induced Raman
- Filamentation instability

3 waves $\omega_0 = \omega_1 + |\omega|$

EM → EM + Density wave

Growth Rate

$$\Gamma_{\rm C}^{\rm max} = \sqrt{\frac{\pi}{32{\rm e}} \frac{\omega_{\rm p}^2 a_{\rm e}^2}{\omega_0} \frac{m_{\rm e} c^2}{k_{\rm B} T_{\rm e}}},$$

Scattering rate

$$\left(t_{{\rm c},\|}^{\rm broad}\right)^{-1} = \pi \frac{\omega_{\rm p}^2 a_{\rm e}^2}{\omega_0} \left(\frac{\omega_0}{\Delta\omega}\right)^2$$
$$= 1.1 \times 10^{20}\,{\rm s}^{-1} \frac{\mathcal{M}_6 R_6^3 B_{{\rm p},14} L_{38}}{P_{\rm sec} r_8^5 \nu_9^2} \left(\frac{\Delta\nu/\nu_0}{1}\right)^{-2} \gg \Delta t^{-1}$$

Strength parameter (dimensionless amplitude) $a_{\rm e} \equiv \frac{2e\,|\boldsymbol{A}_0|}{m_{\rm e} c^2},$

Plasma frequency $\omega_{\rm p} \equiv \sqrt{\frac{8\pi e^2 n_{\rm e0}}{m_{\rm e}}},$

Frequency of the most growing waves

$$\omega_1(\nu, \theta_{kB}) \simeq \omega_0 \left(1 - \sqrt{2(1-\nu)\cos^2\theta_{kB} \frac{k_{\rm B} T_{\rm e}}{m_{\rm e} c^2}}\right)$$ (for $\mathbf{A}_0 \| \mathbf{B}_0$)

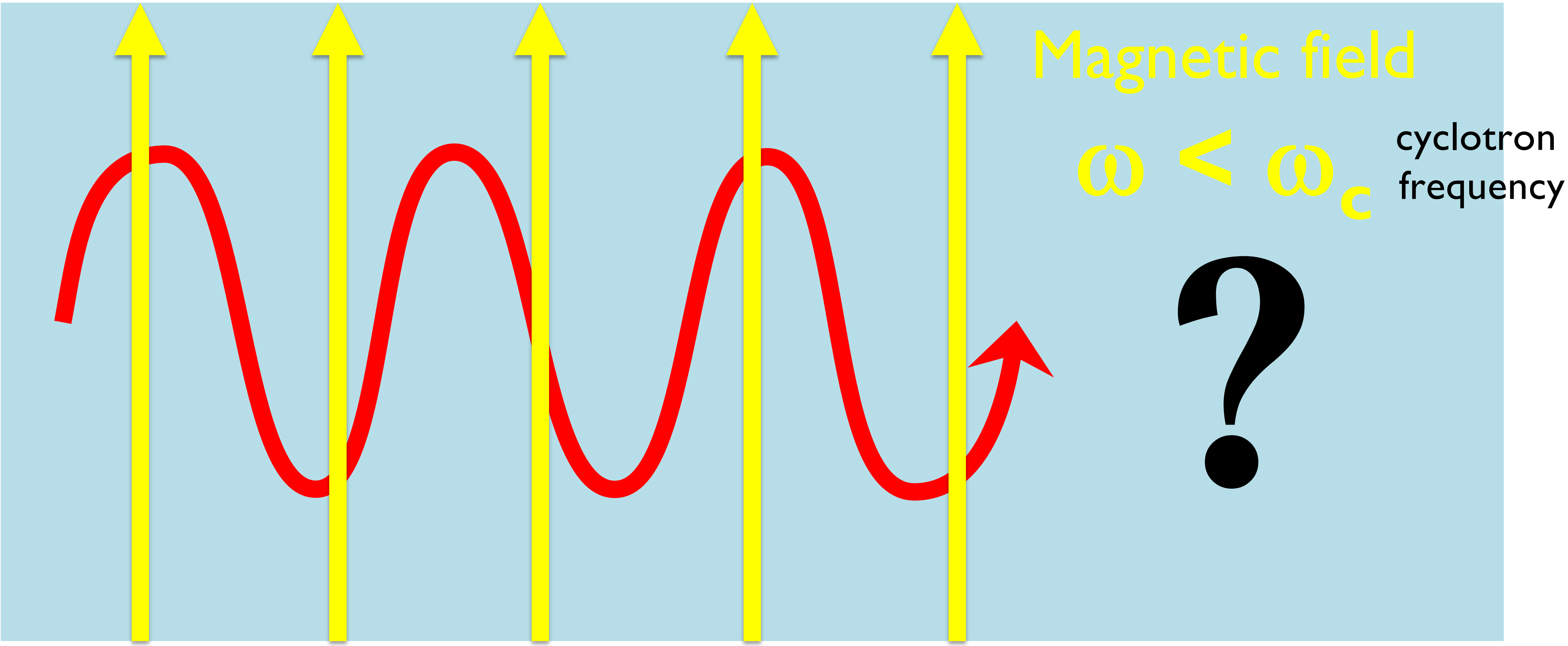
Inverse of the burst duration

$$\Delta t^{-1} = 10^3\ {\rm s}^{-1}$$

➡ Many scatterings

➡ Dissipation

Wave in Plasma



$\omega < \omega_p$ (plasma frequency) can propagate

Growth Rate

Induced Compton scattering for $A_{0\perp}=0$ (narrow band)

① Ordinary mode

$$\Gamma_{\rm C}^{\max}=\sqrt{\frac{\pi}{32{\rm e}}}\frac{\omega_{\rm p}^2 a_{\rm e}^2}{\omega_0}\frac{m_{\rm e}c^2}{k_{\rm B}T_{\rm e}}$$

Induced Compton scattering for $A_{0\parallel}=0$ (narrow band)

② Charged mode

$$\Gamma_{\rm C}^{\max}=\sqrt{\frac{\pi}{32{\rm e}}}\left(\frac{\omega_0}{\omega_{\rm c}}\right)^2\frac{\omega_{\rm p}^2 a_{\rm e}^2}{\omega_0}\frac{m_{\rm e}c^2}{k_{\rm B}T_{\rm e}}\times\begin{cases}1 & \dfrac{8k_{\rm B}T_{\rm e}}{m_{\rm e}c^2}\left(\dfrac{\omega_0}{\omega_{\rm p}}\right)^2\geq 1\\ \dfrac{\rm e}{2\pi}\left(\dfrac{\omega_0}{\omega_{\rm p}}\right)^4\left(\dfrac{8k_{\rm B}T_{\rm e}}{m_{\rm e}c^2}\right)^2 & \dfrac{8k_{\rm B}T_{\rm e}}{m_{\rm e}c^2}\left(\dfrac{\omega_0}{\omega_{\rm p}}\right)^2\ll 1\end{cases}$$

Gyroradius effect

Debye screening effect

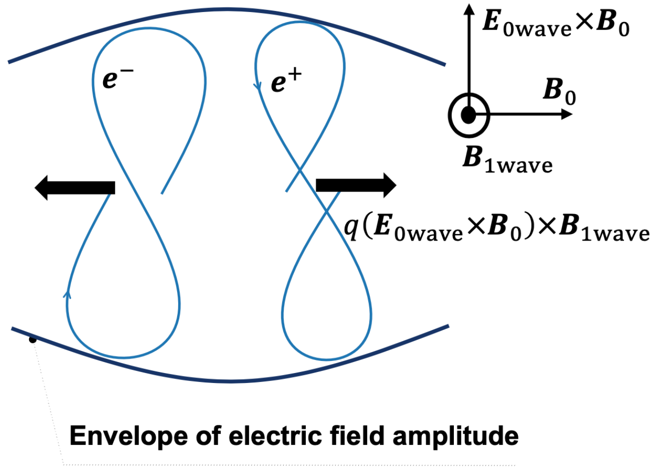
③ Neutral mode

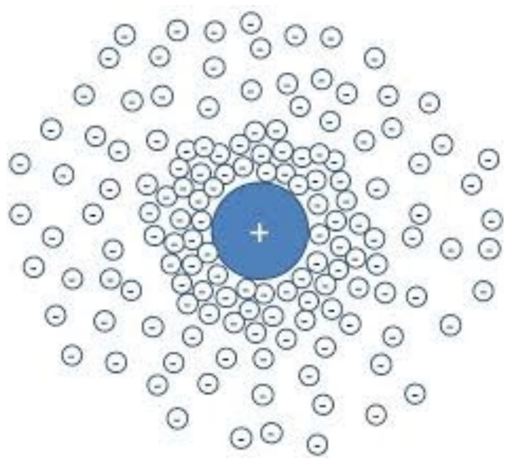
$$\Gamma_{\rm C}^{\max}=\sqrt{\frac{\pi}{32{\rm e}}}\left(\frac{\omega_0}{\omega_{\rm c}}\right)^4\frac{\omega_{\rm p}^2 a_{\rm e}^2}{\omega_0}\frac{m_{\rm e}c^2}{k_{\rm B}T_{\rm e}}$$

(Gyroradius effect)^2

$$\frac{8k_{\rm B}T_{\rm e}}{m_{\rm e}c^2}\left(\frac{\omega_0}{\omega_{\rm p}}\right)^2=32\pi^2\left(\frac{\lambda_{\rm De}}{\lambda_0}\right)^2$$

Charged mode





Scattering Rate

Scattering rate

$$\left(t_{\mathrm{c},\parallel}^{\mathrm{broad}}\right)^{-1} = \pi \frac{\omega_{\mathrm{p}}^2 a_{\mathrm{e}}^2}{\omega_0} \left(\frac{\omega_0}{\Delta\omega}\right)^2$$
$$= 1.1 \times 10^{20}\,\mathrm{s}^{-1} \frac{\mathcal{M}_6 R_6^3 B_{\mathrm{p},14} L_{38}}{P_{\mathrm{sec}} r_8^5 \nu_9^2} \left(\frac{\Delta\nu/\nu_0}{1}\right)^{-2} \gg \Delta t^{-1}$$

Inverse of the burst duration

$$\Delta t^{-1} = 10^3\ \mathrm{s}^{-1}$$

→ $$\left(t_{\mathrm{charged}}^{\mathrm{broad}}\right)^{-1} = 32\pi \left(\frac{\omega_0}{\omega_{\mathrm{c}}}\right)^2 \frac{\omega_{\mathrm{p}}^2 a_{\mathrm{e}}^2}{\omega_0} \left(\frac{k_{\mathrm{B}} T_{\mathrm{e}}}{m_{\mathrm{e}} c^2}\right)^2 \left(\frac{\omega_0}{\omega_{\mathrm{p}}}\right)^4 \left(\frac{\omega_0}{\Delta\omega}\right)^2$$
$$= 9.3 \times 10^2\,\mathrm{s}^{-1} \frac{P_{\mathrm{sec}} r_8^7 L_{38} T_{80\mathrm{keV}}^2}{\mathcal{M}_6 \nu_9^2 R_6^9 B_{\mathrm{p},14}^3} \left(\frac{\Delta\nu/\nu_0}{1}\right)^{-2} \sim \Delta t^{-1},$$

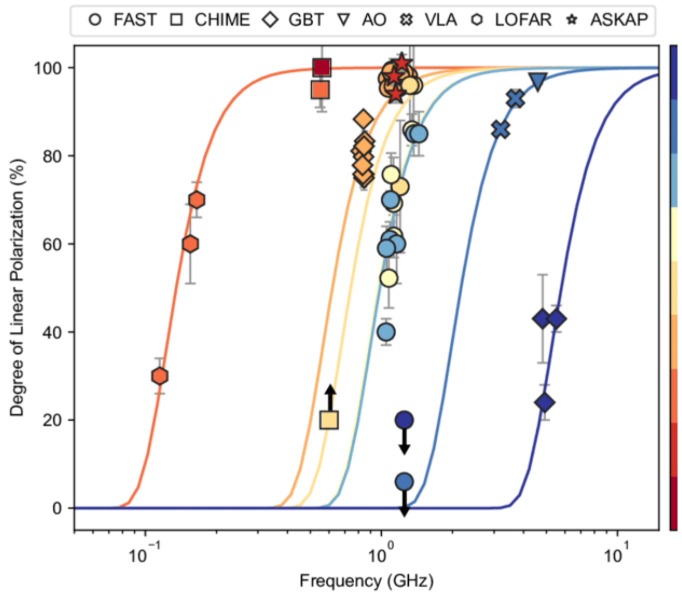
$$t_{\mathrm{C,wrong}}^{-1} \sim \frac{(\omega_{\mathrm{p}} a_{\mathrm{e}})^2}{\omega_0} \left(\frac{\omega_0}{\omega_{\mathrm{c}}}\right)^2$$
$$\sim 4.5 \times 10^8\,\mathrm{s}^{-1} \frac{r_8\, L_{38}\, \mathcal{M}_6\, R_{\mathrm{NS},6}^3}{B_{\mathrm{p},14}\, \nu_9\, P_{\mathrm{sec}}}.$$

→ $$\left(t_{\mathrm{neutral}}^{\mathrm{broad}}\right)^{-1} = \pi \frac{\omega_{\mathrm{p}}^2 a_{\mathrm{e}}^2}{\omega_0} \left(\frac{\omega_0}{\omega_{\mathrm{c}}}\right)^4 \left(\frac{\omega_0}{\Delta\omega}\right)^2$$
$$= 1.8 \times 10^{-2}\,\mathrm{s}^{-1} \frac{\mathcal{M}_6 L_{38} \nu_9^2 r_8^7}{P_{\mathrm{sec}} R_6^9 B_{\mathrm{p},14}^3} \left(\frac{\Delta\nu/\nu_0}{1}\right)^{-2} \ll \Delta t^{-1}$$

Waves can escape!

Polarization

	Scatt. angle	Escaping polarization	Max
Ordinary mode	$\mathbf{E}_1 \parallel \mathbf{E}_0$	$\mathbf{E} \perp \mathbf{B}_0$	***100%***
Charged mode	$\mathbf{E}_1 \perp \mathbf{E}_0$	$\mathbf{E} \perp \mathbf{B}_0$	***~50%***
Neutral mode	$\mathbf{E}_1 \parallel \mathbf{E}_0$	$\mathbf{E} \perp \mathbf{B}_0$	***~50%***



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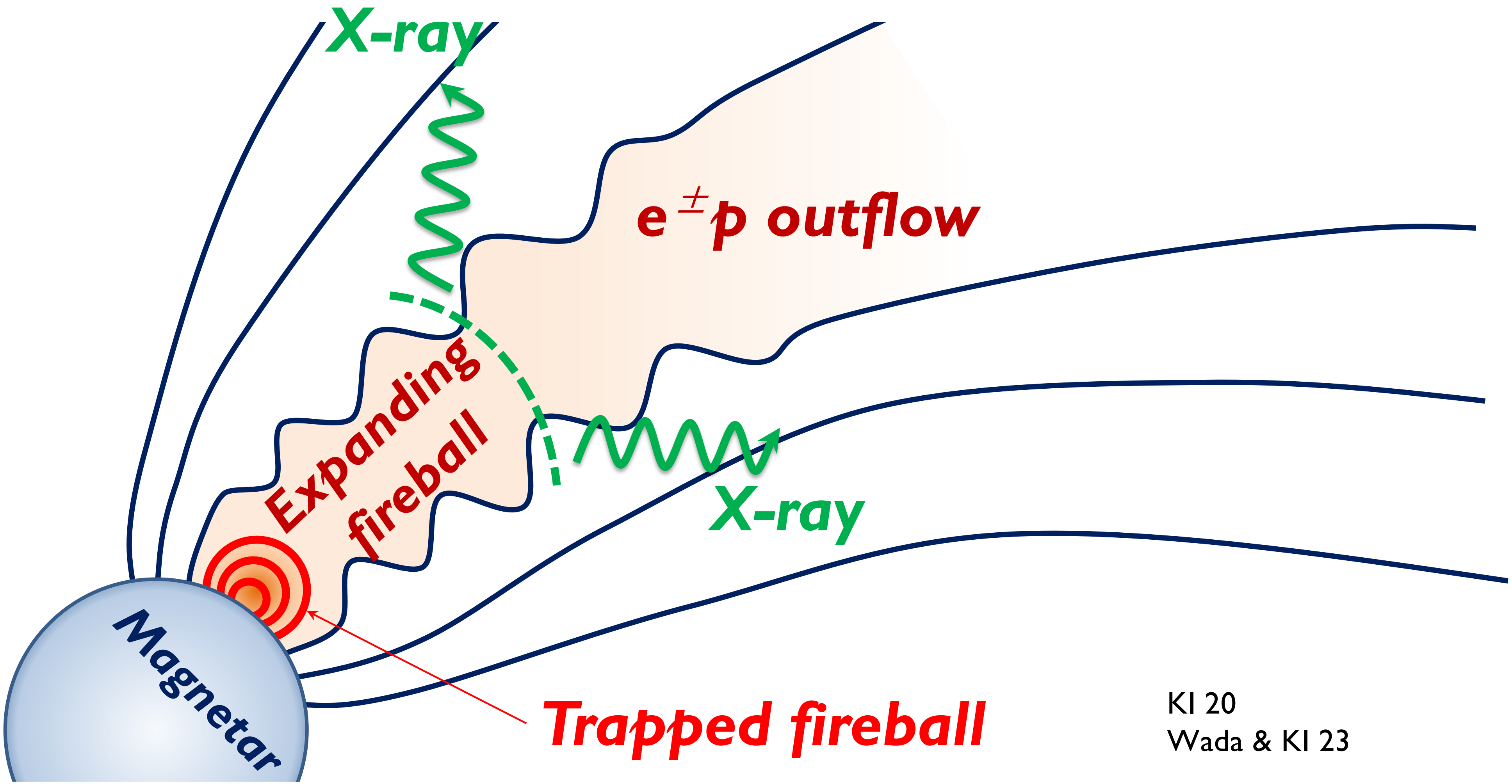
Summary

- ***Induced Compton scattering in B_0 for pairs*** is formulated for the first time
- ***Ordinary, Charged & Neutral modes***
- ***Suppression of scatterings***
  - Gyroradius effect
  - Debye screening
- ***FRB can escape from a neutron star magnetosphere***
- ***Polarization $\perp B_0$ 100% to ~50%***

Implications for binary neutron star mergers?

Thank You

X-ray from Fireball



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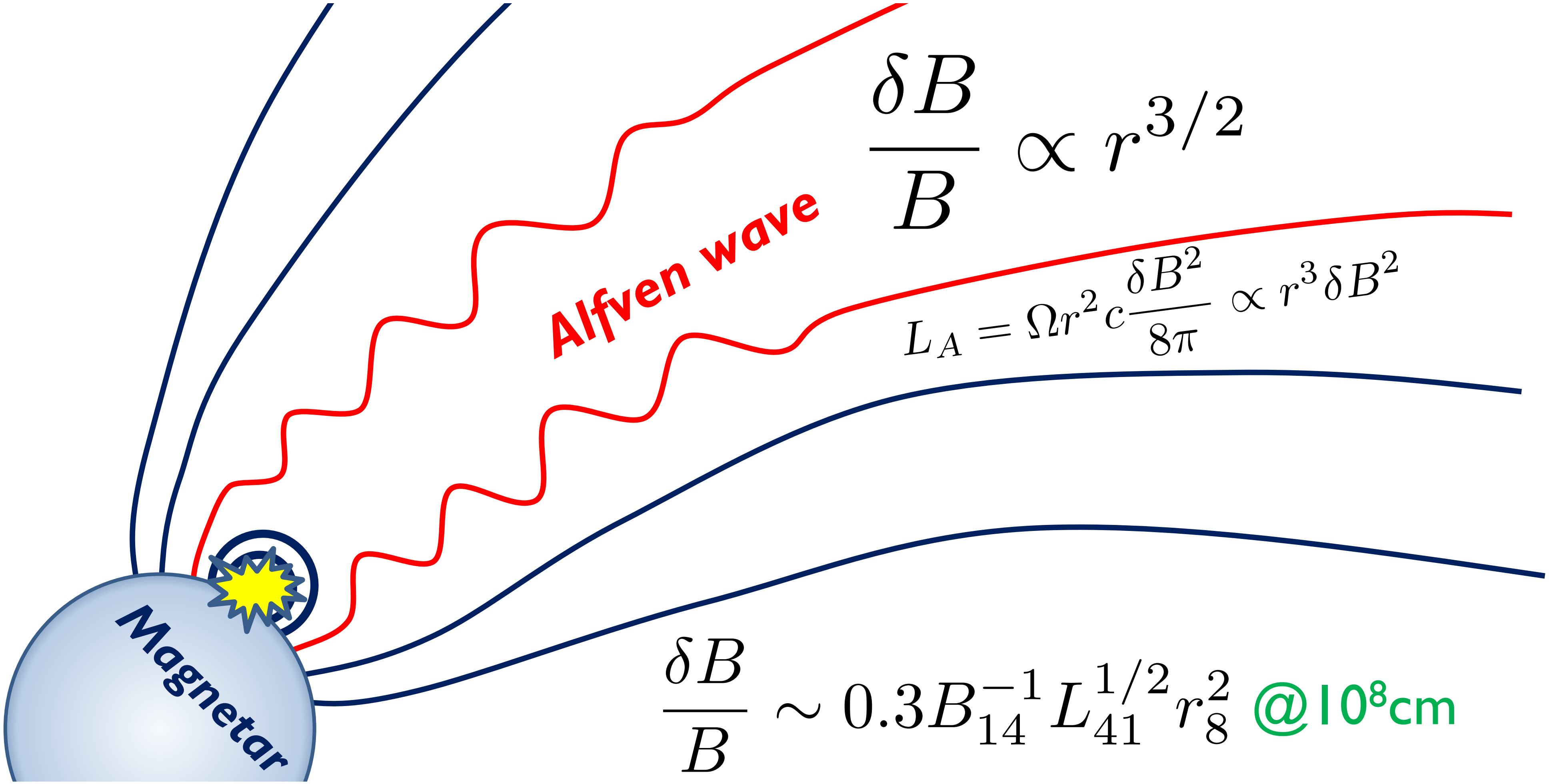
Wave Amplitude

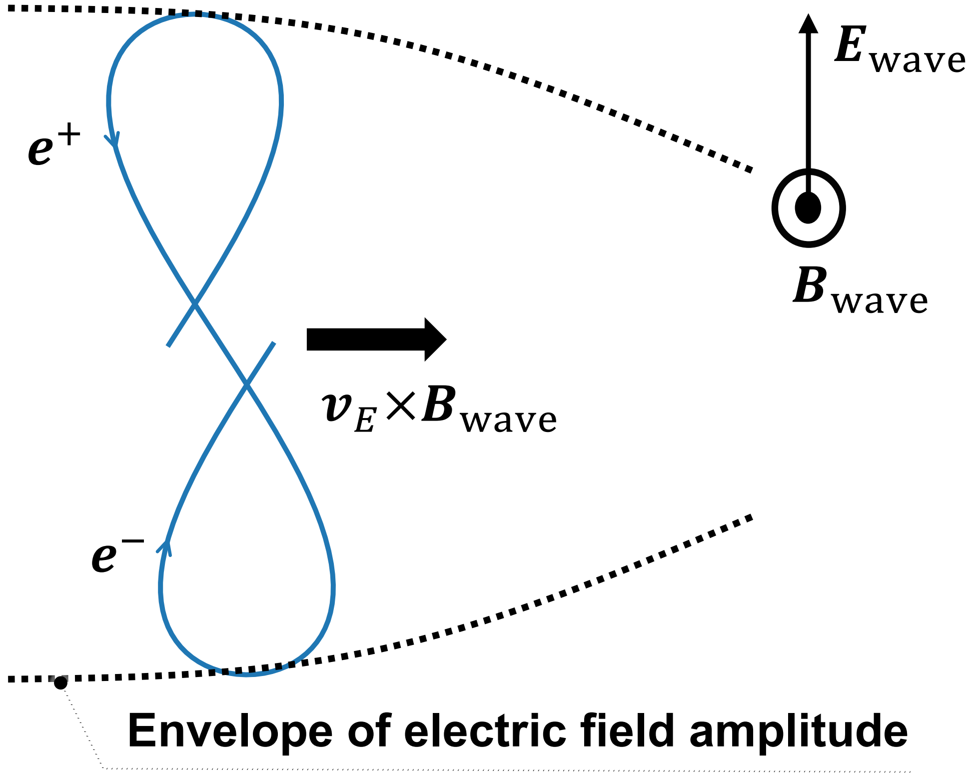


$$\frac{\delta B}{B} \propto r^{3/2}$$

$$L_A = \Omega r^2 c \frac{\delta B^2}{8\pi} \propto r^3 \delta B^2$$

$$\frac{\delta B}{B} \sim 0.3 B_{14}^{-1} L_{41}^{1/2} r_8^2$$ @10^8cm

Ponderomotive Force



$$\frac{d^2\boldsymbol{r}}{dt^2} = q\boldsymbol{E}(\boldsymbol{r}) + \frac{q}{c}\boldsymbol{v} \times \boldsymbol{B}$$

$$\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{r}_1$$

$\boldsymbol{r}_0$: oscillation center

$\boldsymbol{r}_1$: fast oscillation

$$\frac{d^2\boldsymbol{r}_0}{dt^2} \simeq -\nabla\phi_{\rm p}$$

ponderomotive potential

$$\phi_{\rm p} = \frac{e^2}{2m\omega^2}\langle|\boldsymbol{E}(\boldsymbol{r}_0)|^2\rangle_{\rm time}$$

Independent of the charge sign

Basic Equations

Maxwell eq.

$$\frac{\partial^2 \boldsymbol{A}}{\partial t^2} - c^2 \Delta \boldsymbol{A} = 4\pi c \boldsymbol{j}$$

Equations of motion ($\omega \sim \omega_{0,1}$)

$$\frac{d\boldsymbol{v}_\pm}{dt} = \pm \frac{e}{m_e}\left(\boldsymbol{E} + \frac{\boldsymbol{v}_\pm \times \boldsymbol{B}_0}{c}\right)$$

Vlasov equation ($\omega \ll \omega_{0,1}$)

$$\frac{\partial f_\pm}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{\nabla} f_\pm + \boldsymbol{F}\cdot\frac{\partial f_\pm}{\partial \boldsymbol{p}} = 0$$

$$\boldsymbol{F} = -\boldsymbol{\nabla}\phi_\pm \pm e\left(\boldsymbol{E} + \frac{\boldsymbol{v}\times\boldsymbol{B}_0}{c}\right)$$

Ponderomotive force

$$\boldsymbol{\nabla}\cdot\boldsymbol{E} = \sum_{q=\pm e} 4\pi q n_{e0} \int \delta f_\pm d^3\boldsymbol{v}$$

EM waves $\boldsymbol{A}(\boldsymbol{r},t) = \boldsymbol{A}_0 e^{i(\boldsymbol{k_0}\cdot\boldsymbol{r}-\omega_0 t)} + \boldsymbol{A}_1 e^{i(\boldsymbol{k_1}\cdot\boldsymbol{r}-\omega_1 t)} + \text{c.c.},$

Density fluctuation $\widetilde{\delta n_\pm}(\boldsymbol{k},\omega)$

$$= n_{e0}\int d^3\boldsymbol{v}\ \widetilde{\delta f_\pm}(\boldsymbol{k},\boldsymbol{v},\omega)$$

Current

$$\boldsymbol{j} = \sum_{q=\pm e} q n_\pm(\boldsymbol{r},t)\boldsymbol{v}_\pm(\boldsymbol{r},t)$$

➡ Dispersion relation for (ω_1,k_1)

(for $\mathbf{A}_0 \| \mathbf{B}_0$)

$$c^2k_1^2 - \omega_1^2 + \omega_p^2 = \frac{1}{4}c^2\left(\omega_p a_e \mu\right)^2 \times \sum_{\ell=-\infty}^{+\infty}\int d^3\boldsymbol{v}\frac{J_\ell^2\left(k_\perp r_L\right)\boldsymbol{k}\cdot\frac{\partial f_{0\pm}}{\partial \boldsymbol{v}^*}}{\omega - k_\parallel v_\parallel - \ell\omega_c}.$$

Detail Calculations

Solution of density fluctuations

$$\widetilde{\delta n_\pm}(\boldsymbol{k},\omega) = n_{e0}\int \mathrm{d}^3\boldsymbol{v}\ \widetilde{\delta f_\pm}(\boldsymbol{k},\boldsymbol{v},\omega)$$
$$= -\frac{n_{e0}}{m_e}\left\{\widetilde{\phi_\pm}(\boldsymbol{k},\omega)\sum_{\ell=-\infty}^{+\infty}\int \mathrm{d}^3\boldsymbol{v}\frac{J_\ell^2(k_\perp r_{L\pm})\,\boldsymbol{k}\cdot\frac{\partial f_{0\pm}}{\partial \boldsymbol{v}^*}}{\omega-k_\parallel v_\parallel+\ell\omega_{c\mp}}\right\}$$
$$\pm\frac{n_{e0}H_\pm}{m_e\varepsilon_L}\left\{\widetilde{\phi_+}(\boldsymbol{k},\omega)\sum_{\ell=-\infty}^{+\infty}\int \mathrm{d}^3\boldsymbol{v}\frac{J_\ell^2(k_\perp r_{L+})\,\boldsymbol{k}\cdot\frac{\partial f_{0\pm}}{\partial \boldsymbol{v}^*}}{\omega-k_\parallel v_\parallel+\ell\omega_{c-}}\right.$$
$$\left.-\widetilde{\phi_-}(\boldsymbol{k},\omega)\sum_{\ell=-\infty}^{+\infty}\int \mathrm{d}^3\boldsymbol{v}\frac{J_\ell^2(k_\perp r_{L-})\,\boldsymbol{k}\cdot\frac{\partial f_{0\pm}}{\partial \boldsymbol{v}^*}}{\omega-k_\parallel v_\parallel+\ell\omega_{c+}}\right\},$$

longitudinal electric susceptibility
$$H_\pm \equiv \int \mathrm{d}^3\boldsymbol{v}\frac{4\pi e^2 n_{e0}}{m_e k^2}\sum_{\ell=-\infty}^{+\infty}\frac{J_\ell^2(k_\perp r_{L\pm})\,\boldsymbol{k}\cdot\frac{\partial f_{0\pm}}{\partial \boldsymbol{v}^*}}{\omega-k_\parallel v_\parallel+\ell\omega_{c\mp}},$$

longitudinal dielectric constant
$$\varepsilon_L(\boldsymbol{k},\omega) = 1+H_+(\boldsymbol{k},\omega)+H_-(\boldsymbol{k},\omega).$$

Solution of EOM (v<<c)

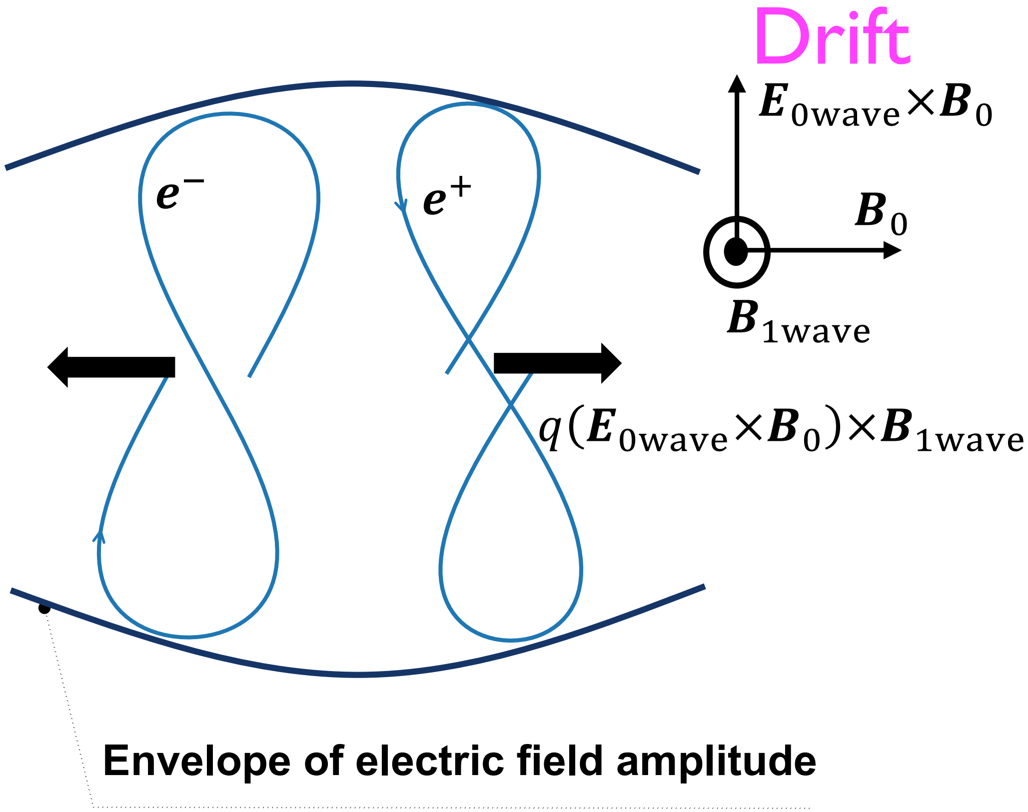
$$\boldsymbol{v}_{0\pm}^{(1)} = \mp\frac{e}{m_e c}\boldsymbol{A}_{0\parallel}\mp\frac{e}{m_e c}\frac{\omega_0^2}{\omega_0^2-\omega_c^2}\boldsymbol{A}_{0\perp} - \mathrm{i}\frac{e}{m_e c}\frac{\omega_0\omega_c}{\omega_0^2-\omega_c^2}\boldsymbol{A}_0\times\hat{\boldsymbol{B}}_0,$$

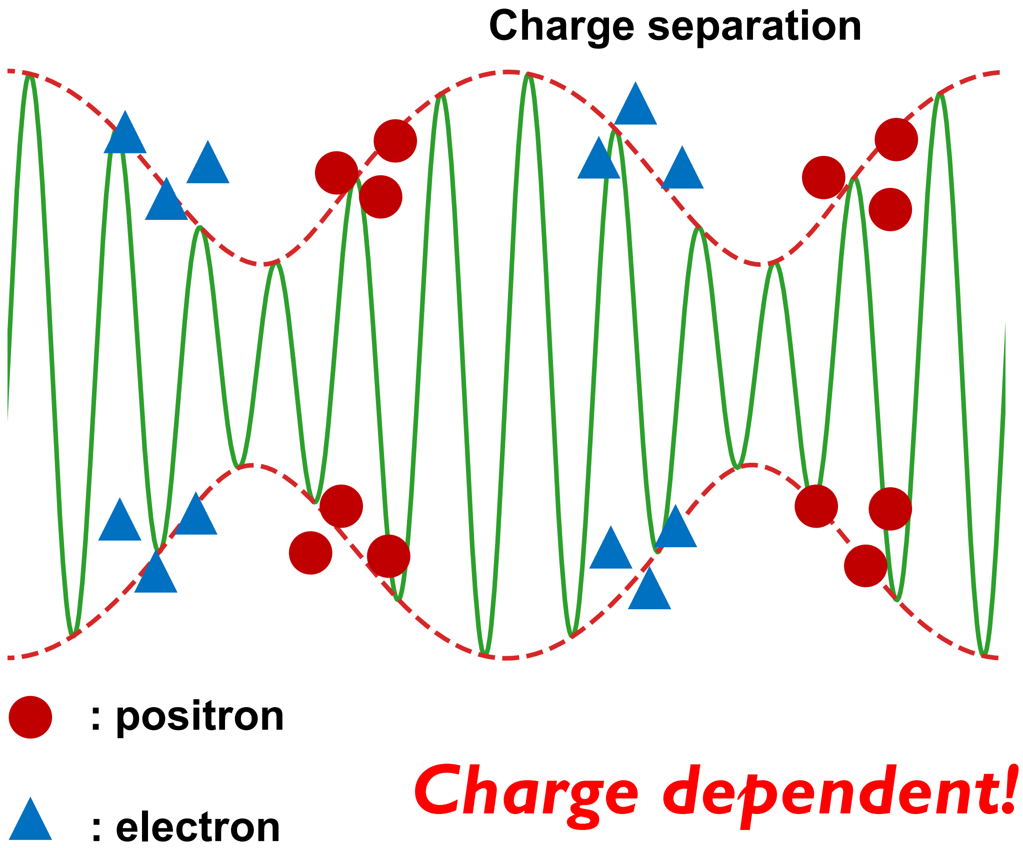
For thermal distributions

$$\sum_{\ell=-\infty}^{+\infty}\int \mathrm{d}^3\boldsymbol{v}\frac{J_\ell^2(k_\perp r_L)\,\boldsymbol{k}\cdot\frac{\partial f_0}{\partial \boldsymbol{v}^*}}{\omega-k_\parallel v_\parallel-\ell\omega_c} = \frac{m_e k^2}{4\pi e^2 n_{e0}}H_+$$
$$\overset{\ell=0}{\sim}\frac{2}{v_{th}^2}\left\{1+\frac{\omega}{k_\parallel v_{th}}I_0\left[\frac{1}{2}\left(\frac{k_\perp v_{th}}{\omega_c}\right)^2\right]\times e^{-\frac{1}{2}(k_\perp v_{th}/\omega_c)^2}Z\left(\frac{\omega}{k_\parallel v_{th}}\right)\right\}$$
$$\sim\frac{2}{v_{th}^2}\left\{1+\frac{\omega}{k_\parallel v_{th}}Z\left(\frac{\omega}{k_\parallel v_{th}}\right)\right\}$$

Ponderomotive Force in $B_0(\perp A_0)$

Charged mode





Charge dependent!

Nonlinear Current

Nonlinear current as functions of $\widetilde{\phi_\pm}$ and $\boldsymbol{v}_{0\pm}^{(1)}$

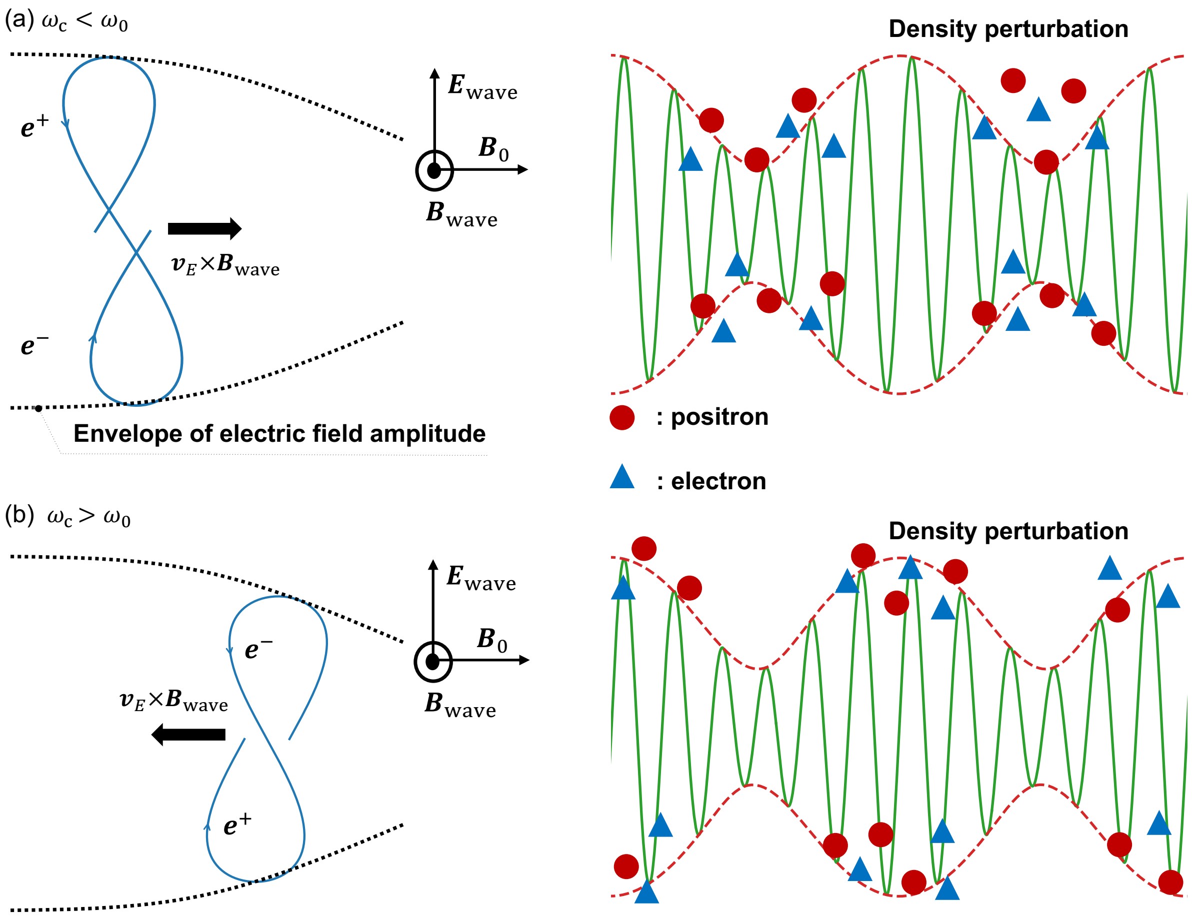
$$\tilde{\boldsymbol{j}}_1^{*\mathrm{nonlinear}}(\boldsymbol{k}_1,\omega_1) = e\widetilde{\delta n_+}\,\boldsymbol{v}_{0+}^{(1)*} - e\widetilde{\delta n_-}\boldsymbol{v}_{0-}^{(1)*}$$

$$= (\cdots)\widetilde{\phi_+}\,\boldsymbol{v}_{0+}^{(1)*} + (\cdots)\widetilde{\phi_-}\,\boldsymbol{v}_{0+}^{(1)*} + (\cdots)\widetilde{\phi_-}\,\boldsymbol{v}_{0-}^{(1)*} + (\cdots)\widetilde{\phi_+}\,\boldsymbol{v}_{0-}^{(1)*}$$

ponderomotive potential	$\phi_\pm =$	$\frac{e^2}{2m_\mathrm{e}}\left\langle\frac{E_\parallel^2}{\omega_0^2}\right\rangle$	$-\frac{e^2}{2m_\mathrm{e}}\left\langle\frac{E_\perp^2}{\omega_\mathrm{c}^2-\omega_0^2}\right\rangle$	$\pm\mathrm{i}\frac{e^2}{2m_\mathrm{e}}\left\langle\frac{\omega_\mathrm{c}\left(E_z^*E_y - E_y^*E_z\right)}{\omega_0(\omega_\mathrm{c}^2-\omega_0^2)}\right\rangle$
fast velocity oscillation	$\boldsymbol{v}_{0\pm}^{(1)} =$	$\mp\frac{e}{m_\mathrm{e}c}\boldsymbol{A}_{0\parallel}$	$\mp\frac{e}{m_\mathrm{e}c}\frac{\omega_0^2}{\omega_0^2-\omega_\mathrm{c}^2}\boldsymbol{A}_{0\perp}$	$-\mathrm{i}\frac{e}{m_\mathrm{e}c}\frac{\omega_0\omega_\mathrm{c}}{\omega_0^2-\omega_\mathrm{c}^2}\boldsymbol{A}_0\times\widehat{\boldsymbol{B}}_0$
Excited mode		**Ordinary mode**	**Neutral mode**	**Charged mode**
Polarization of incident wave		**Parallel** $\boldsymbol{A}_{0\perp}=\boldsymbol{0}$	**Perpendicular** $\boldsymbol{A}_{0\parallel}=\boldsymbol{0}$	**Perpendicular** $\boldsymbol{A}_{0\parallel}=\boldsymbol{0}$

Ponderomotive Force in $B_0(\perp A_0)$

Neutral mode



Ponderomotive potential

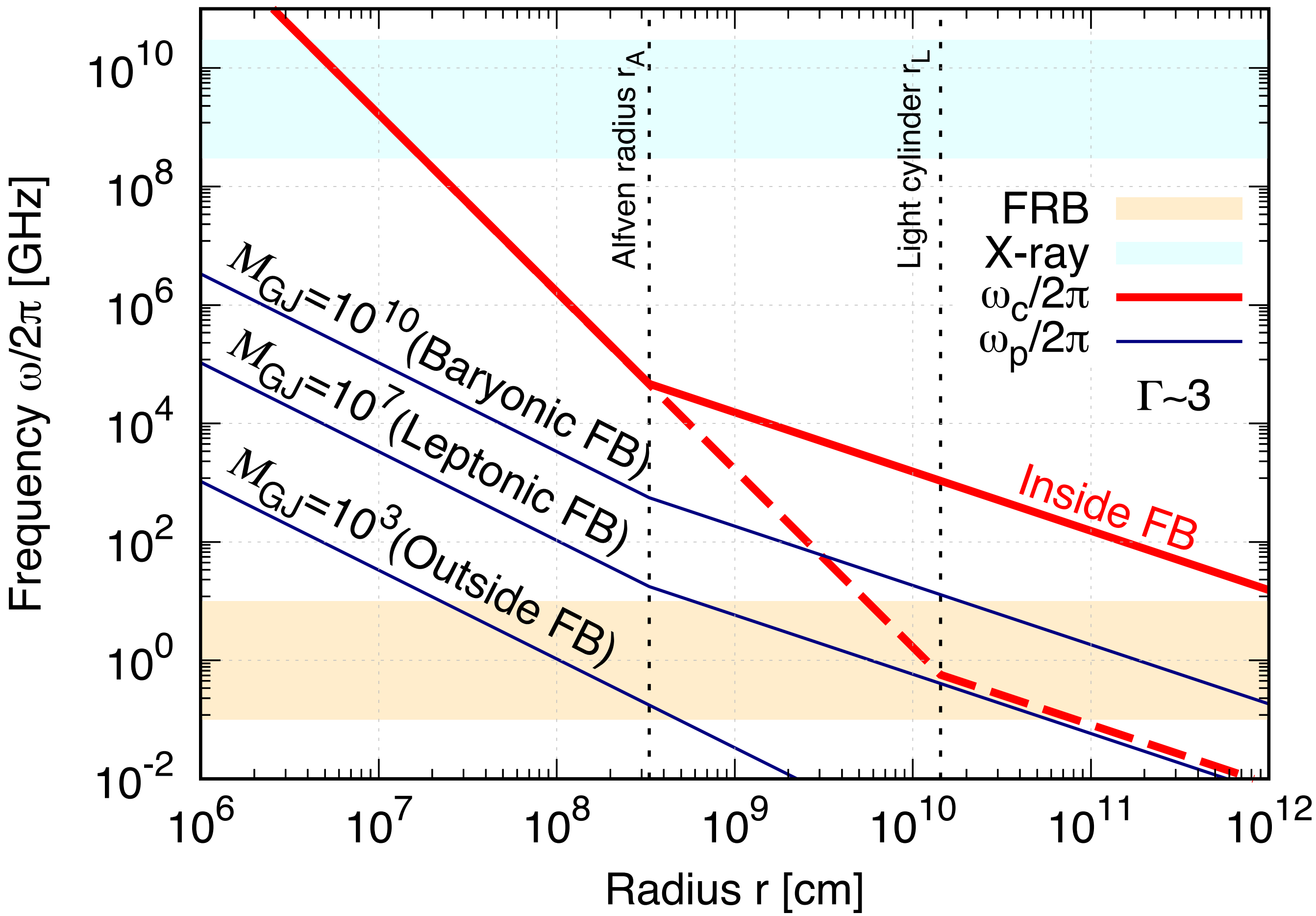
$$\phi_\pm = \frac{e^2}{2m_e}\left\langle \frac{E_\parallel^2}{\omega_0^2} - \frac{E_\perp^2}{\omega_c^2-\omega_0^2} + \mathrm{i}\frac{\omega_{c\pm}\left(E_z^* E_y - E_y^* E_z\right)}{\omega_0\left(\omega_c^2-\omega_0^2\right)}\right\rangle$$

Neutral mode $\sim O(\omega_0/\omega_c)^2$

Charged mode $\sim O(\omega_0/\omega_c)$

Charge independent

Cyclotron & Plasma Frequency



$$\omega_c = \Gamma \frac{qB}{mc}$$

$$\omega_p = \Gamma \left(\frac{4\pi q^2 n}{m} \right)^{1/2}$$

$\omega_c > \omega_{FRB}$

$\omega_p \sim \omega_{FRB}$