Statistical properties of parameter estimation in scalar-tensor polarization search

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Introduction

Gravitational waves (GWs) are described by a superposition of independent modes called polarization, which represent the degrees of freedom of the gravitational field. In General Relativity (GR), only two tensor modes exist, however more generally, modified gravity theories predict the existence of non-tensor modes [1]. Therefore, the exploration of polarization modes allows us to test general relativity with focus on the degrees of freedom of the gravitational field.

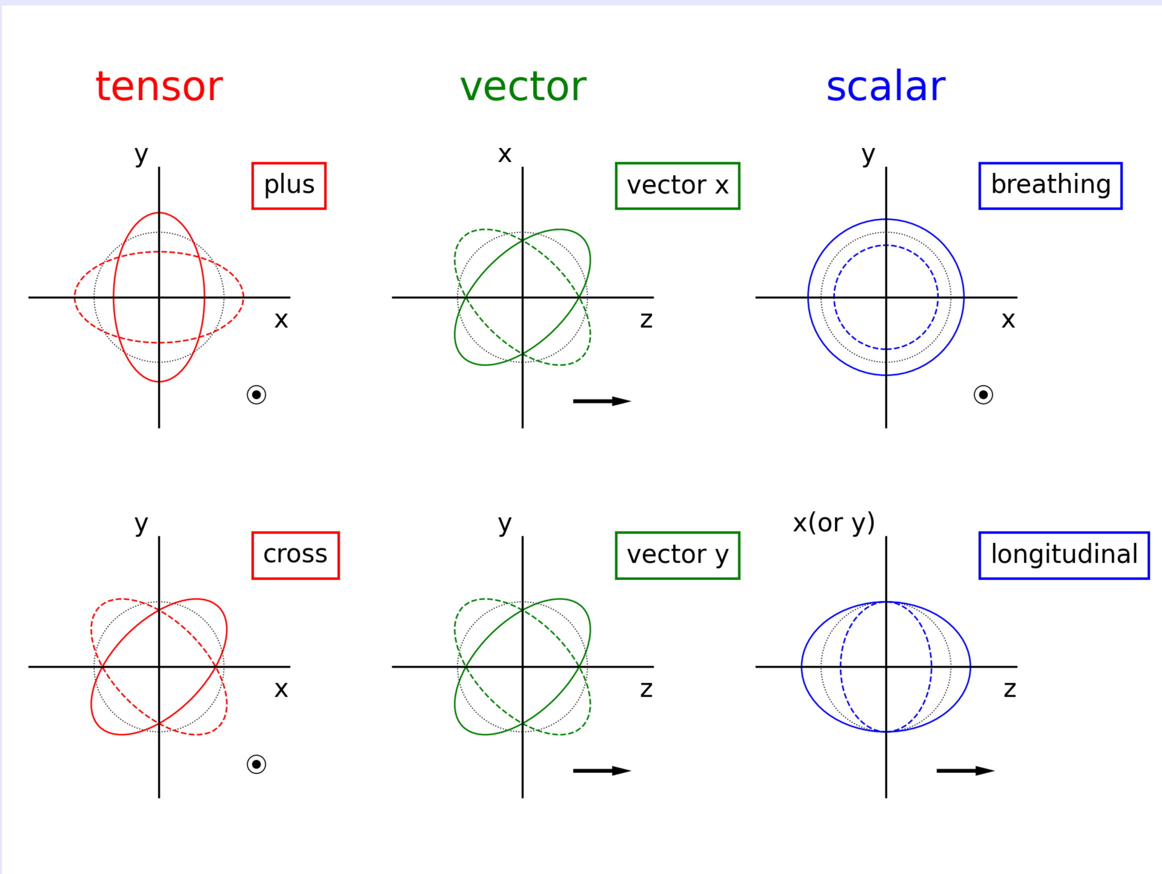

Figure 1. Polarization modes of the GW traveling the z-axis direction. From left to right: tensor modes (plus and cross), vector modes (vector x and vector y), scalar modes (breathing and longitudinal). The black circles are the position of test masses before the GW arrives, and the GW causes them to oscillate between the solid and dashed lines.

Motivation

- Final aim
  - Analyze multiple GW events in the tensor+scalar framework and give a single constrain on the scalar mode.
- ∗ By analyzing multiple events and combining their results, a reduction in statistical error can be expected.
- ∗ **It is needed to investigate the properties of parameter estimation.**

Waveform model

Introduce the scalar breathing mode in addition to the tensor mode in the theoretical waveform.

- Modification of the binary dynamics
  - The orbital evolution is faster due to scalar mode radiation.
  - Extend the previous research [2], introduce γ based on [3].
  - The stress energy tensor is modified from GR by the scalar mode and γ.
  - While the tensor mode is radiated through quadrupole, the scalar mode is radiated through quadrupole and dipole.

$$\dot{E}_{\rm GW} = -\frac{d_L^2}{16\pi}\int \mathrm{d}\Omega \left\{ \langle \dot{h}_+^2(t) + \dot{h}_\times^2(t)\rangle + \gamma\langle \dot{h}_{b1}^2(t) + \dot{h}_{b2}^2(t)\rangle \right\}. \quad (1)$$

$h_{b1}(t)$: The dipole of scalar mode.
$h_{b2}(t)$: The quadrupole of scalar mode.
γ : The coupling parameter between the scalar mode and matters.

- Modification of the waveform

The frequency domain waveform can be obtained as follows

$$\tilde{h}(f) = \tilde{h}_T^{(2)}(f) + \tilde{h}_b^{(2)}(f) + \tilde{h}_b^{(1)}(f), \quad (2)$$

$$\tilde{h}_T^{(2)}(f) = -\left(F_+(1+\cos^2\iota)\tilde{h}_+^{(2)} + F_\times 2i\cos\iota\tilde{h}_\times^{(2)}\right)\left(1-\delta A^{(2)}\right)e^{-i\delta\Psi^{(2)}}, \quad (3)$$

$$\tilde{h}_b^{(2)}(f) = F_b A_{b2}\, 2\sin^2\iota\, \tilde{h}_+^{(2)} e^{-i\delta\Psi^{(2)}}, \quad (4)$$

$$\tilde{h}_b^{(1)}(f) = F_b A_{b1}\, 2\sin\iota\, \tilde{h}_+^{(1)} e^{-i\delta\Psi^{(1)}}. \quad (5)$$

- Introduce parameters A_{b1},A_{b2}, characterizing the amplitude of the scalar dipole and quadrupole, respectively.
- Consider the modification of the phase evolution due to the scalar radiation not only for the tensor mode but also for the scalar mode.
- The inclination angle dependence is $\sin\iota$ for dipole of scalar mode and $\sin^2\iota$ for quadrupole of scalar mode.
- $\delta A^{(2)}, \delta\Psi^{(1)}, \delta\Psi^{(2)}$ are correction terms for the amplitude and phase, respectively, and are functions of $\sqrt{\gamma}A_{b1} := \tilde{A}_{b1}$, $\sqrt{\gamma}A_{b2} := \tilde{A}_{b2}$.

Degeneracy of luminosity distance and inclination

For the accurate inference of the scalar mode amplitude, we need to accurately estimate the inclination angle, since each mode has a different inclination angle dependence. However, it is known that the luminosity distance d_L and the inclination angle ι are strongly correlated in the GW data analysis, since the GWs amplitude can be written roughly as follows,

$$h \propto \cos\iota/d_L. \quad (6)$$

Furthermore, we use the uniform-in-volume prior, i.e. the prior probability of the luminosity distance is expressed as

$$p(d_L) \propto d_L^2. \quad (7)$$

This means that larger distance and smaller inclination angle are more preferred.

Pure polarization

As a first step, we check inclination angle recovery for pure tensor mode case and pure scalar mode case.

- Analyze 50 mock signals which have same parameters except radec.
- Plot the median of the $\cos\iota$ posterior as a histogram from each analysis.

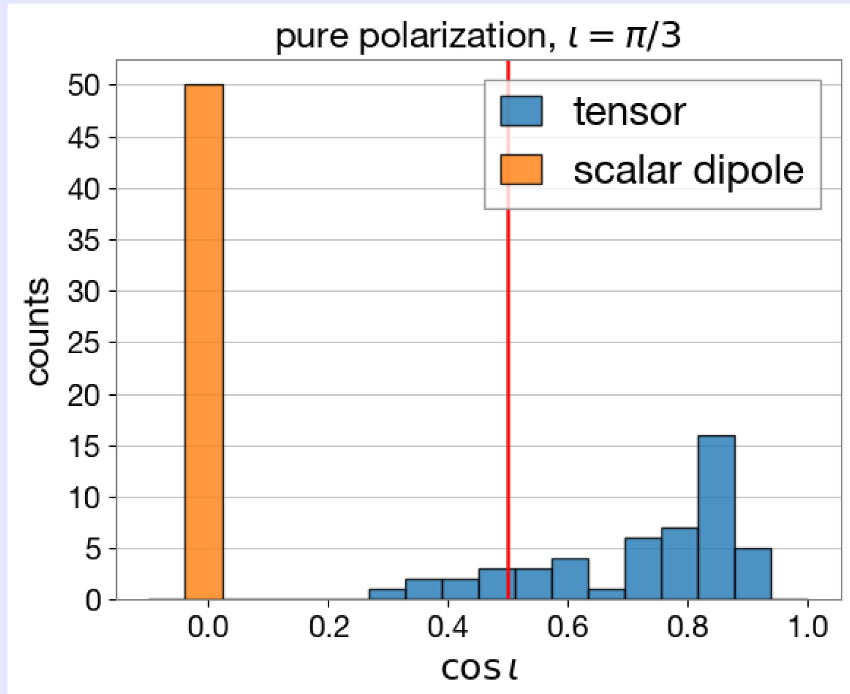

Figure 2. The histogram of the median of the $\cos\iota$ posterior. The red line shows the inject value.

- In the case of pure tensor mode, $\iota \sim 0$ is favored, since $h \propto \cos\iota/d_L$.
- In the case of pure scalar dipole, $\iota \sim \pi/2$, since $h \propto \sin\iota/d_L$.

Mixed polarization

- Inject 3 times each as $A_{b1} = 0, 0.1, 0.3, 0.5, 1$.
- Plot the median of the $\cos\iota$ and A_{b1} posterior from each analysis.

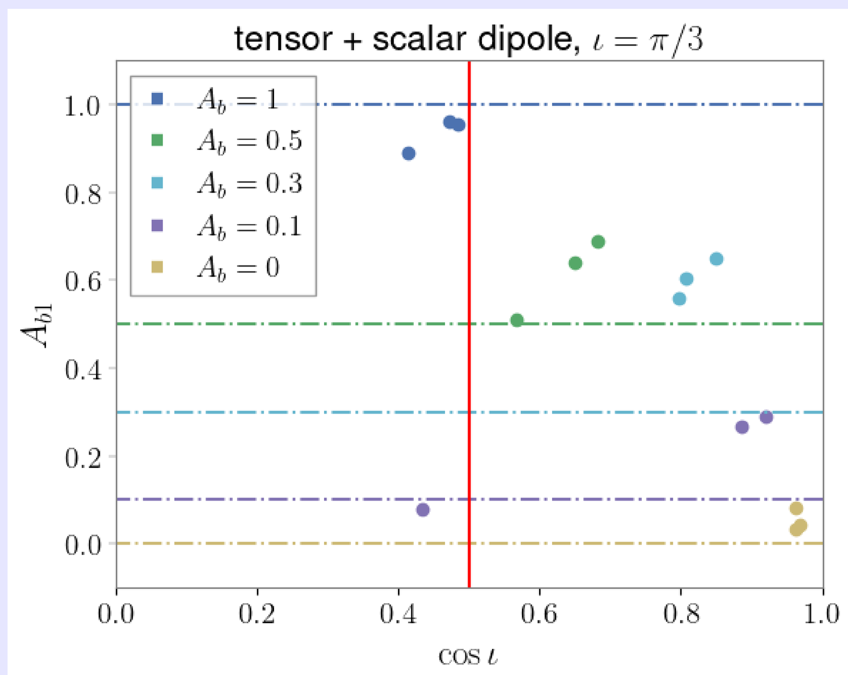

Figure 3. The median of the $\cos\iota$ and A_{b1} posterior. The vertical red line and each horizontal colored line represent the inject value of $\cos\iota$ and A_{b1}, respectively.

- As A_{b1} is larger, the correct $\cos\iota$ is estimated.
- A_{b1} is overestimated when it is small.

Summary

- $\iota \sim 0$ is more favored in the pure tensor case, $\iota \sim \pi/2$ is more favored in the pure scalar case.
- The competition of these tendencies determines the tensor+scalar case.

References

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