Simulation-Based Inference for gravitational-wave data

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Introduction

One major goal in gravitational-wave(GW) astronomy is to get the source's information as soon as the GW is detected.

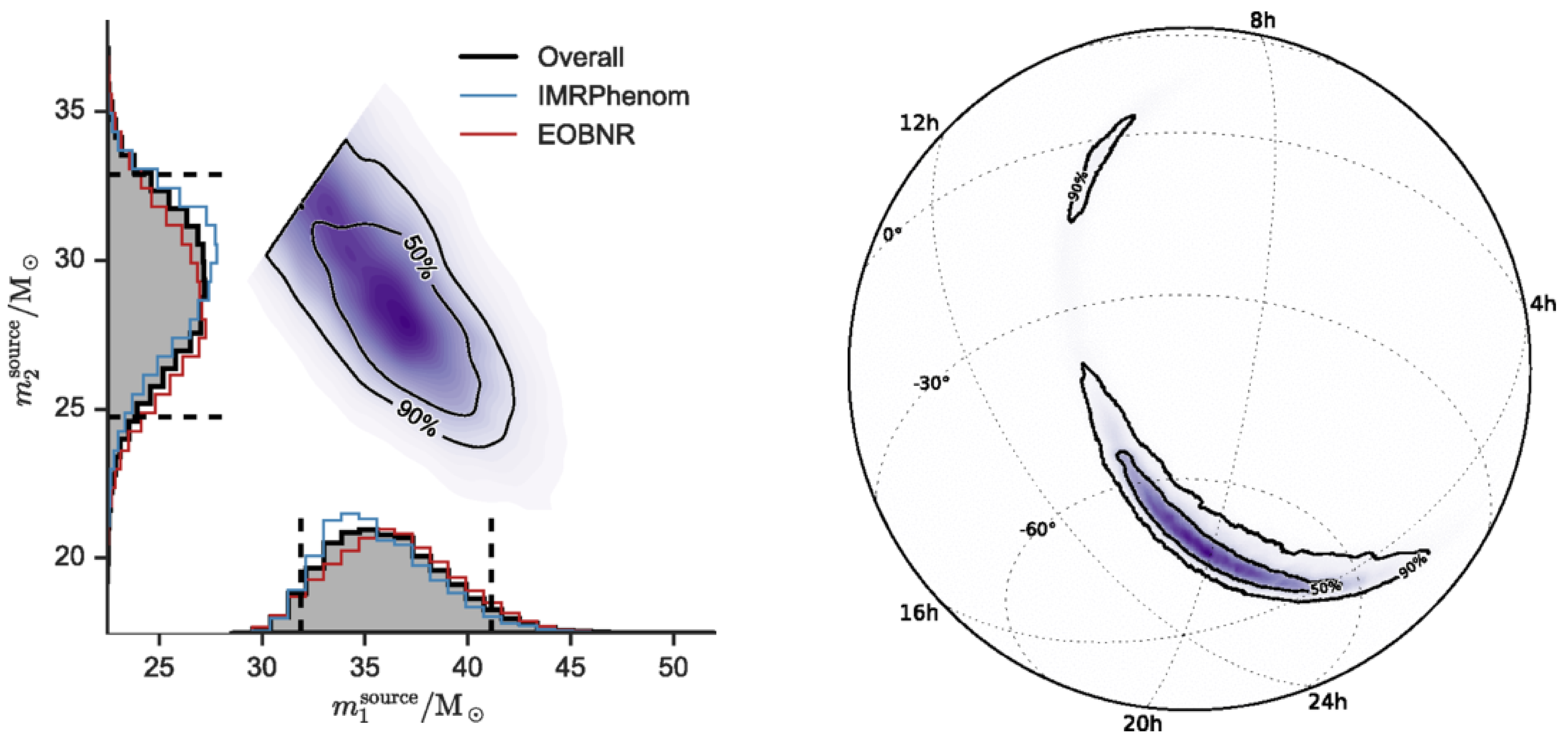

Figure 1: *parameter estimation of GW150914 (Abbott et al. 2016)*

This figure is a example of parameter estimation of the first detected event GW150914, left is about masses and the right is sky localization.

These parameter probability distributions are gained by "Bayesian inference". In conventional Bayesian inference, when we want to estimate the true parameters behind a model, we need to calculate the likelihood $p(x|\theta)$ of data x given parameters θ. Then we use the Bayes' theorem(1) to get the posterior $P(\theta|x)$.

$$P(\theta|x) = \frac{P(x|\theta)P(\theta)}{Z} \tag{1}$$

where $P(\theta)$ is a prior.

However, in many cases it is extremely computationaly expensive to get the posterior because likelihood involves so many parameters, high-dimensional data and intractable integrals. For example, running time of computer for black hole systems takes a few days and for neutron star systems weeks.

In order to avoid this drawback, some new approaches to execute rapid estimation combined with deep learning were developed [1]. Recently, another inference approach using deep learning called "Simulation-Based Inference"(SBI) was proposed [2]. This method doesn't use the analytically expressed likelihood. SBI just uses parameters and data set to train the neural net weight parameters. SBI is expected to reduce high computational cost and produce the exact posterior given new data much faster than usual methods.

Mechanism of SBI

SBI uses another pobability function $q_\phi(\theta|x)$ which is called "posterior estimator". ϕ represents neural net weights.

We prepare a proposal prior $\tilde{p}(\theta)$ and take N samples $\theta_1, \theta_2, \dots, \theta_N$. And then we generate simulated data like $x_1, x_2, \dots, x_N$ for each parameters.

Then it can be used to train the estimator"$q_{F(x,\phi)}(\theta|x)$".Training is executed to maximaize the below quantity over ϕ.(2)

$$\Pi_{n=1}^{N} q_{F(\phi,x)}(\theta_n|x_n) \tag{2}$$

If the expressiveness of estimator is enough, the maximized estimator is given by

$$q_F(\phi,x) = p(\theta|x)\frac{\tilde{p(\theta)}}{p(\theta)}\frac{1}{Z(x,\phi)} \tag{3}$$

where $Z(x,\phi)$ is normalizing constant. From this equation, we can estimate the true posterior $p(\theta|x)$ by

$$\hat{p}(\theta|x) = q_F(\phi,x)\frac{p(\theta)}{\tilde{p}(\theta)}Z(x,\phi) \tag{4}$$

The left side estimator will converges to the true posterior as $N \to \infty$. This algorithm is summarized as follows.

SBI Algorithm

Input: simulator with implicit $p(x|\theta)$, prior $p(\theta)$, density family q_ψ, neural network $F(\phi, x)$, number of simulations N
$\tilde{p}(\theta) \leftarrow p(\theta)$;
for $r = 1$ **to** R **do**
  for $j = 1$ **to** N **do**
    sample $\theta_{r,j} \sim \tilde{p}_r(\theta)$;
    sample $x_{r,j} \sim p(x|\theta_{r,j})$;
  endfor
  $\phi \leftarrow$ argmin$_\phi \sum_{i=1}^{r}\sum_{j=1}^{N} -\log q_{F(x_{r,j},\phi)}(\theta_{r,j})$;
  $\tilde{p}_{r+1}(\theta) \leftarrow q_{F(x_0,\phi)}$;
endfor
Output: Approximated posterior $q_{F(\phi,x_0)}(\theta) \to$ true posterior $p(\theta|x_0)$

Parameter estimation of GW by SBI

Gravitational waves from binary compact star mergers contains several information about the source. Here is the parameters which we want to know.

I estimated the mass ratio, chirp mass, and 7 geometrical parameters of GW using SBI.

Discription	Parameter	Prior
mass ratio	$q = m_1/m_2$	$[0.2, 1.0]$
chirp mass	$M_c = (m_1 m_2)^{\frac{3}{5}}/(m_1+m_2)^{\frac{1}{5}}$	$[80, 120]M_\odot$
luminosity distance	d_L	$[500, 5000]Mpc$
sky position	α, δ	$[0, \pi]$
inclination	θ_{JN}	$[0, \pi]$
polarization	ψ	$[0, \pi]$
phase	ϕ_c	$[0, \pi]$
geocent time	t_c	$\Delta t = 0.3[s]$
spin parameters	$a_1, a_2, \theta_1, \theta_2, \phi_{12}, \phi_{JL}$	$all\ 0$ (No spins)

Table 1: *parameter and prior*

Result

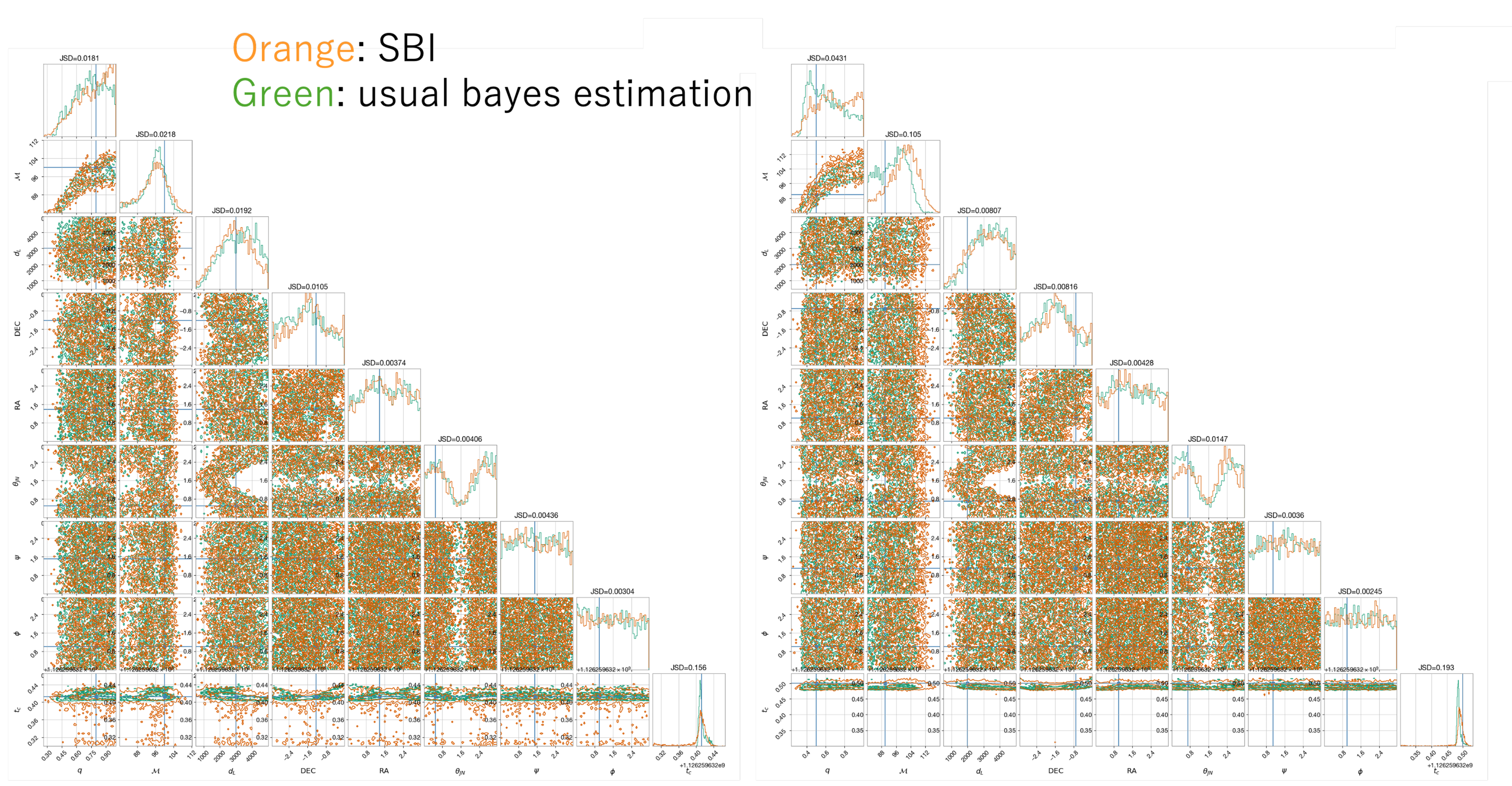

Figure 2: *Two examples of results with different injections*

Fig2 displays the result of inference of parameters in Table1 with different injections. Orange line shows posterior generated by SBI and green line shows the result of conventional Bayes inference by python module "bilby". True parameters are shown as a blue line.

We can see that SBI and bilby posterior are almost same shape but a little different such as mass ratio in the rifht figure.

Future research and expectation

This research will have two improvement in the future.

1. use multiple detectors' data
2. change the way of parameterization

The above results are gained by one detector's data, so if we expand the SBI code to use several detectors' data, we can expect more accurate correspondence between these methods.Furthermore, more efficient parametrization methods are proposed [3][4]. Implementation of these methods in SBI will enable us to get accurate posterior quickly.

References

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