Comparison of steady-state flow calculations with M1 and FLD

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What is PNS and PNSC simulation

Proto-Neutron Star (**PNS**) is an object produced by a **core collapse supernova** and it is **cooled** by the **emission of neutrinos** (PNS cooling, **PNSC**), which evolves into the Neutron Star (NS).

This PNSC simulation requires solving the **Einstein equation** (spacetime), the **hydrodynamic equation** (other than neutrinos) and the **Boltzmann equation** (neutrinos).

Due to the **multidimensionality** of the Boltzmann equation, this equation cannot be computed for long periods of time without **the use of approximations**.

Numerical cost is too high!

1.Objective and Background

- ✓ A code describing the **cooling process of proto-neutron stars** (PNSC) was created by H.Suzuki (1993) using multi-energy flux limited diffusion (FLD) scheme to solve Boltzmann equation approximately.
- ✓ To prepare for **future neutrino observations**, the current code needs to be improved and updated to make it **more accurate** and capable of **long-time** calculations.
- ✓ Therefore, taking these points into account, I created a code to solve the Boltzmann equation by **M1** and compared it to FLD.

2.Formalism

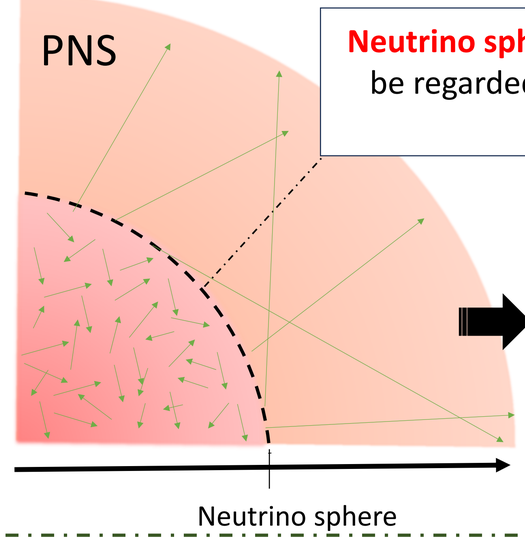
Difference between FLD and M1

- ✓ **Flux limited diffusion (original code)**

  Assume diffusion limit of neutrinos (good assumption inside neutrino sphere) and set a limit of the flux in transparent region.

- ✓ **M1-closure (Created Code)**

  The two states of the neutrino, **thermal equilibrium** and **free propagation**, can be solved exactly. The states in between are complementary (closure).



Neutrino sphere: Inside the neutrino sphere, neutrinos can be regarded as **in thermal equilibrium**, outside it can be regarded as **freely propagating.**

- ➢ FLD
  - can be solved
  - extrapolate is needed
- ➢ M1
  - interpolate is needed (**closure**)
  - can be solved
  - can be solved

Equation for moments (Steady-flow for simplicity)

$$\partial_t \vec{U} + \partial_m\left(4\pi r^2 \rho \vec{\mathcal{F}}\right) = \vec{S}$$

f_ν: distribution function of neutrino

Moment

$$n_\nu = \frac{\omega^2}{(hc)^3}\int f_\nu d\Omega$$

$$F_\nu = \frac{\omega^2}{(hc)^3}\int f_\nu \mu d\Omega$$

$$P_\nu = \frac{\omega^2}{(hc)^3}\int f_\nu \mu^2 d\Omega$$ ——— Closure : Levermore, ME, etc.

Variables

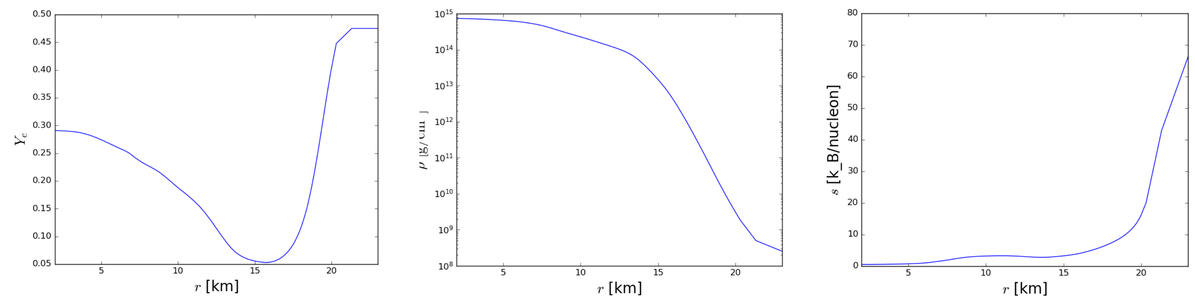
$$\vec{U} = \frac{1}{\rho}(n_\nu, F_\nu) \qquad \vec{\mathcal{F}} = \frac{1}{\rho}(F_\nu, P_\nu)$$

$$\vec{S} = \frac{1}{\rho}\left(S^0, S^1 - 4\pi r c \rho e^{\phi}(n_\nu - P_\nu)\partial_m\left(r e^{-\phi}\right)\right)$$

3.Numerical situation

- ✓ To do the numerical simulations of steady flow, I imposed the following situation.
- ✓ These profile obtained from the result of the PNSC calculation (original code).

■ after 600 msec of shock revival



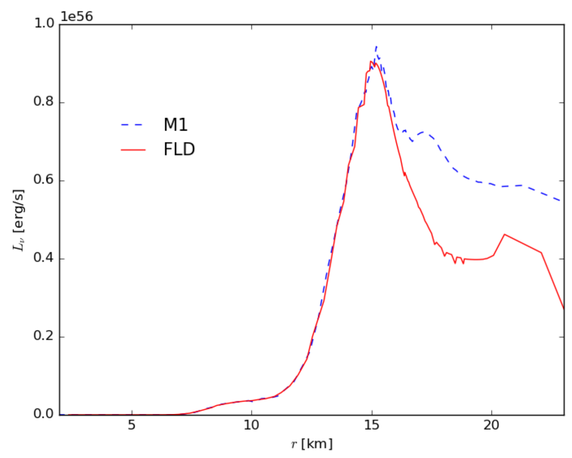
4.Calculation results

- ✓ Compare M1 and FLD in steady flow with a fixed distribution of fluid after 600 msec of proto-neutron star cooling.
- ✓ Compare the results of different closures.

Details

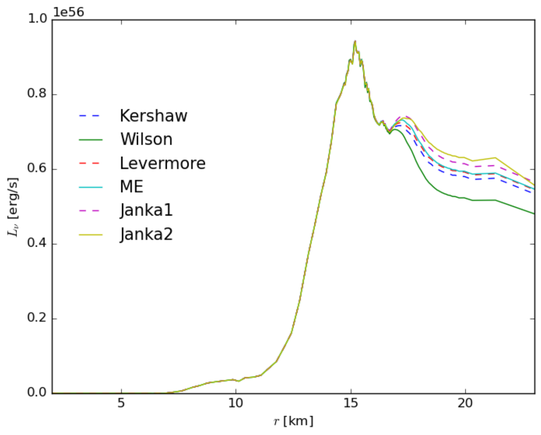
I. Compare M1 and FLD

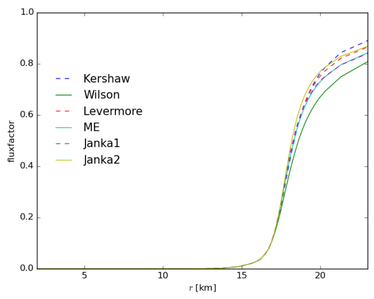
Luminosity ($\frac{dL_\nu}{dE_\nu}\Delta E_\nu$, observed as a neutrino of 20 MeV at infinity)



- Near the center, the neutrino is in **thermal equilibrium**, so the results for FLD and M1 are the same.
- The difference between FLD and M1 appears **near the neutrino sphere** (~17 km).
- The higher the energy, the larger the neutrino sphere and the smaller the difference between FLD and M1.

II. Compare with different closures





- Each closure behaves differently **near the surface**.
- It is **impossible** to know which closure is better without comparing it to the more accurate solution.

5. Conclusion/Future Plans

- Neutrino luminosity were found to be larger for M1 than for FLD.
- However, the result for M1 is also an **approximation**, and the actual value may be somewhere between M1 and FLD.
- Therefore, we would like to discuss the behavior of M1 by **comparing** the more accurate or direct integration solution of the Boltzmann equation (of course, due to numerical cost, long time calculation is impossible) with M1 in the future.