

LISA can detect dark matter subhalos through **gravitational-wave lensing**

Shin'ichiro Ando

GRAPPA · University of Amsterdam

Paper I — Ando, arXiv:2603.04267

Paper II — Ando, arXiv:2606.21519

Small-scale structure: a dark matter discriminator

Dark matter's identity is written in the smallest gravitationally bound clumps.

CDM cuspy profiles, **very abundant** subhalos

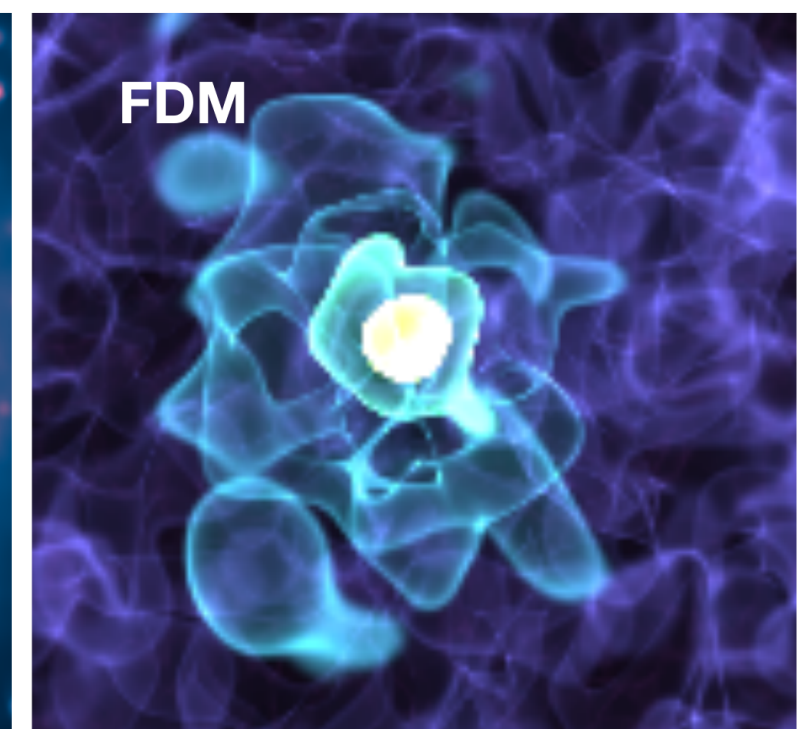
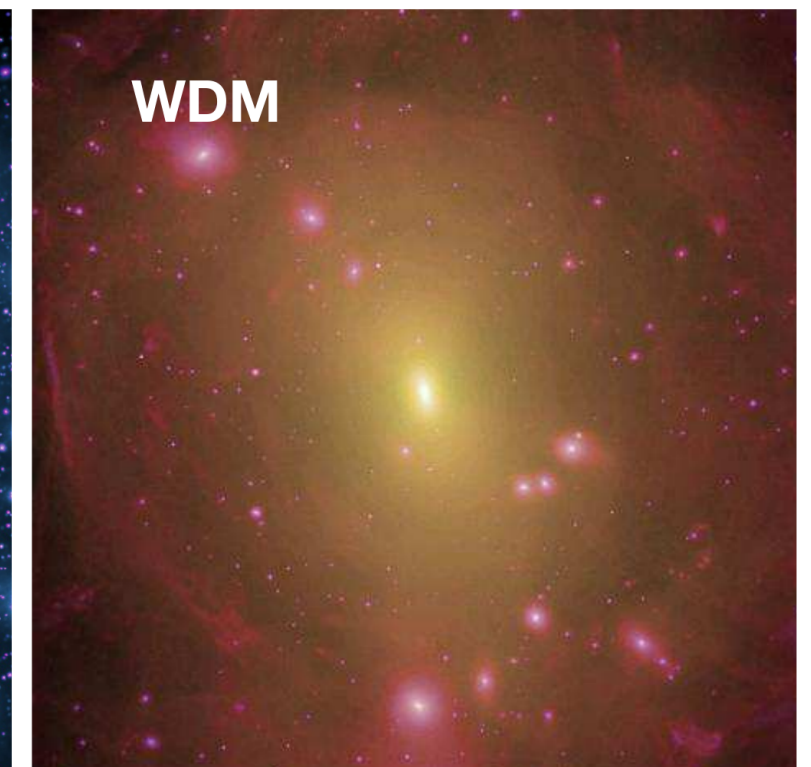
WDM **cutoff** below sub-galaxy scales

SIDM **cored** / **core-collapsed** halos

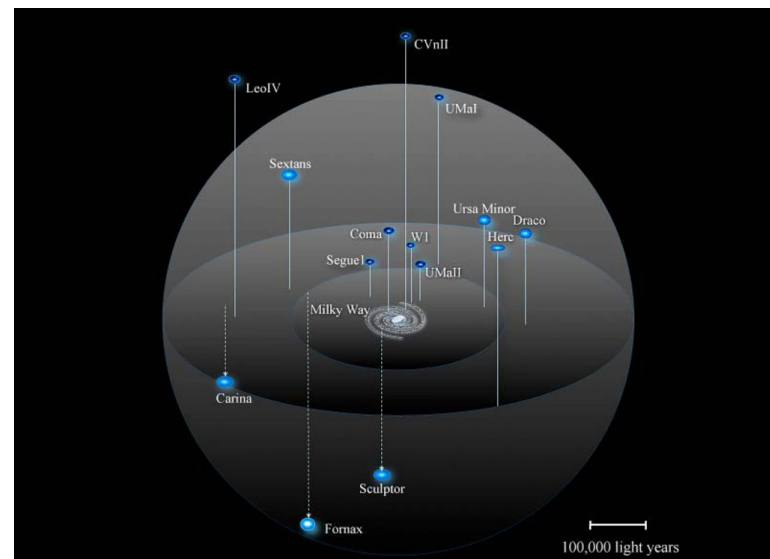
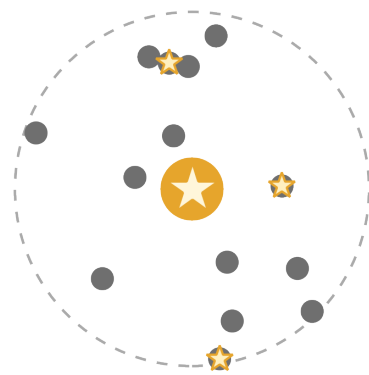
FDM de Broglie **interference** pattern

Models diverge at sub-galactic scales —

regime of **starless** subhalos ($\lesssim 10^8 M_\odot$)

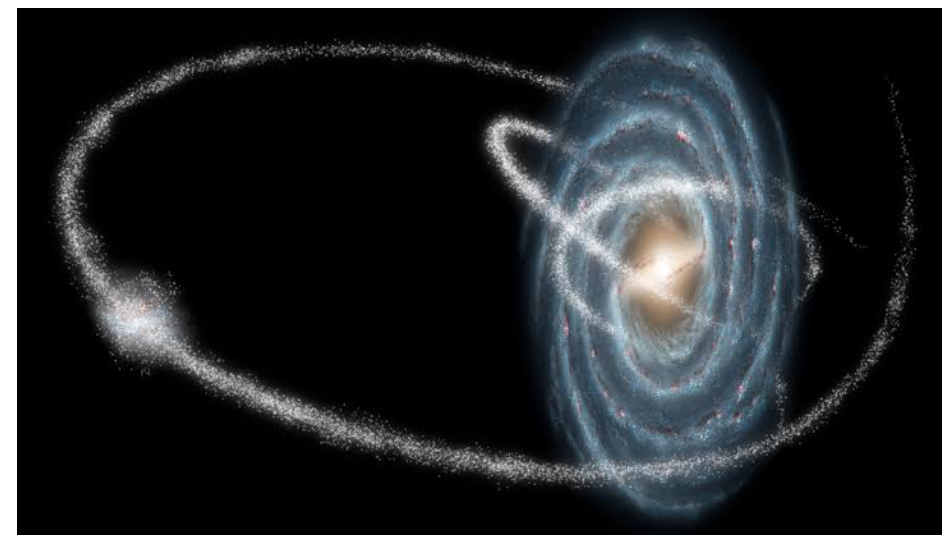
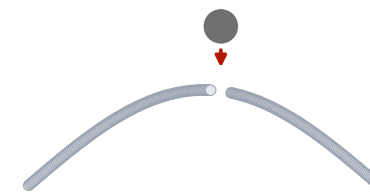


Traditional probes — and their blind spots



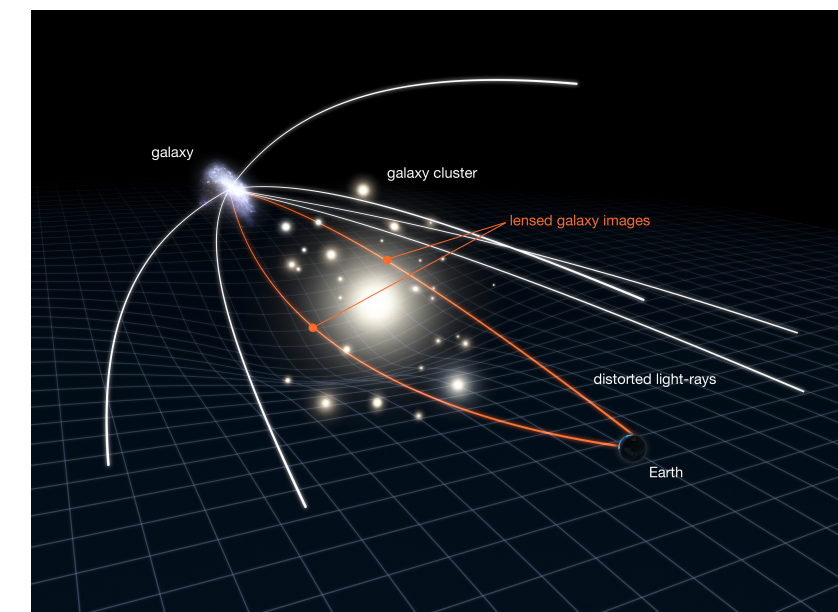
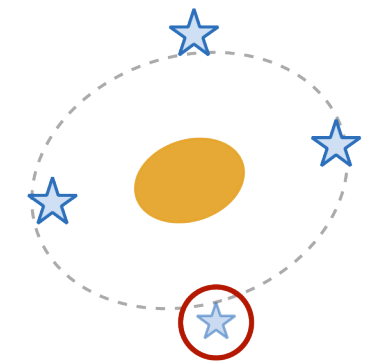
Satellite counts & dwarfs

✗ only star-hosting subhalos



Stellar streams

✗ a few Milky-Way streams



Lensed-quasar flux ratios

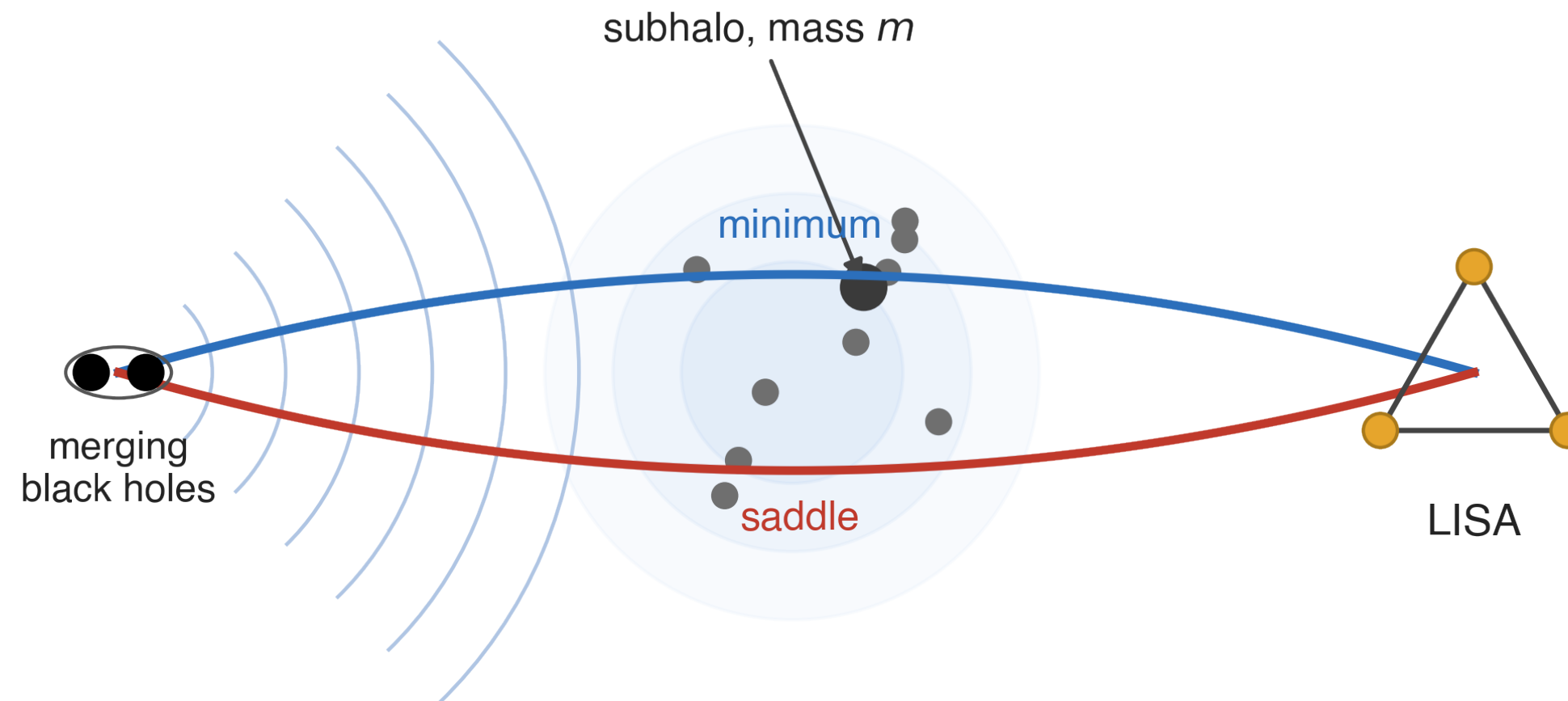
✗ heavy or compact subhalos; baryons

We want the gravity of **individual light dark clumps** — independent of light.

Gravitational lensing of gravitational waves: a light-independent probe

$f \gg 1/(2\pi\Delta t) \rightarrow$ geometric optics
just a magnification (frequency independent)

$f \lesssim 1/(2\pi\Delta t) \rightarrow$ wave optics
a **frequency-dependent imprint**



Lens time delay $\Delta t \sim 2Gm/c^3 \approx 10^{-5} \text{ s } (m/M_\odot)$; LISA band $0.1 - 100 \text{ mHz} \leftrightarrow 10^4 - 10^7 M_\odot$

Lensing of gravitational waves: a quick overview

01

Wave optics is established physics

Frequency-dependent lensing once the lens delay meets the GW period.

Takahashi & Nakamura (2003)

02

Generic lines of sight — too weak

$\lesssim 10^{-6} \text{ yr}^{-1}$ in CDM (LISA). Order-unity rates need compact profiles or SMBHs.

Fairbairn+ (2023); Guo & Lu (2022);
Brando+ (2025); Oguri & Takahashi (2020)

03

Strong lensing — the opening

Magnified images, clumps, caustics — but **no realistic CDM subhalo population.**

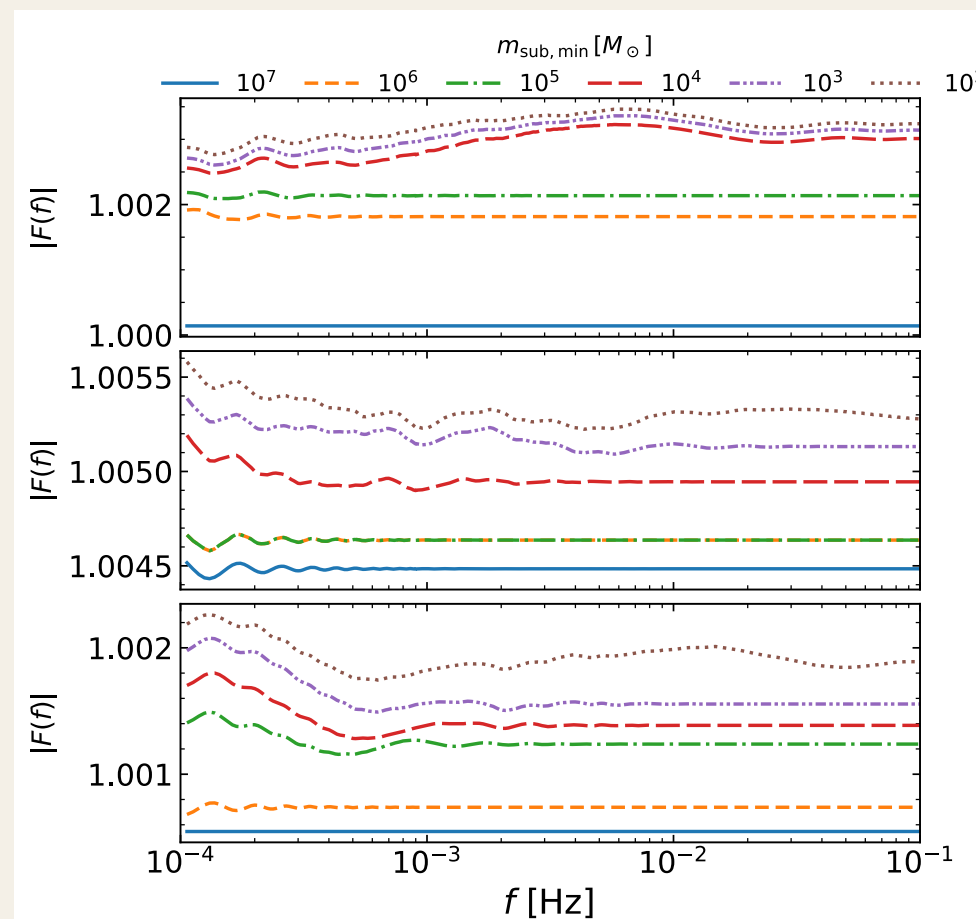
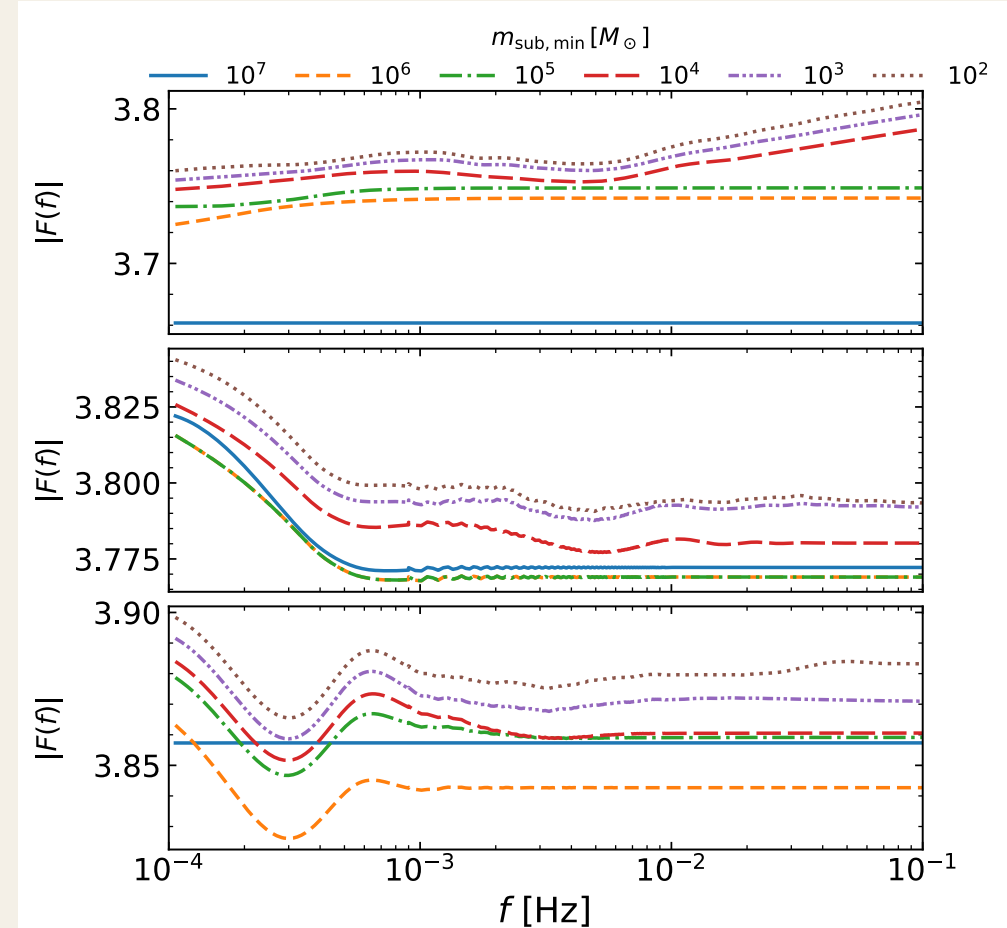
Oguri & Takahashi (2022); Dai+ (2018);
Vujeva+ (2025)

Missing: Lensed GW amplitude for **CDM subhalos** at a **near-critical image**, in the LISA band.

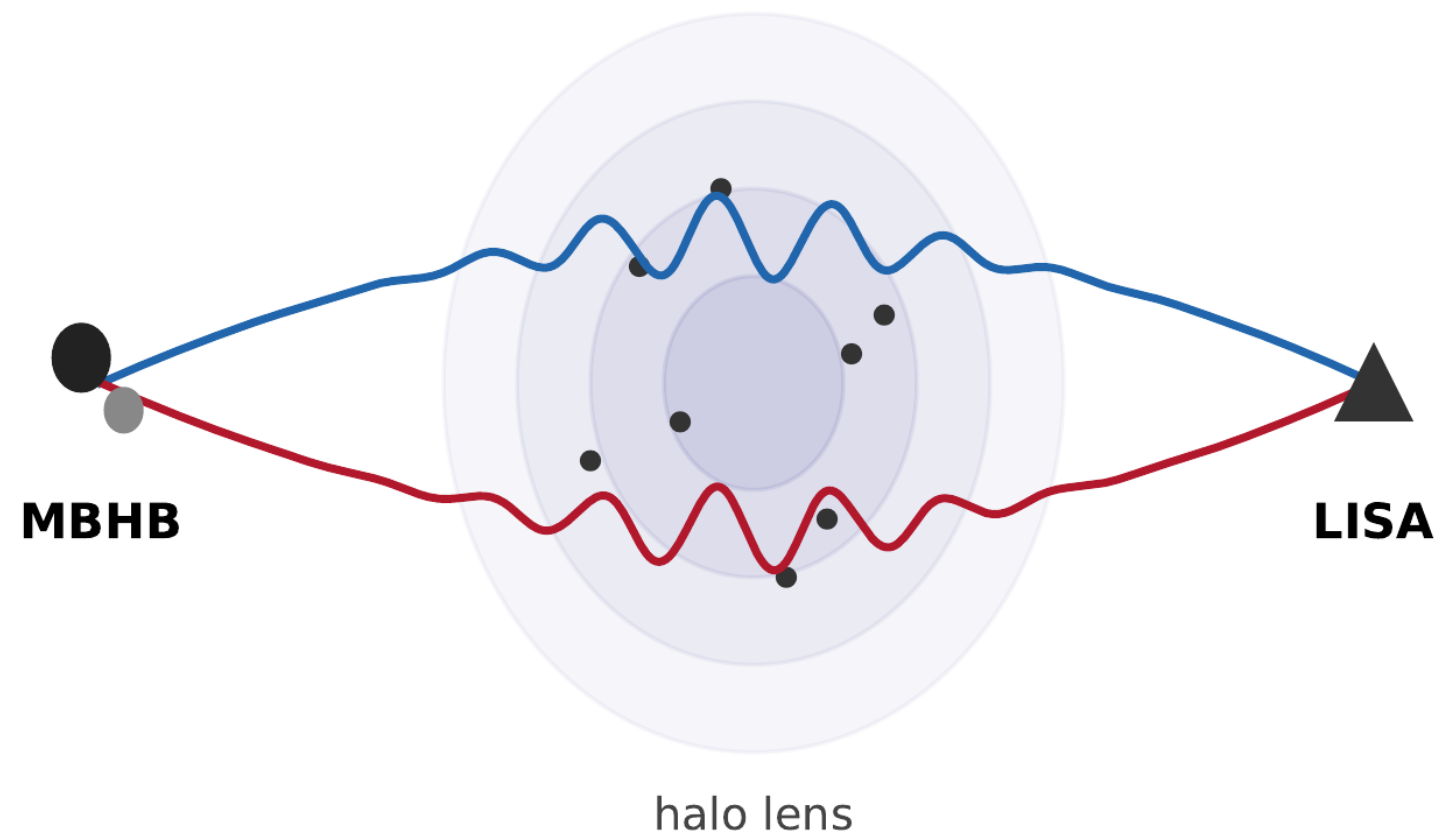
Tool: **GLoW** — diffraction integrals (Villarrubia-Rojo+ 2025; github.com/miguelzuma/GLoW_public)

Why strong lensing: from “too rare” to detectable

- Generic lines of sight (no strong lens): effect $\ll 1\%$
— the raw rate looks small.
- Strong lensing amplifies it to **percent level**:
dense halo core along the line of sight · images near the critical curve · huge magnification $\rightarrow$ high S/N
- MBHBs are loud: **S/N $\sim 100\text{--}1000$** .

Macrolens: **Off**Macrolens: **On**

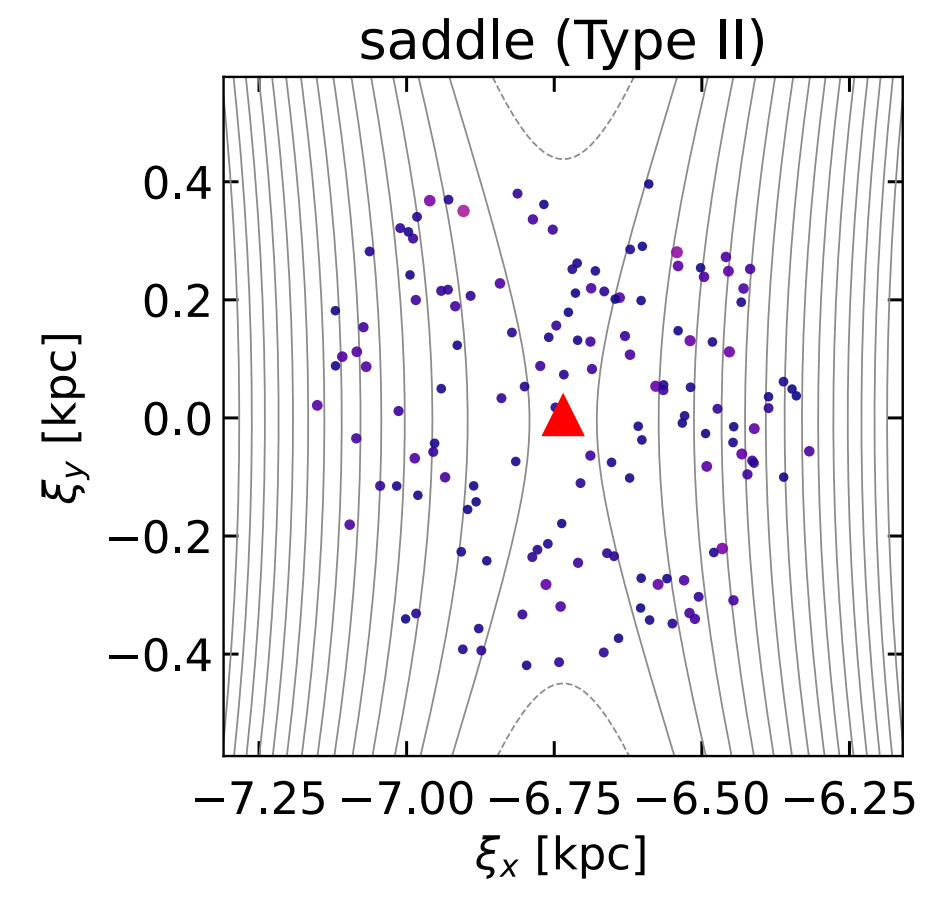
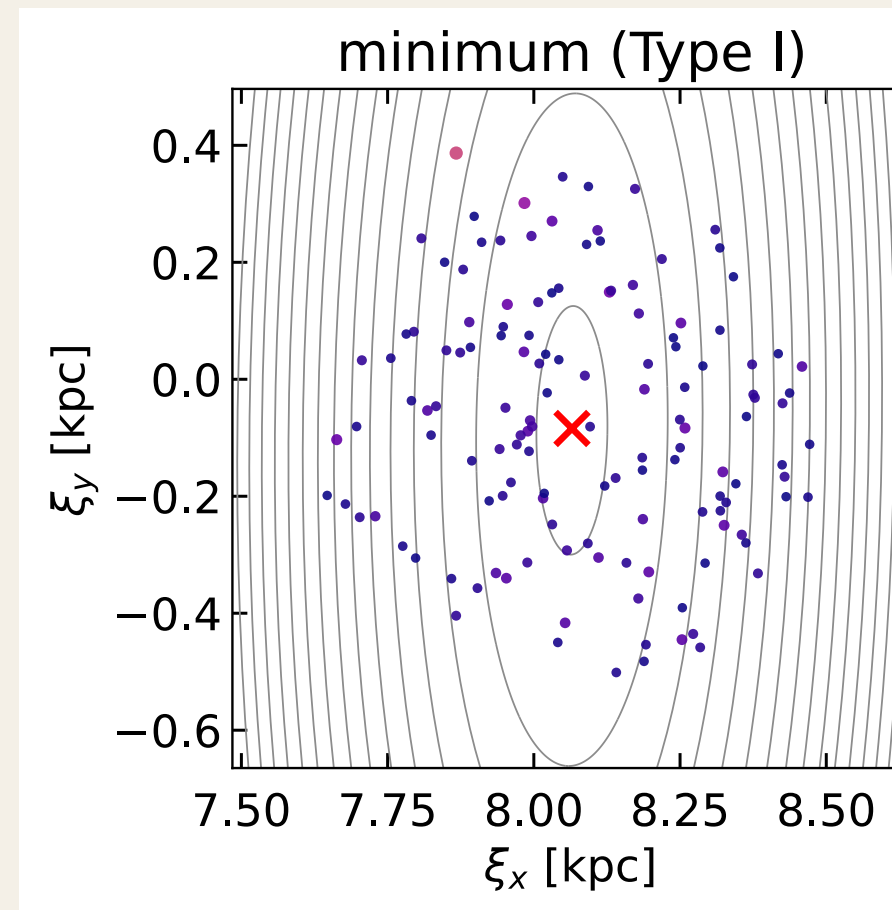
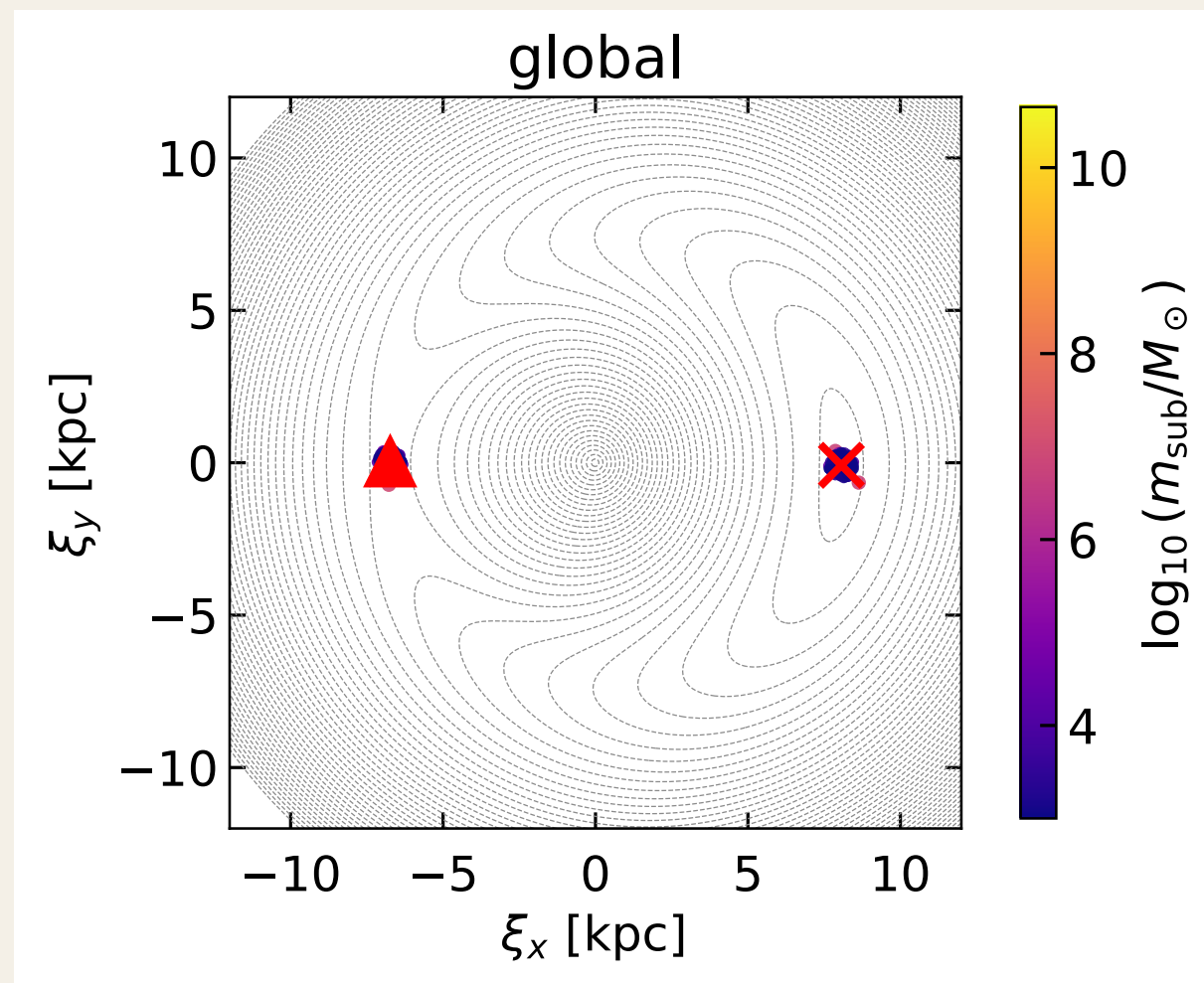
Our system: a galaxy lens dressed in CDM subhalos



- **Galaxy-scale lens:** NFW host ($10^{12}M_{\odot}$, $z = 0.5$) + central galaxy
- **Source:** MBHB at $z = 1.5$, near the caustic ($y = 0.1$)
- **Subhalos:** $10^2 - 10^{11}M_{\odot}$ (truncated NFW)
- **Code available:** Sazanami (漣; *Strong-lensing Amplification at Z, Arrival-time Numerics and Modulation Imprints*)



Zooming in: subhalos around each image



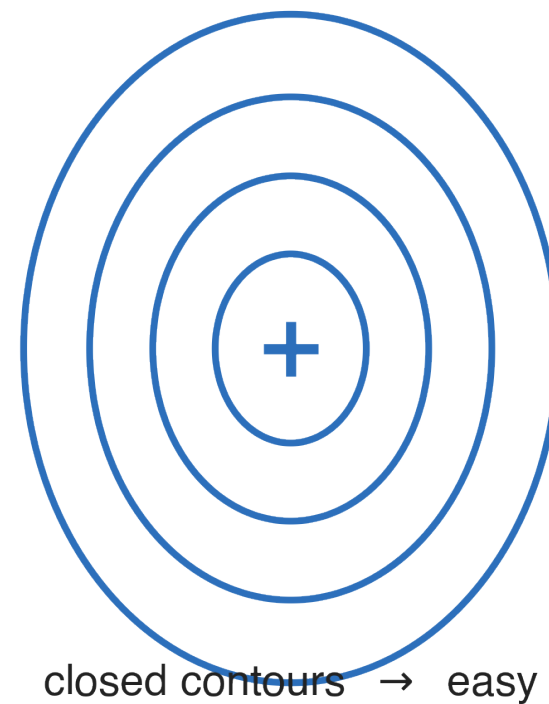
- Fermat potential: $\phi(\mathbf{x}, \mathbf{y}) = \frac{|\mathbf{x} - \mathbf{y}|^2}{2} - \psi(\mathbf{x})$
- Images appear at extrema of ϕ : $\nabla_{\mathbf{x}}\phi(\mathbf{x}, \mathbf{y}) = 0$ (i.e., lens equation)
- Light subhalos ($10^2 - 10^9 M_{\odot}$) generated around images by SASHIMI

Computing the imprint: **minimum** & **saddle**

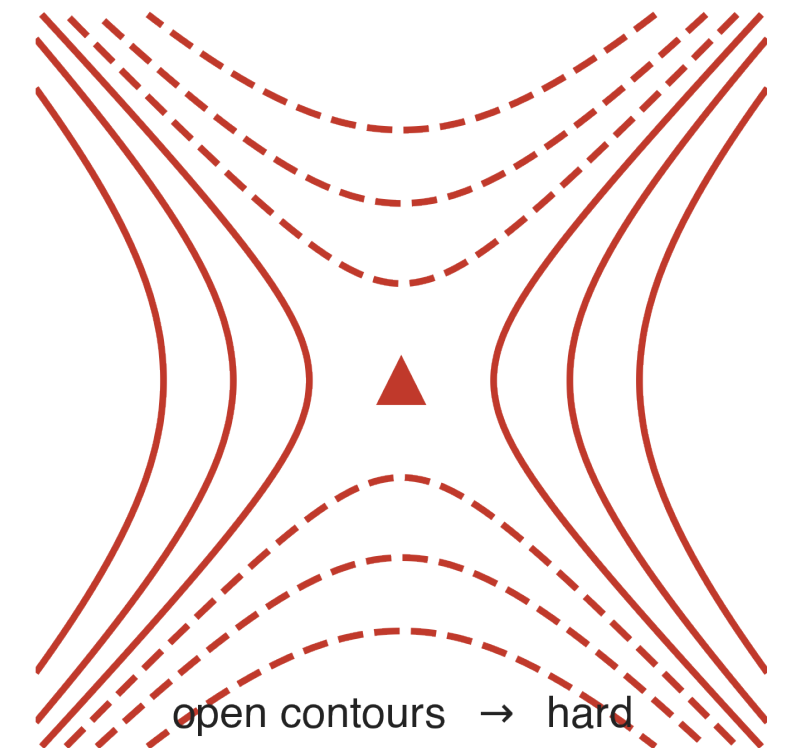
Fresnel-Kirchhoff diffraction integral:

$$F(w) = \frac{w}{2\pi i} \int d^2x \exp [i w \phi(\mathbf{x}, \mathbf{y})]$$

Minimum image



Saddle image



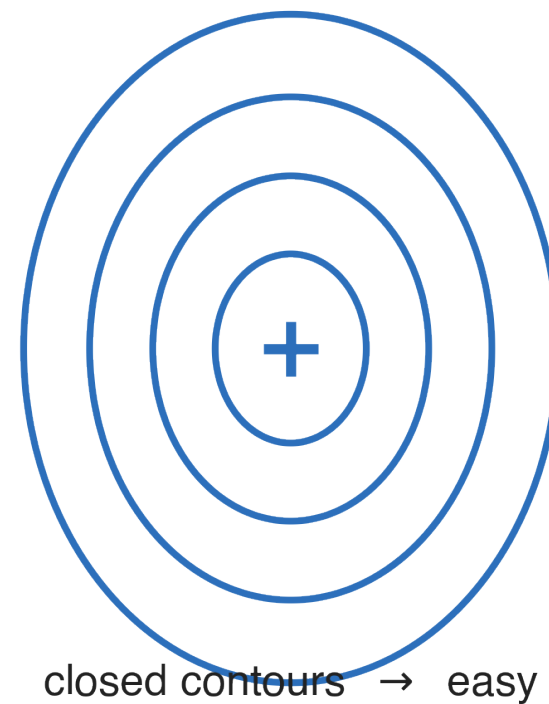
Iso-arrival-time contours around each image

Computing the imprint: **minimum** & **saddle**

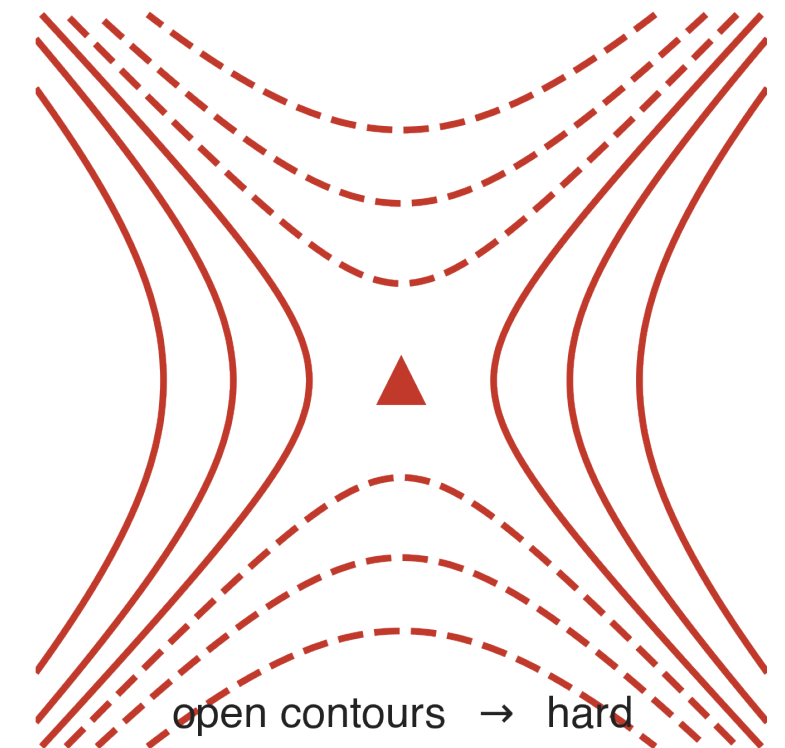
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$$F(w) = \frac{w}{2\pi i} \int d\tau I(\tau) e^{i w \tau}, \quad I(\tau) = \oint_{\phi=\tau} \frac{d\ell}{|\nabla \phi|}$$

Minimum image



Saddle image



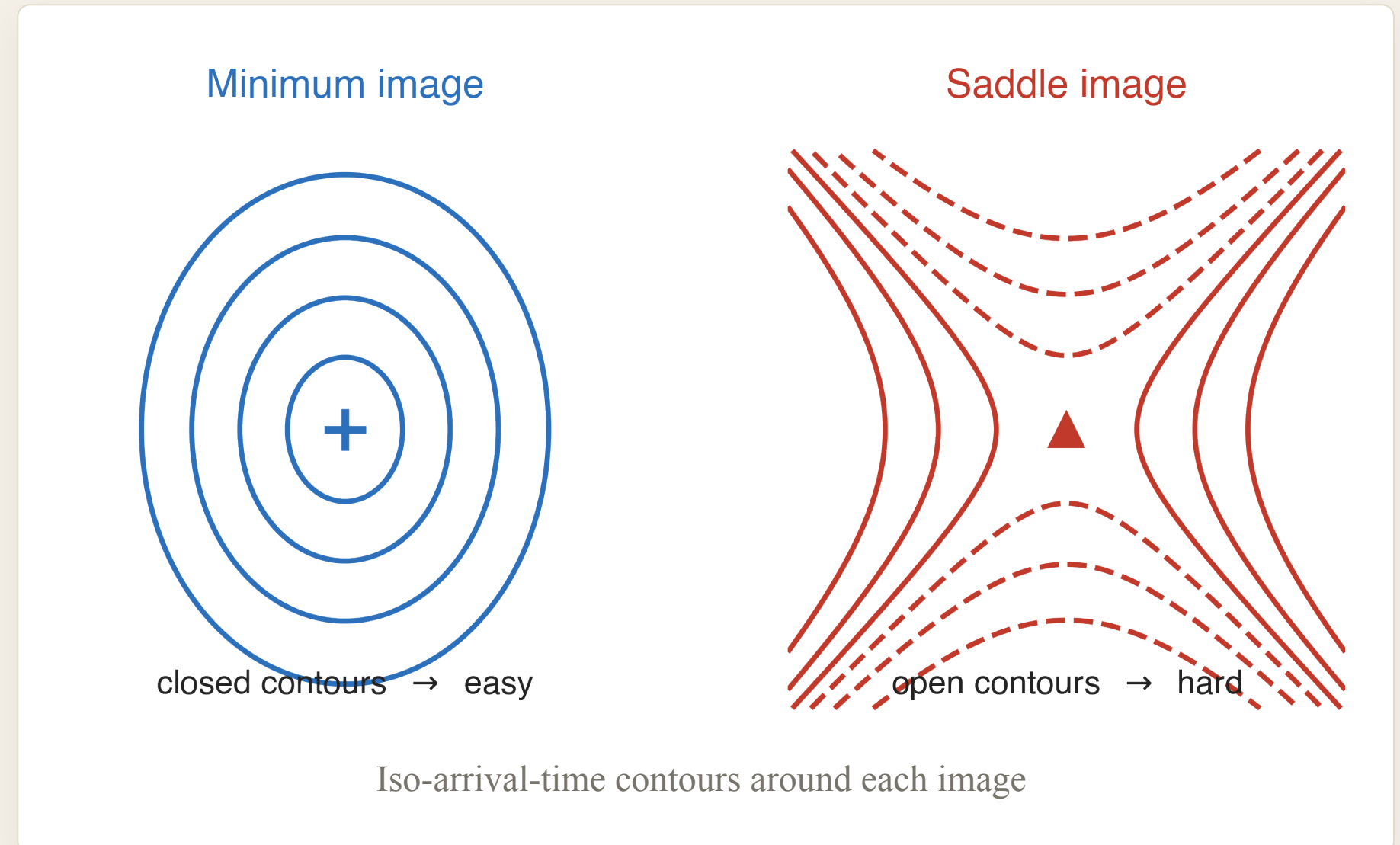
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Amplification = integral over **equal-arrival-time contours** around each image.



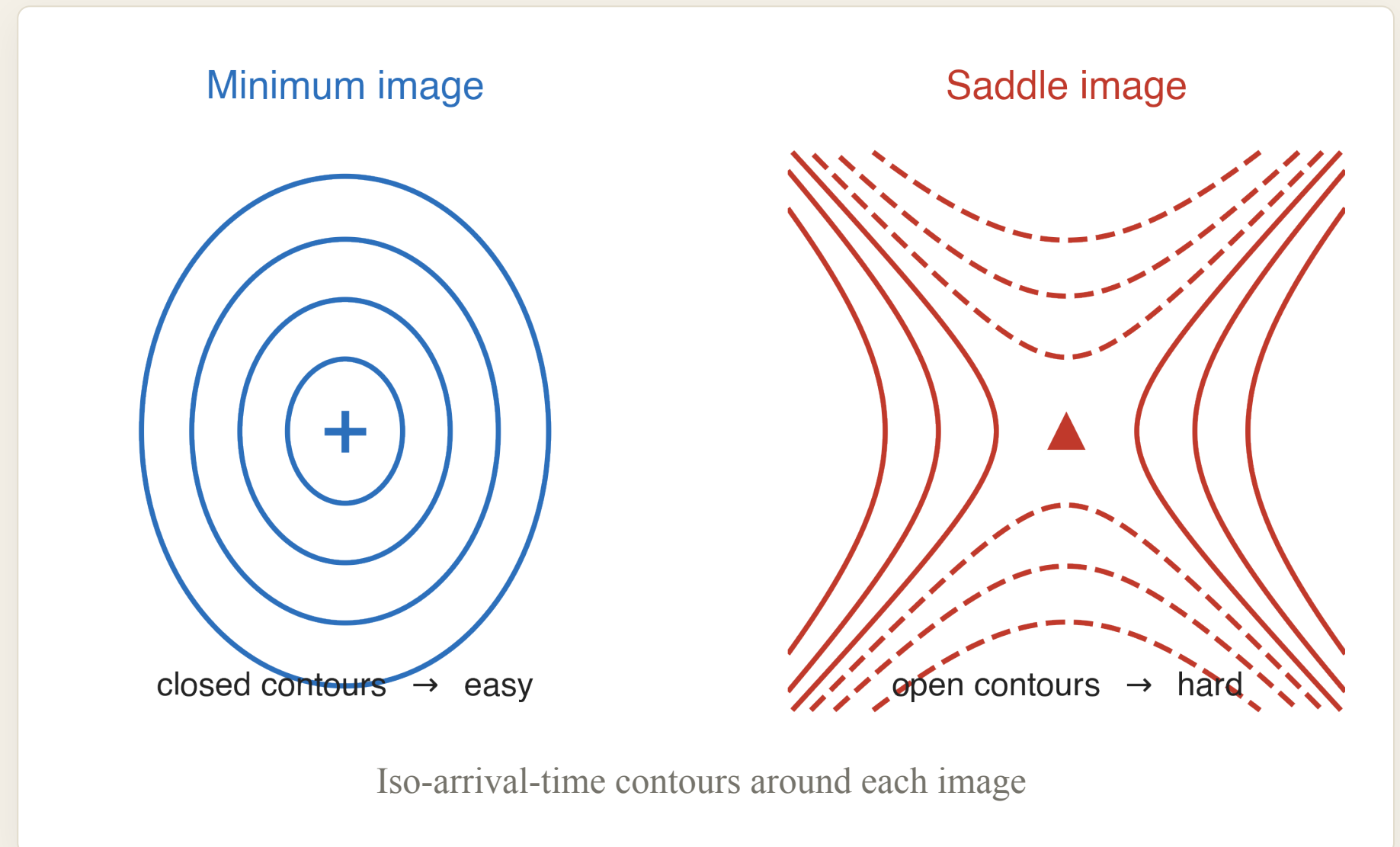
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- **Minimum:** closed loops → straightforward (e.g., GLoW)



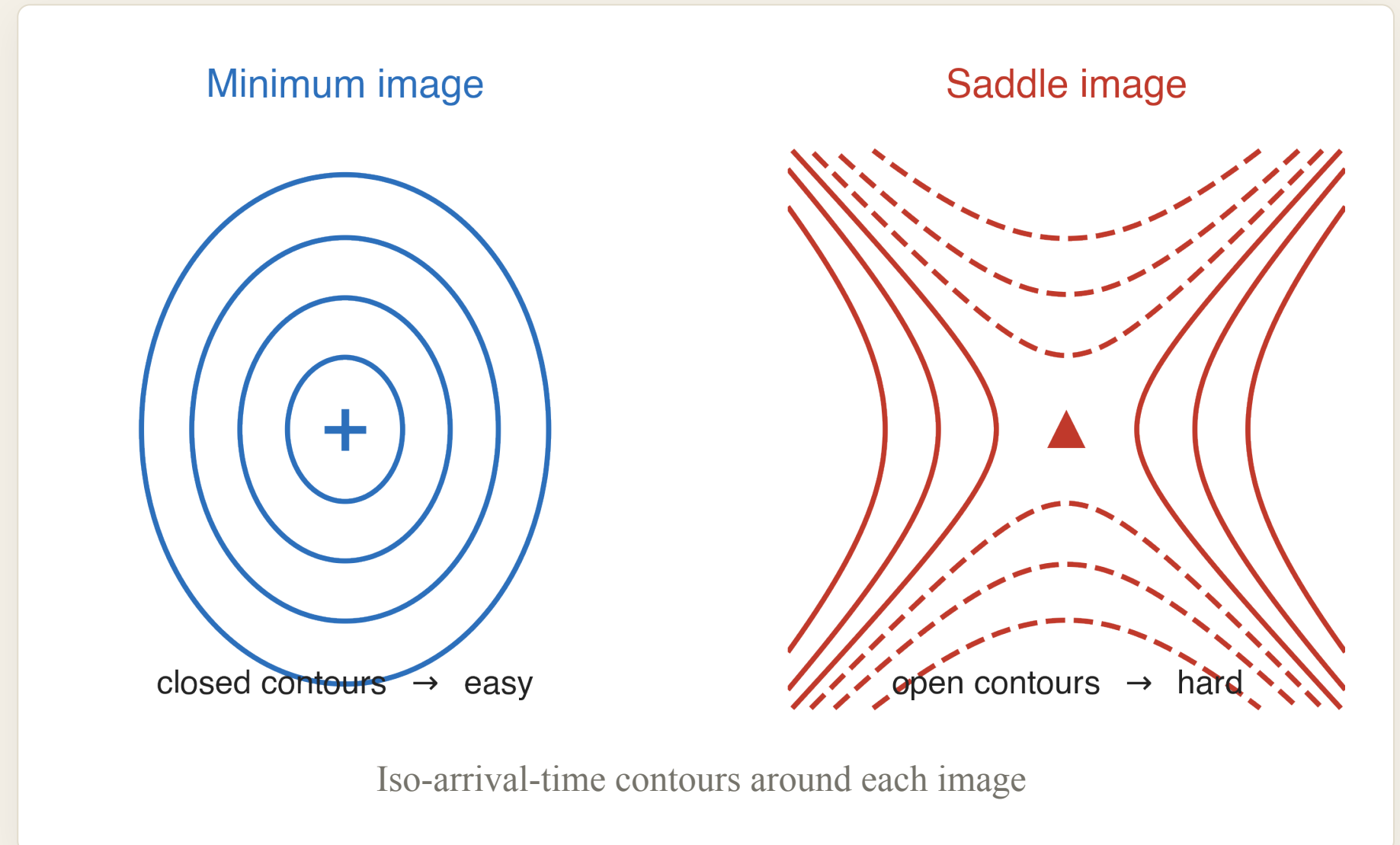
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- **Saddle:** open contours → **numerically hard**



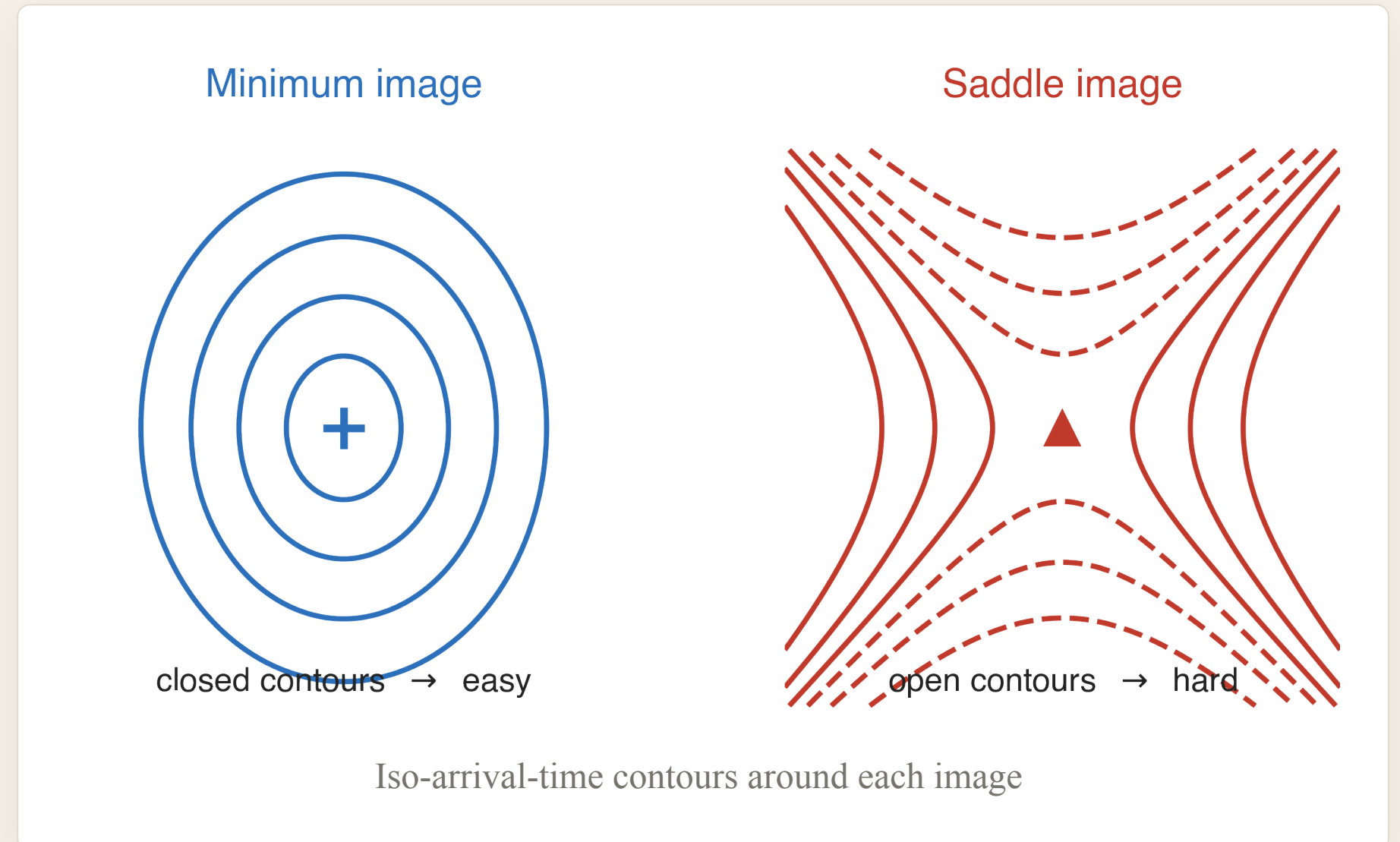
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$$F_{\text{sad}}(w) = e^{i w \phi_{\text{sad}}} \left[-i \sqrt{\mu} + \frac{w}{2\pi i} \left(\text{FFT}[\delta I](w) + \tilde{T}(w) \right) \right]$$

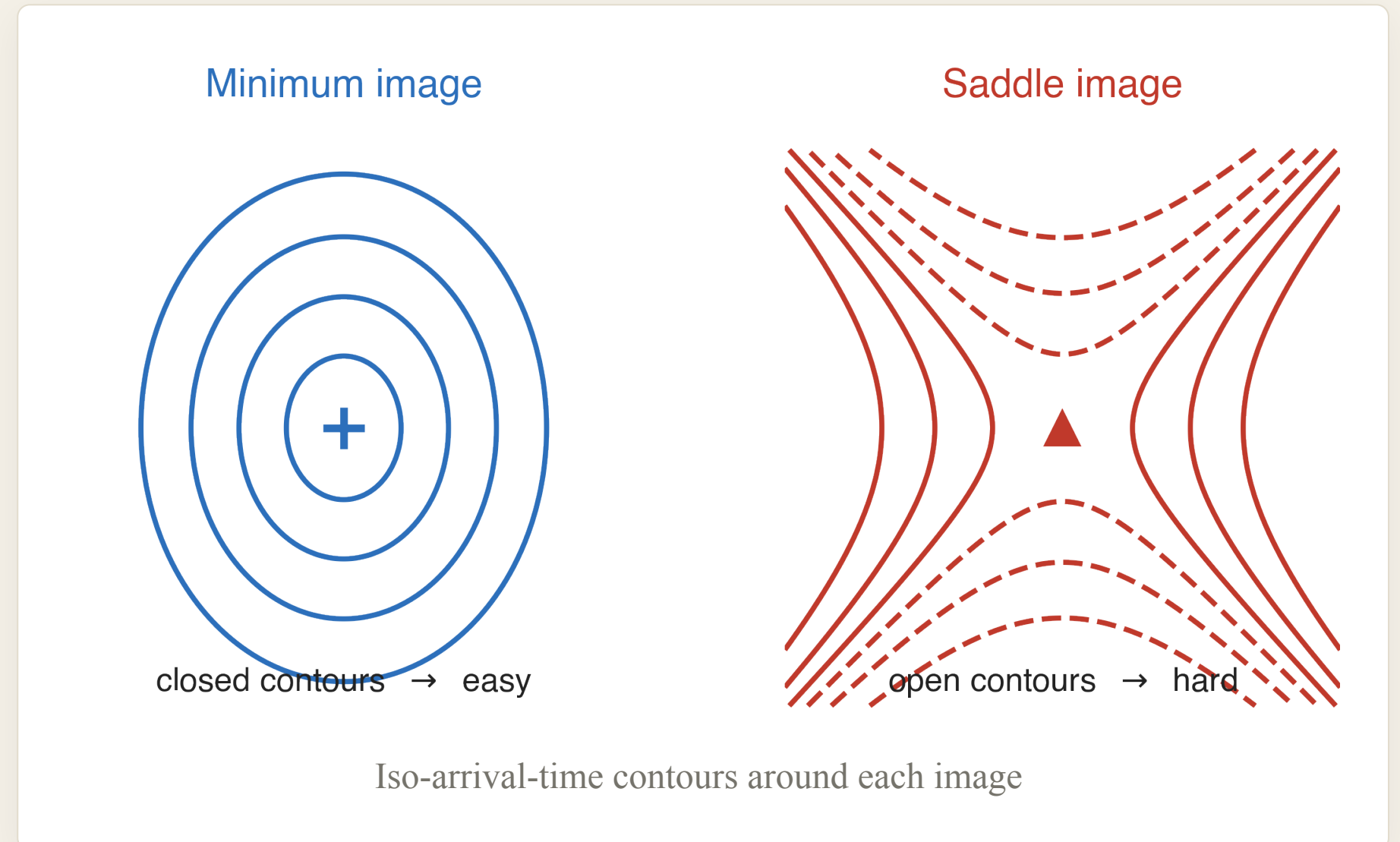
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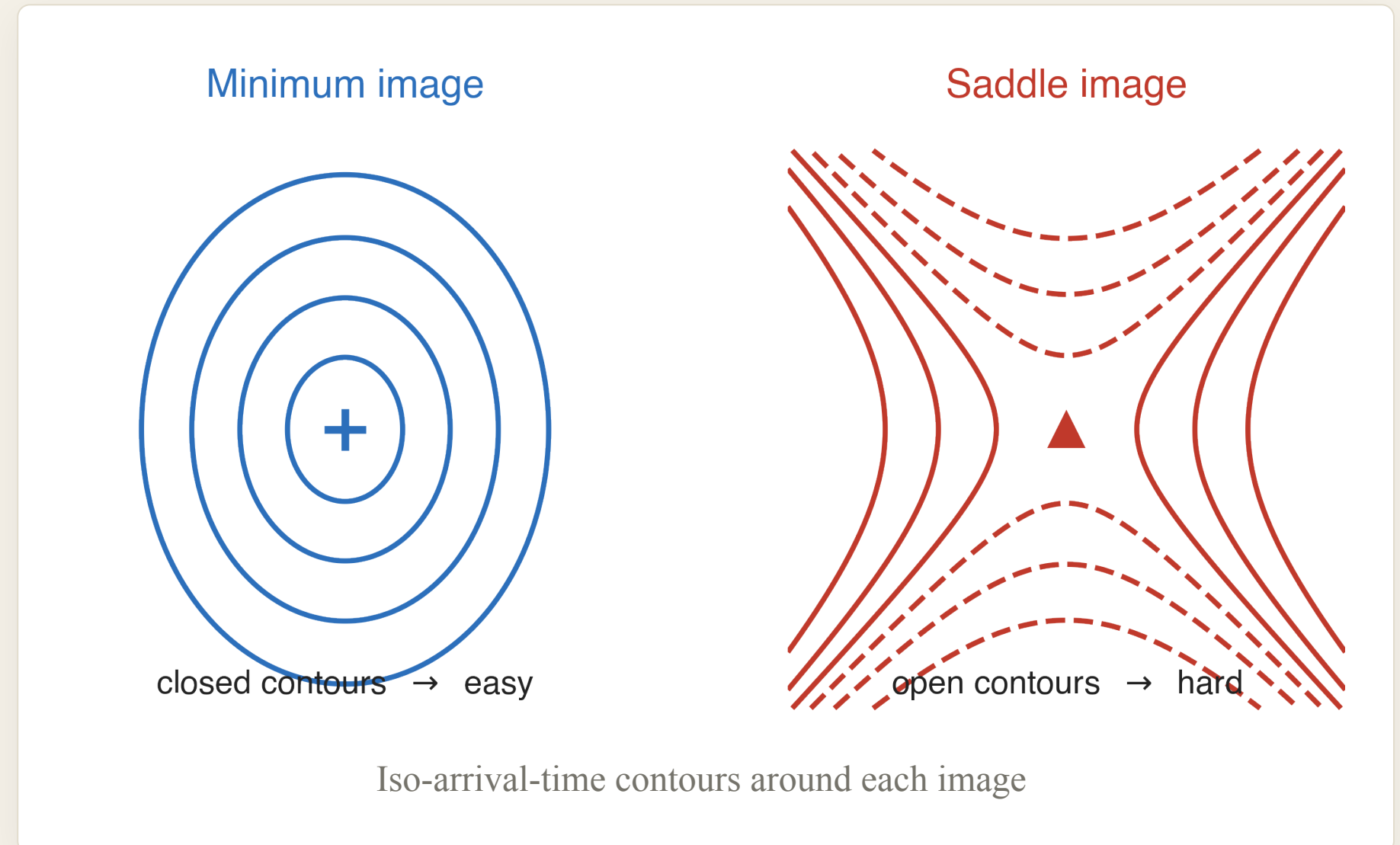
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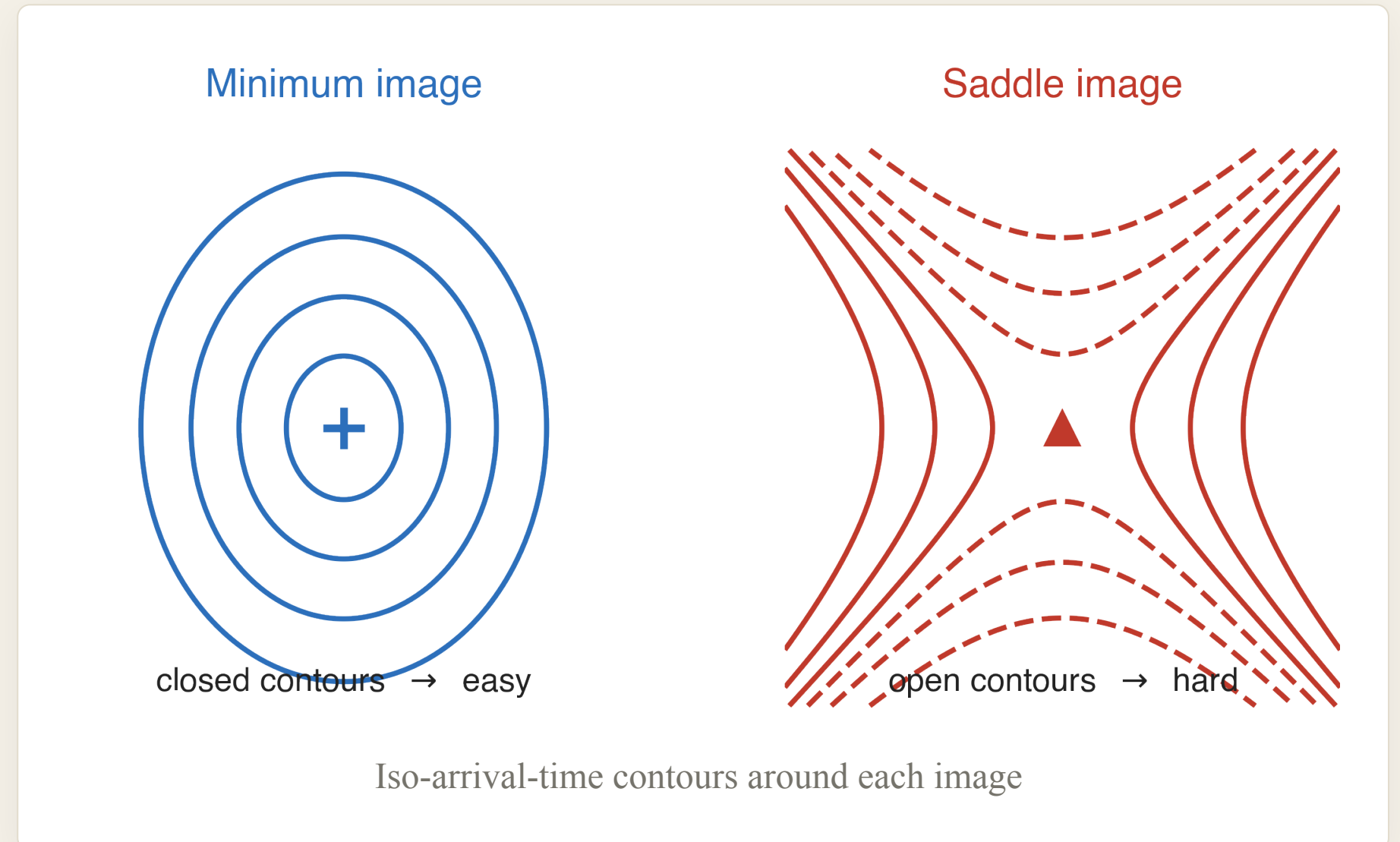
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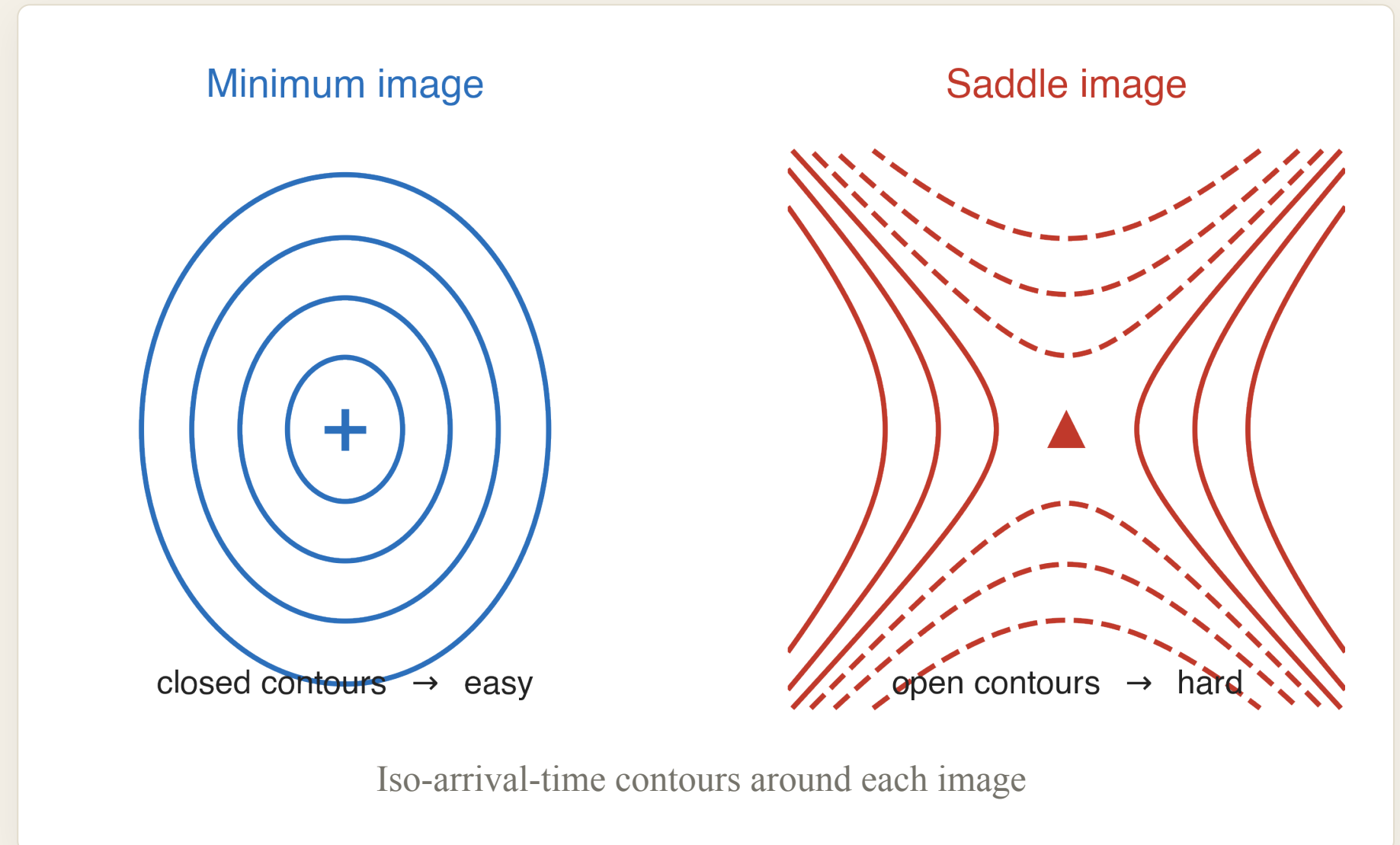
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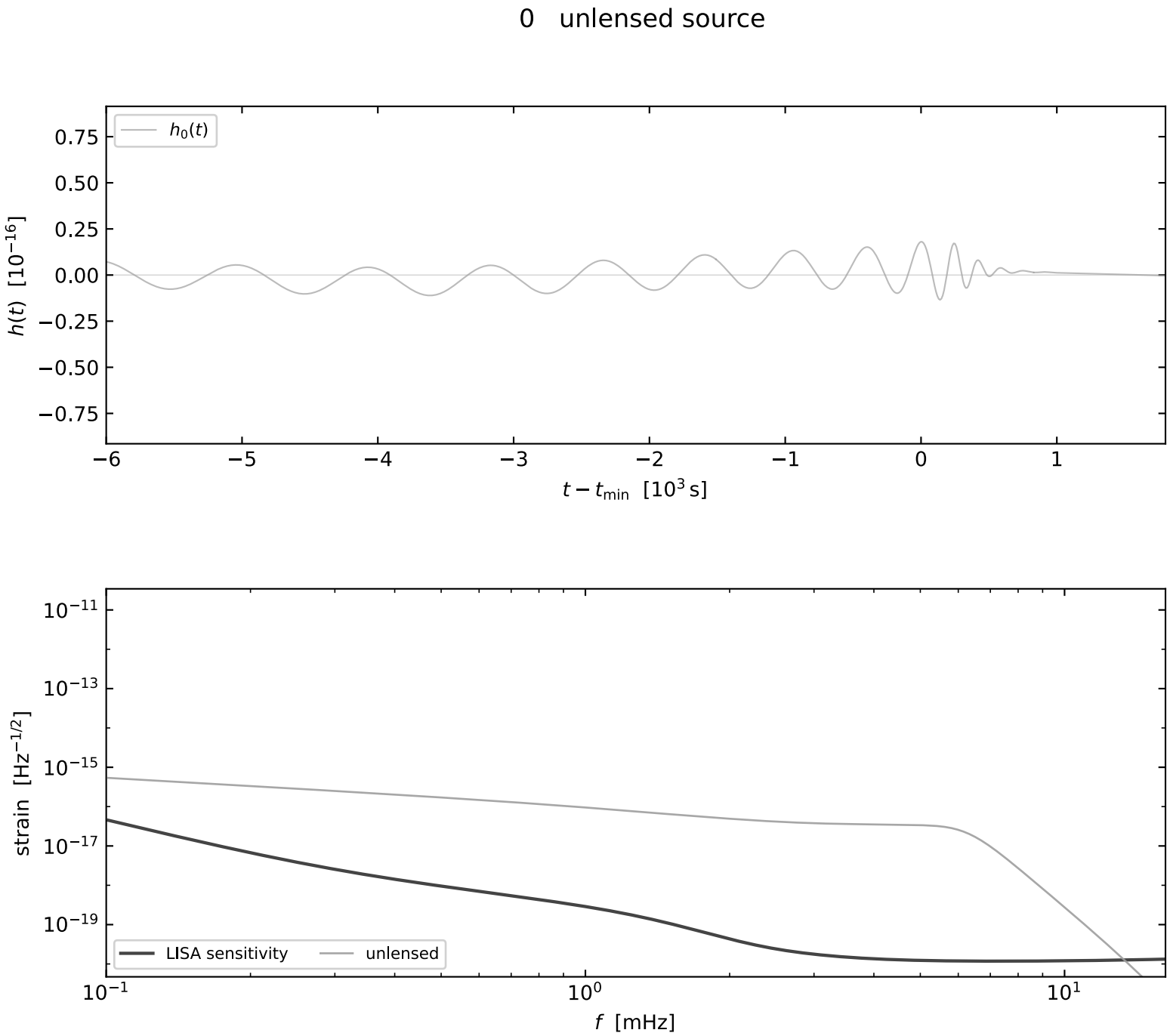
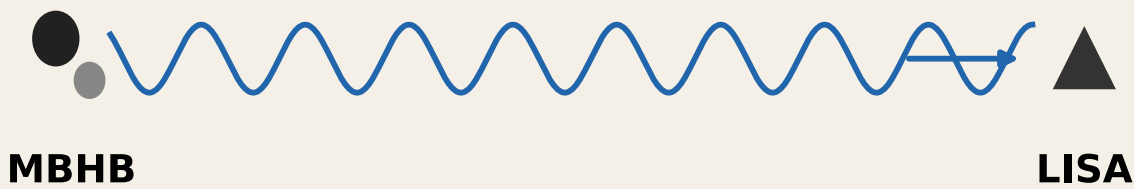
- **Minimum:** closed loops → straightforward (e.g., GLoW)
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- **Sazanami** does both



$$F_{\text{sad}}(w) = e^{i w \phi_{\text{sad}}} \left[\overset{\text{Macrolens term}}{\downarrow} -i\sqrt{\mu} + \frac{w}{2\pi i} \left(\underset{\substack{\text{Subhalos' perturbation} \\ \text{within a given time delay}}}{\uparrow} \text{FFT}[\delta I](w) + \overset{\text{Subhalos' far-field tail}}{\downarrow} \tilde{T}(w) \right) \right]$$

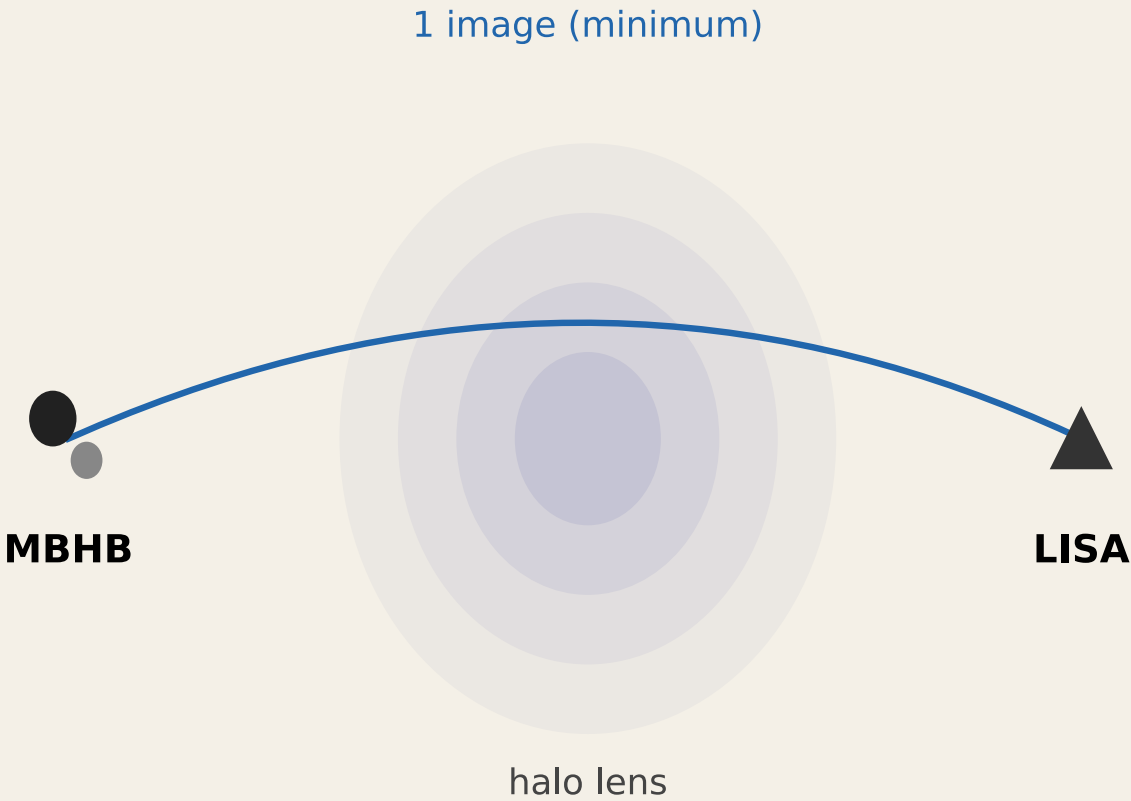
How the lensed signal builds up

0 · unlensed

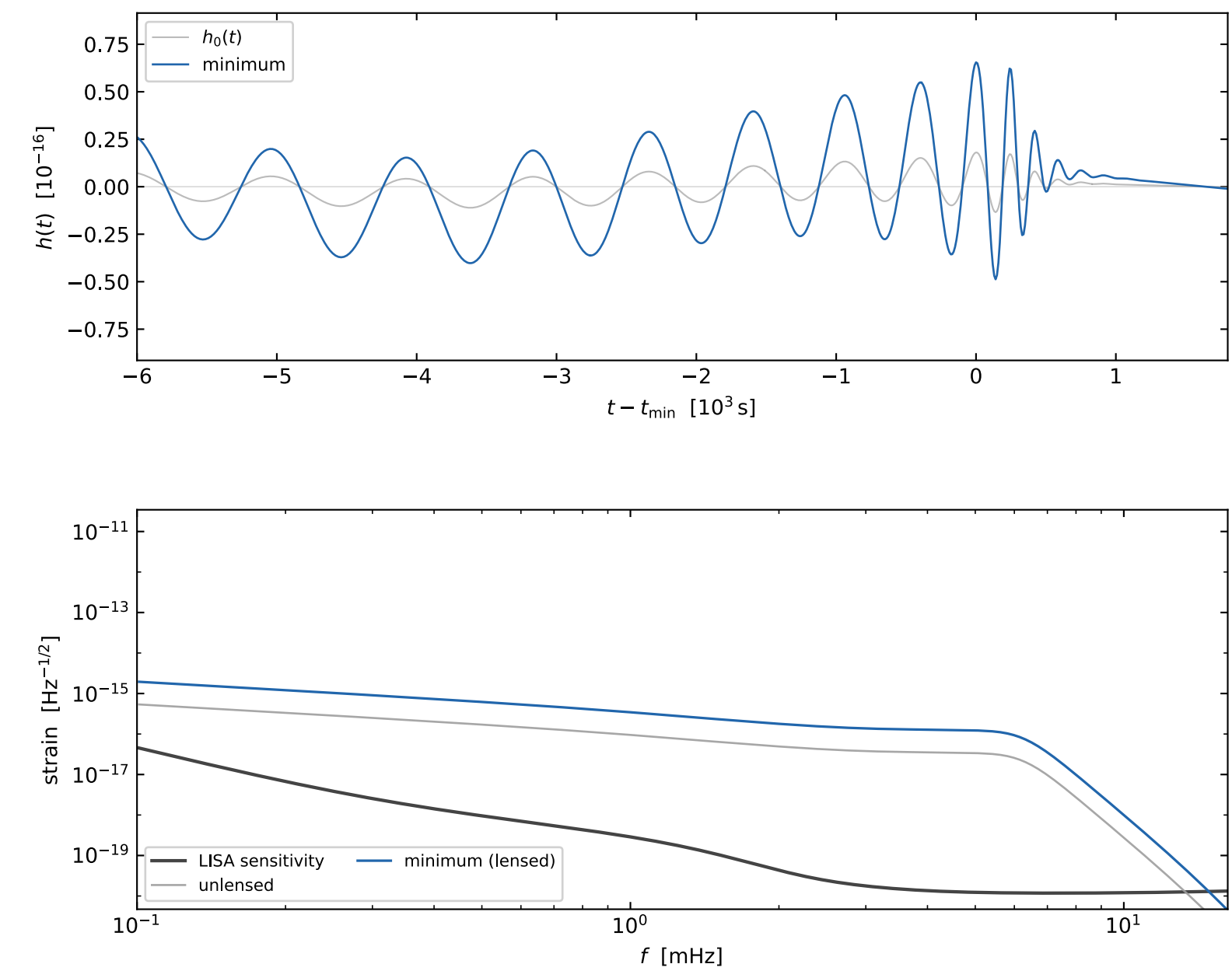


How the lensed signal builds up

1 · + minimum

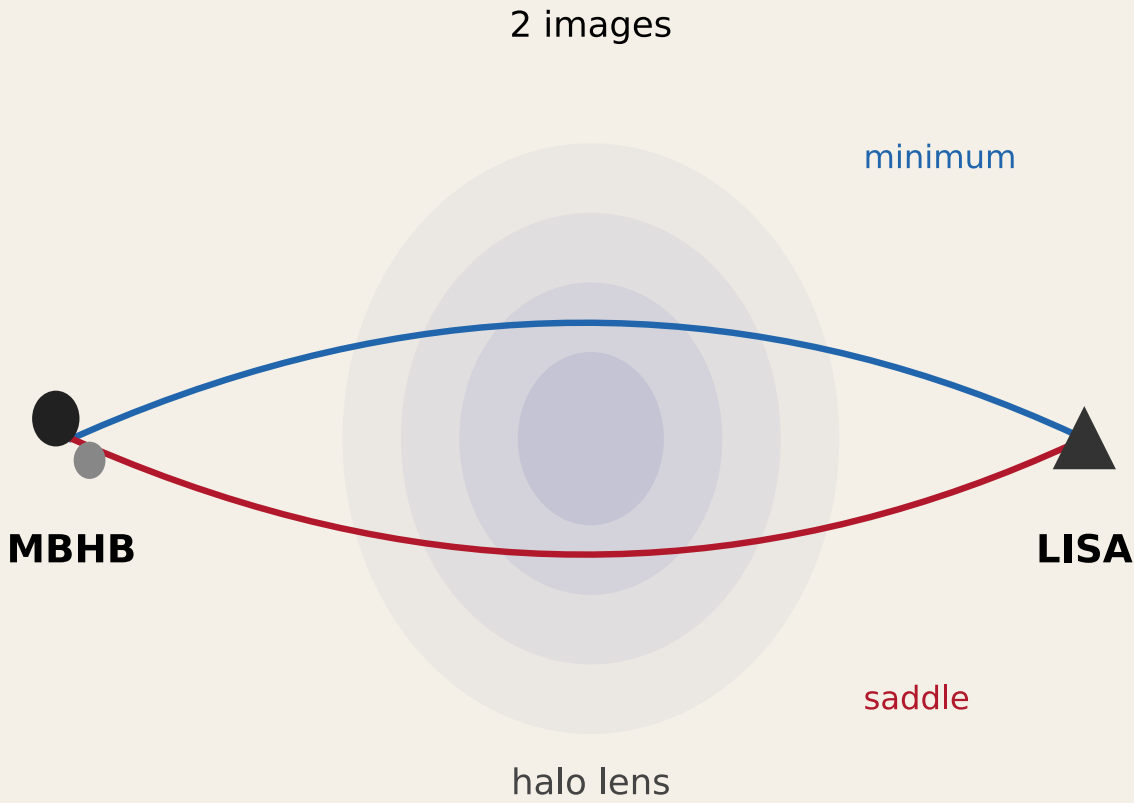


1 minimum image (geometric optics): frequency-independent magnification

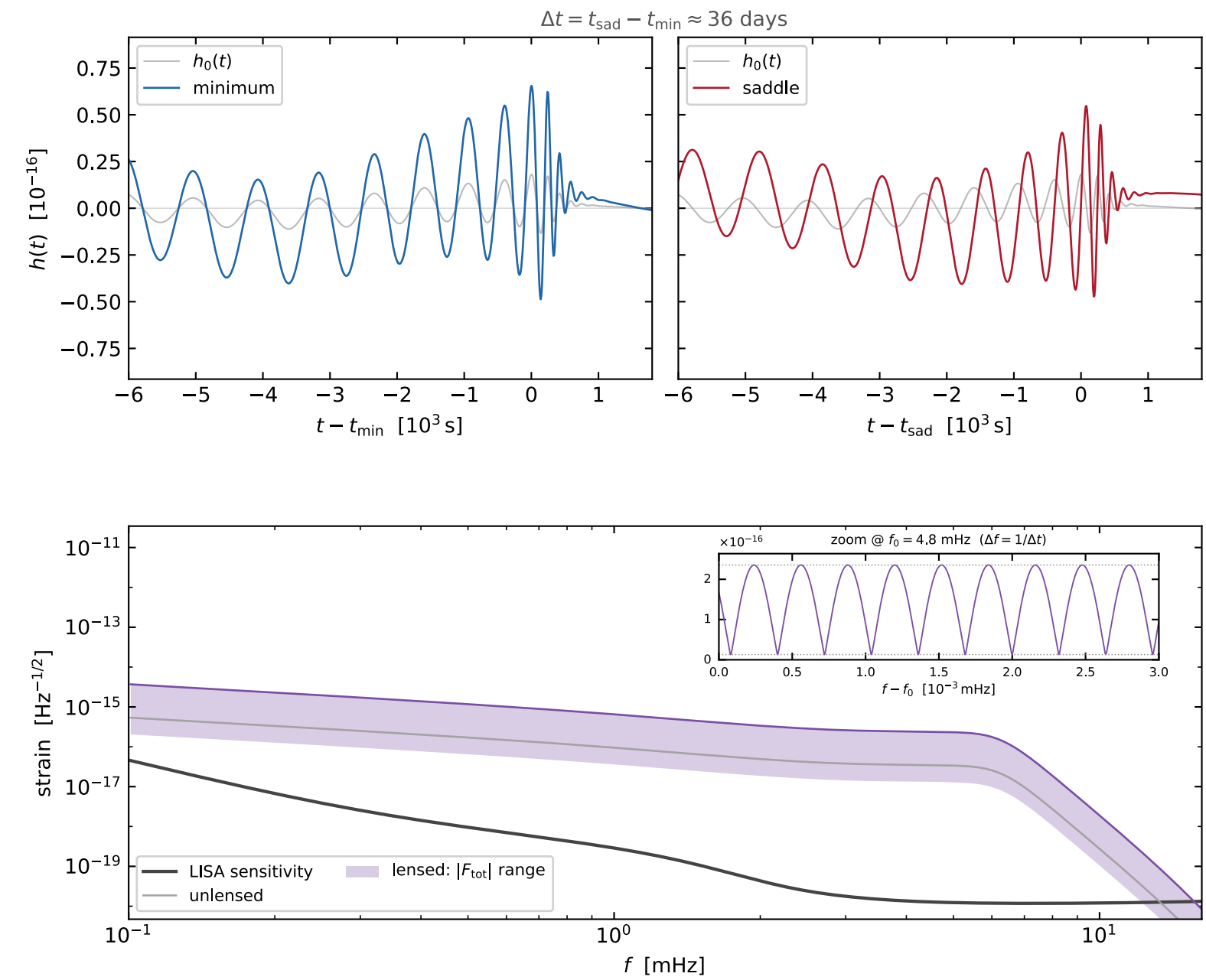


How the lensed signal builds up

2 · + saddle

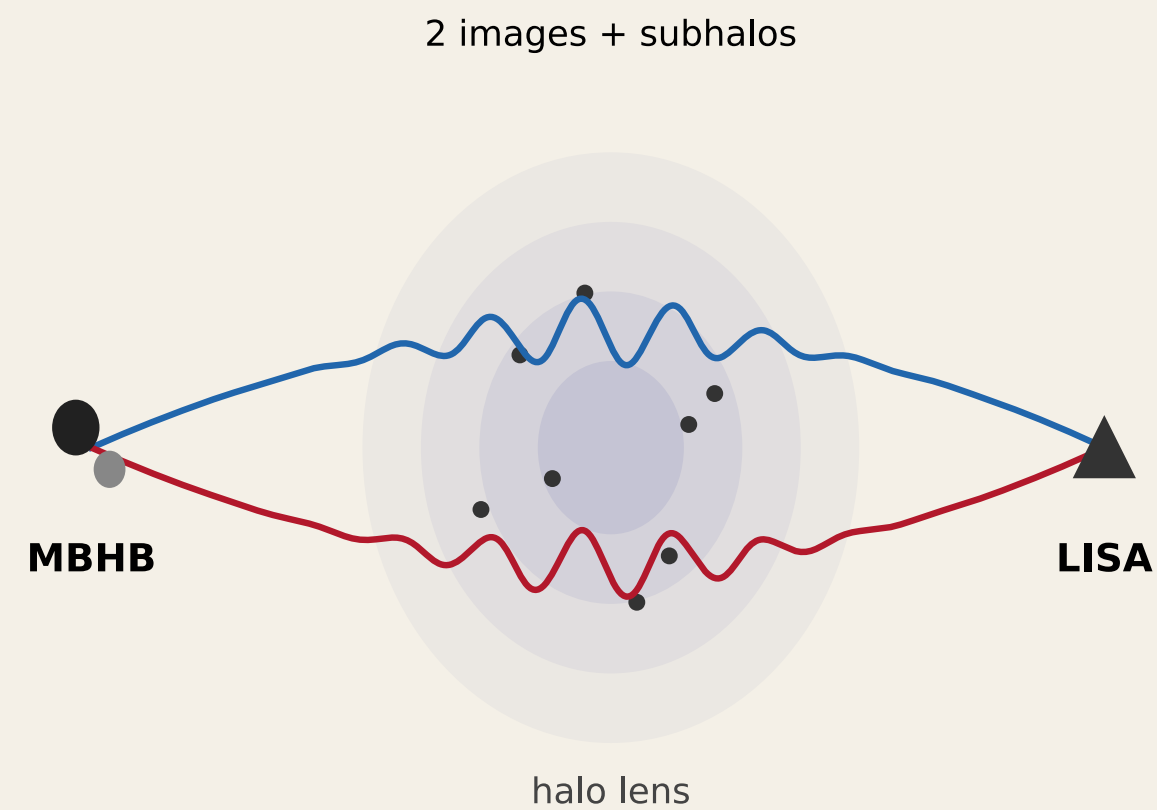


2 + saddle image: two arrivals in time = interference fringes in frequency

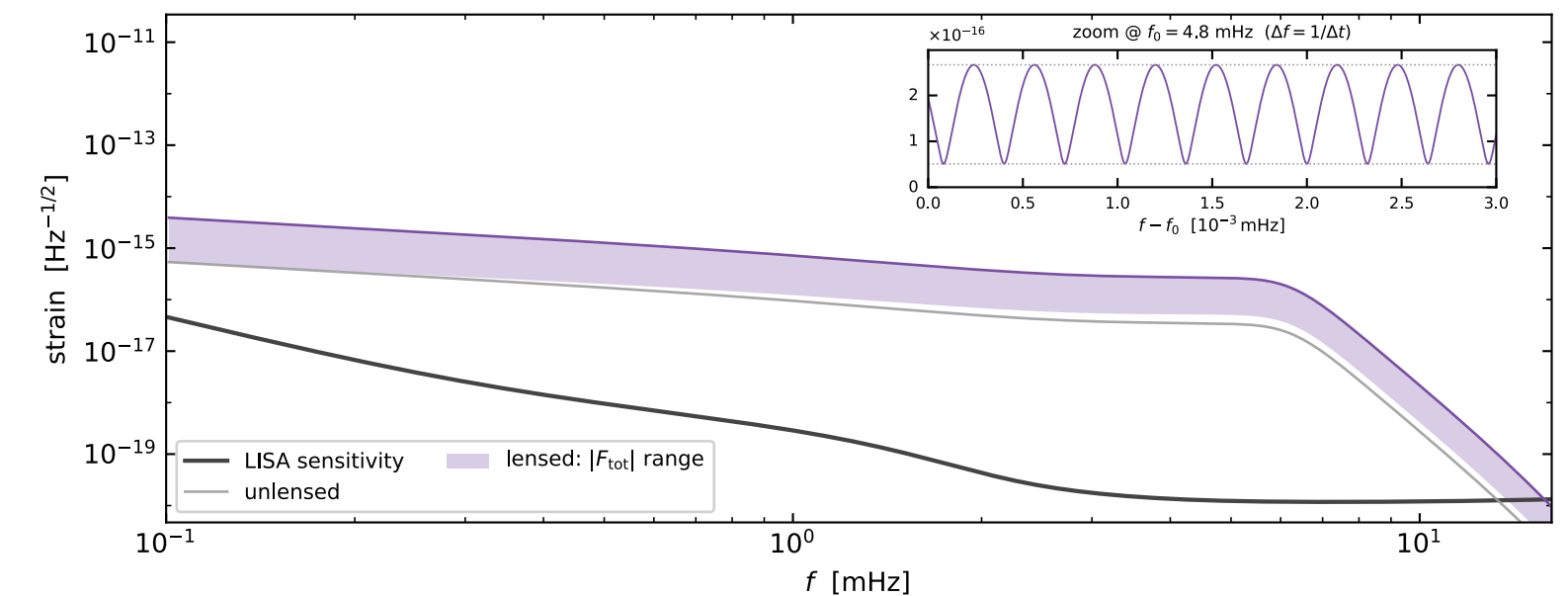
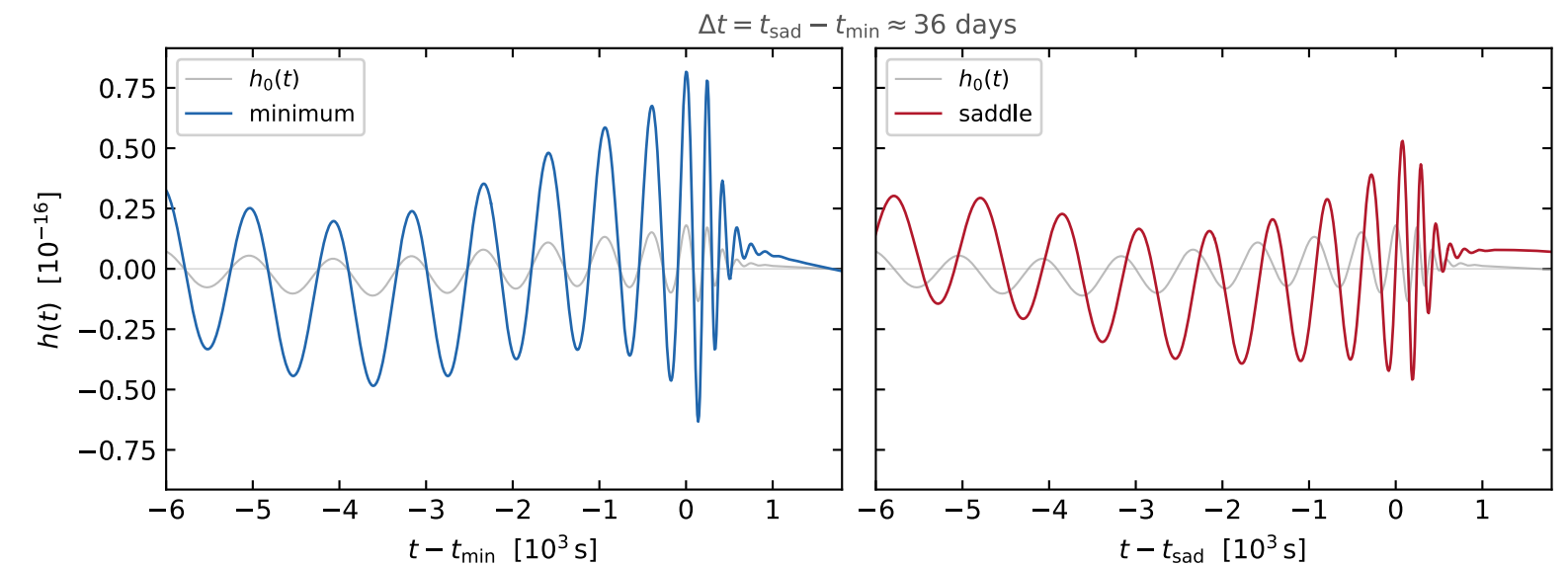


How the lensed signal builds up

3 · + wave optics



3 + wave optics: the interference band's envelope becomes frequency dependent

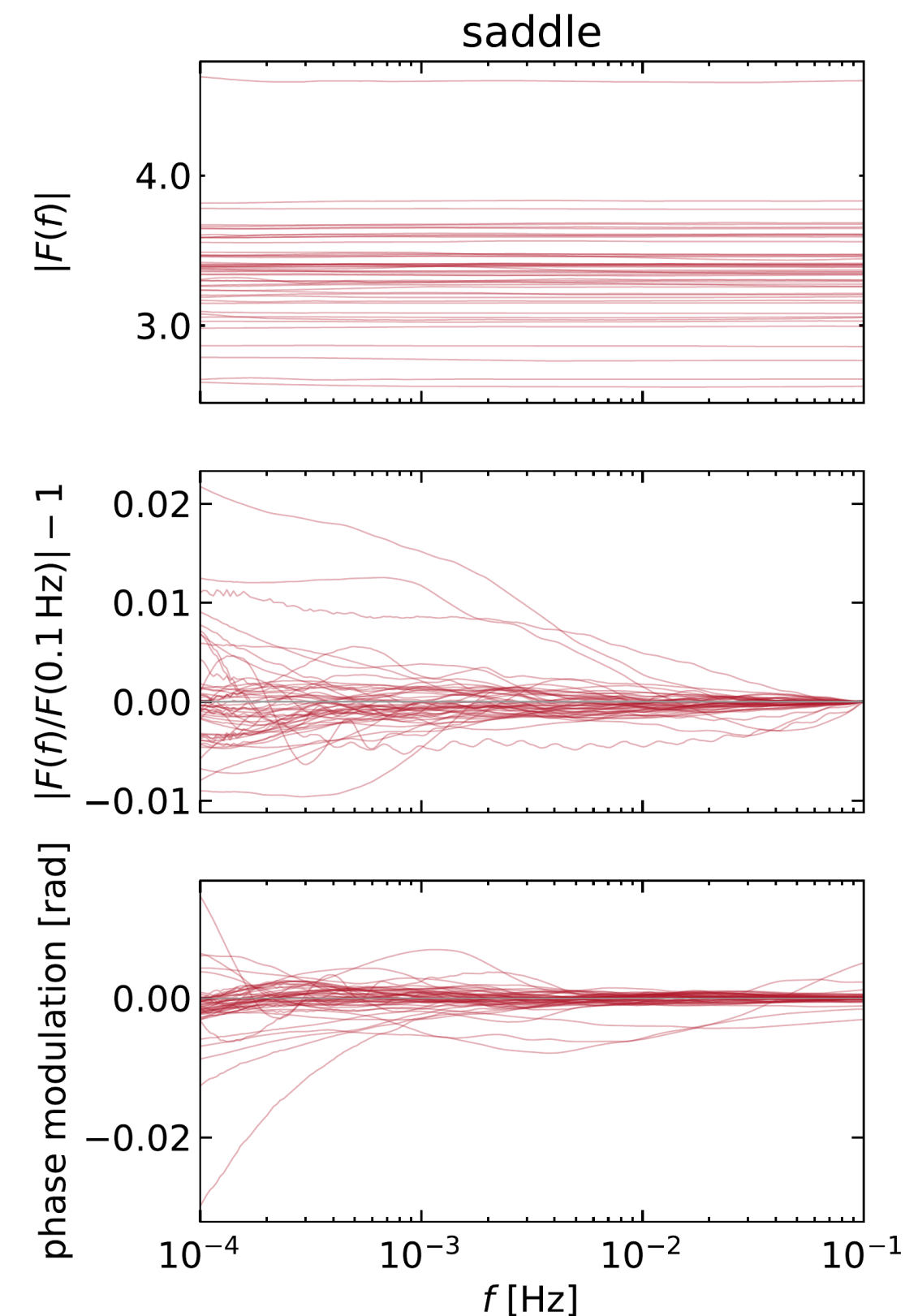
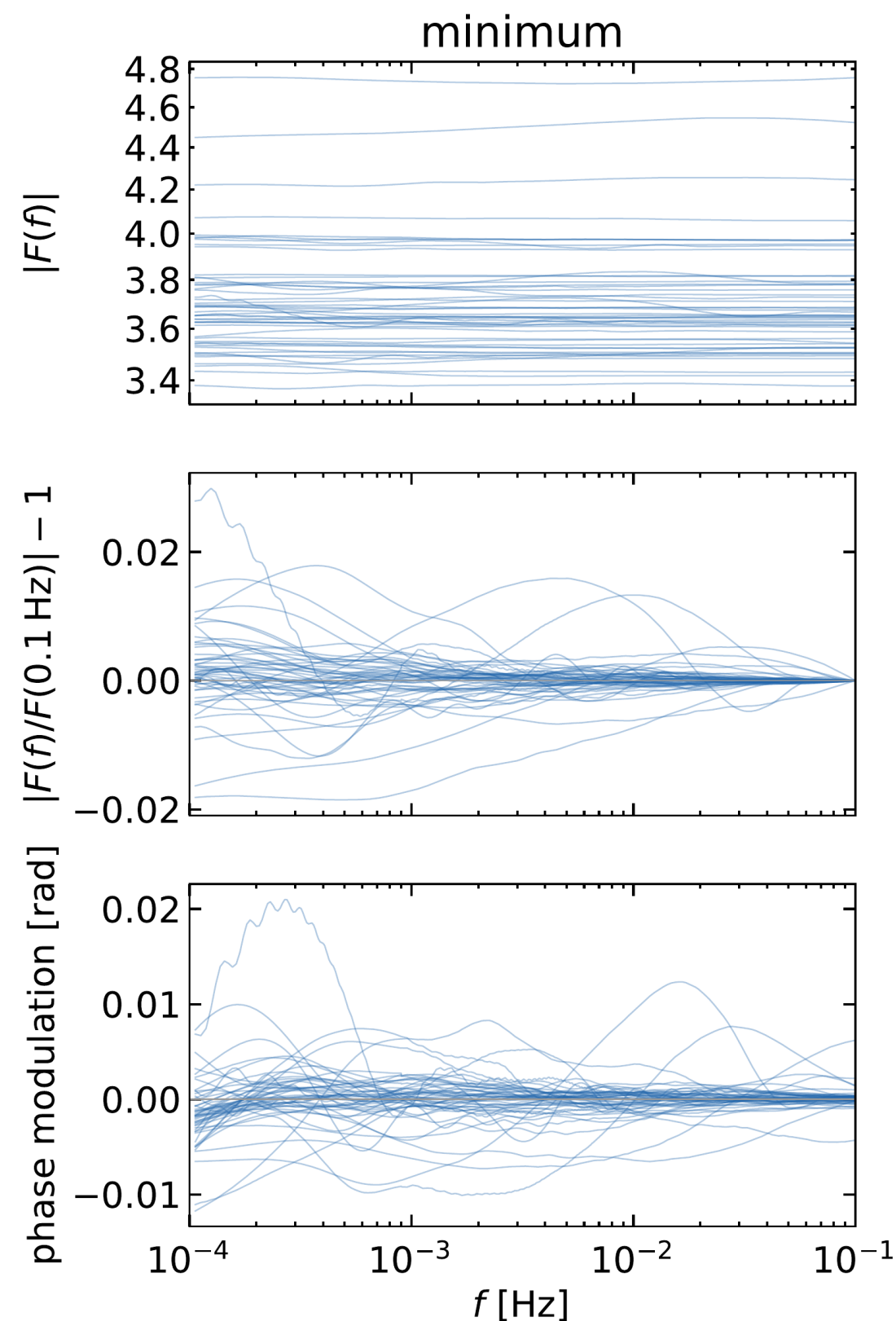


Two cleanly-separated chirps; the **wiggle on each** is the wave-optics subhalo imprint.

Percent-level modulations on both images

1000 CDM realizations

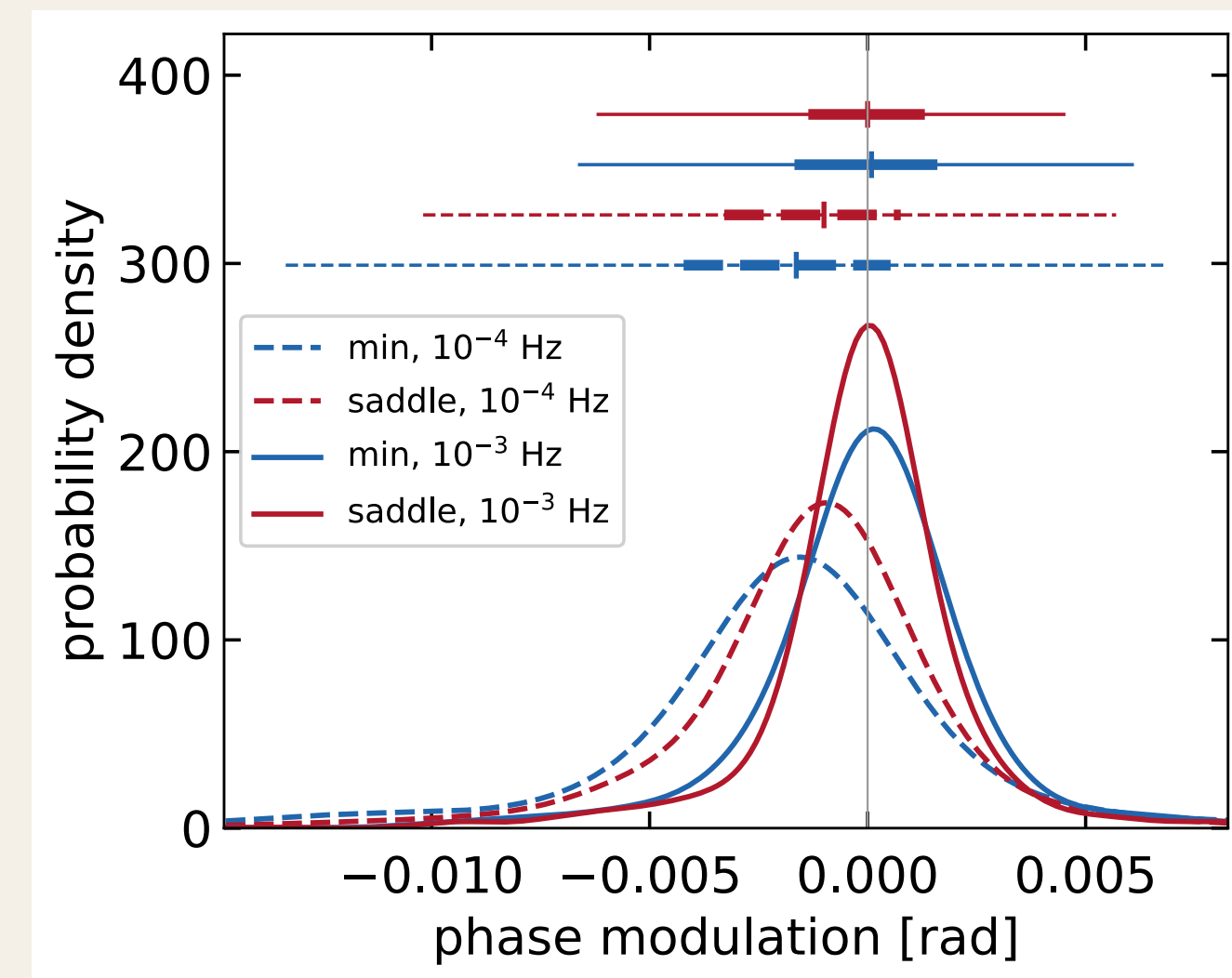
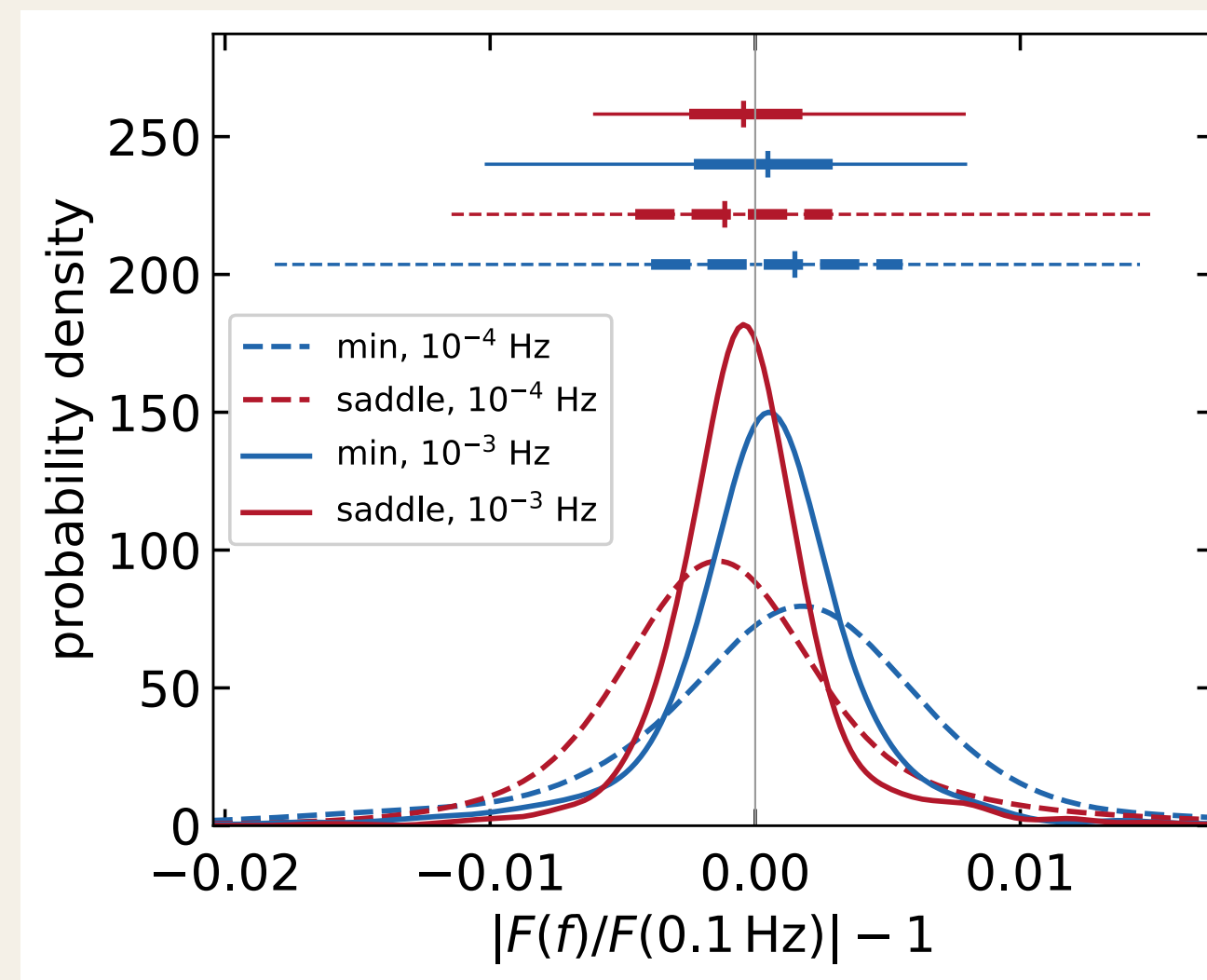
- Amplitude $\sim\%$; phase $\sim$ a few 10^{-3} rad — largest at low frequency
- Both parities imprinted (the fluctuation is parity-blind)
- Dominated by subhalos with $10^4 - 10^7 M_\odot$



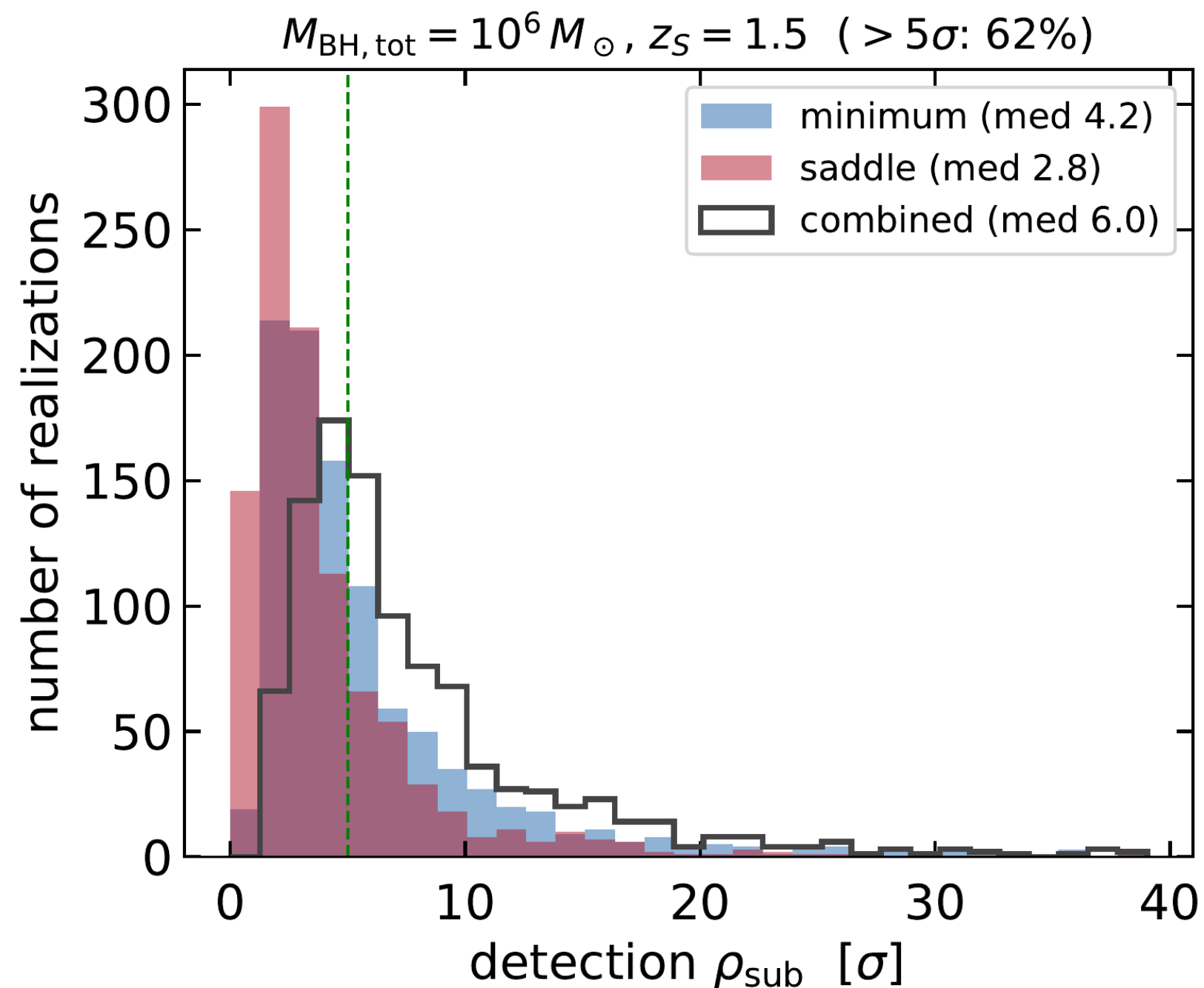
Percent-level modulations on both images

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- Dominated by subhalos with $10^4 - 10^7 M_\odot$



A single loud event detects substructure

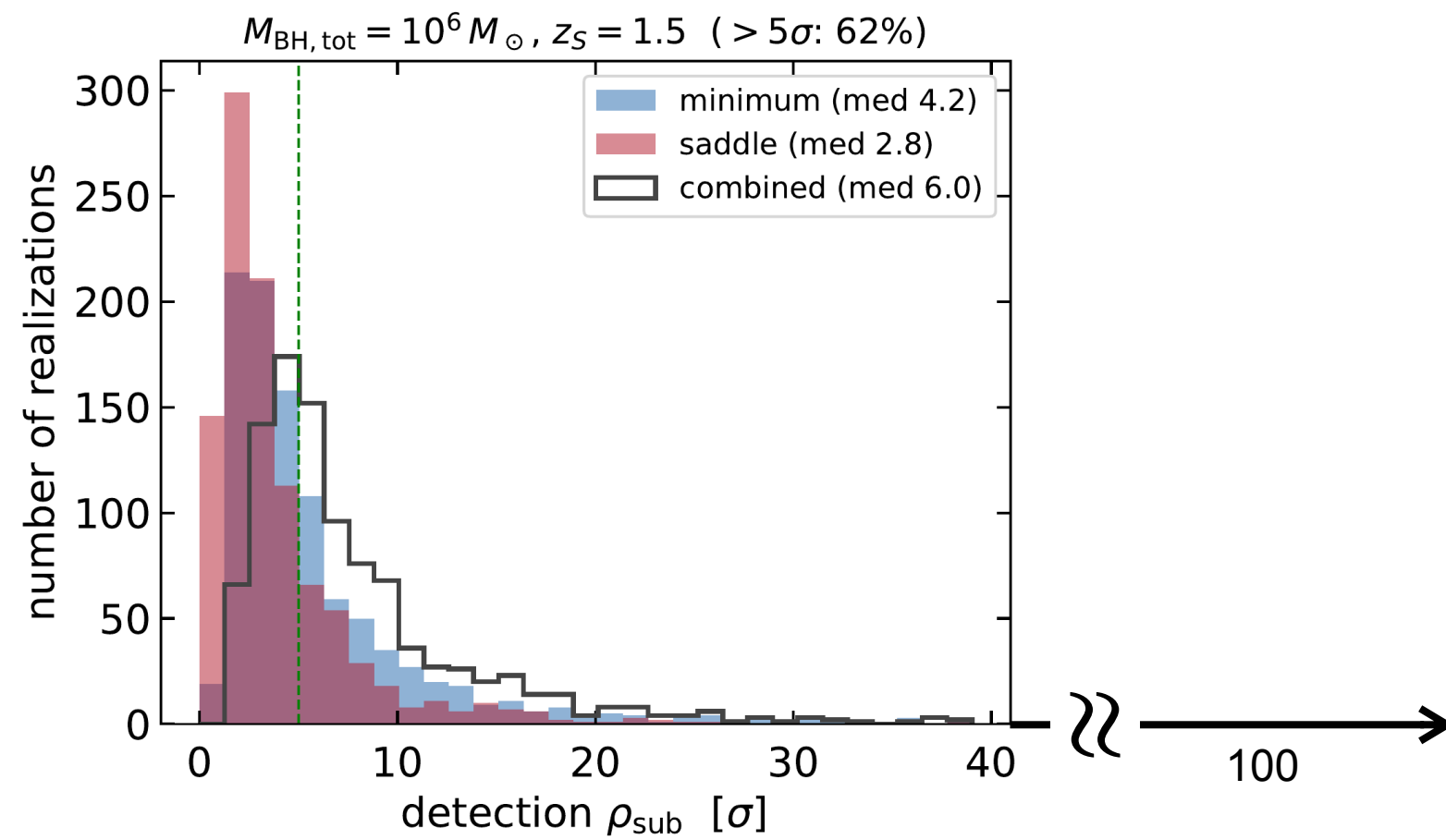


62%

of realizations
detected at $> 5\sigma$

- Project out amplitude, constant phase, arrival time — keep only the frequency structure
- Fiducial $10^6 M_{\odot}$ MBHB near caustic ($y = 0.1$): median $\rho \sim 6$
- Saddle comparable to minimum (5σ : 40 / 23 / 62%)

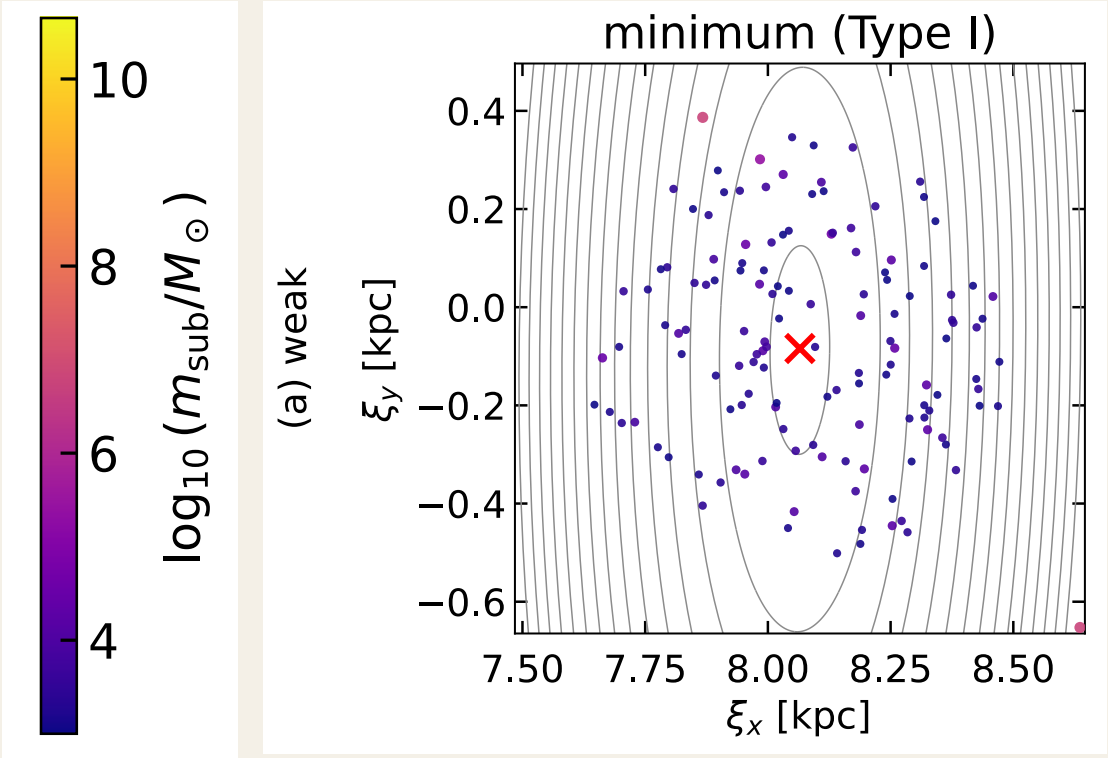
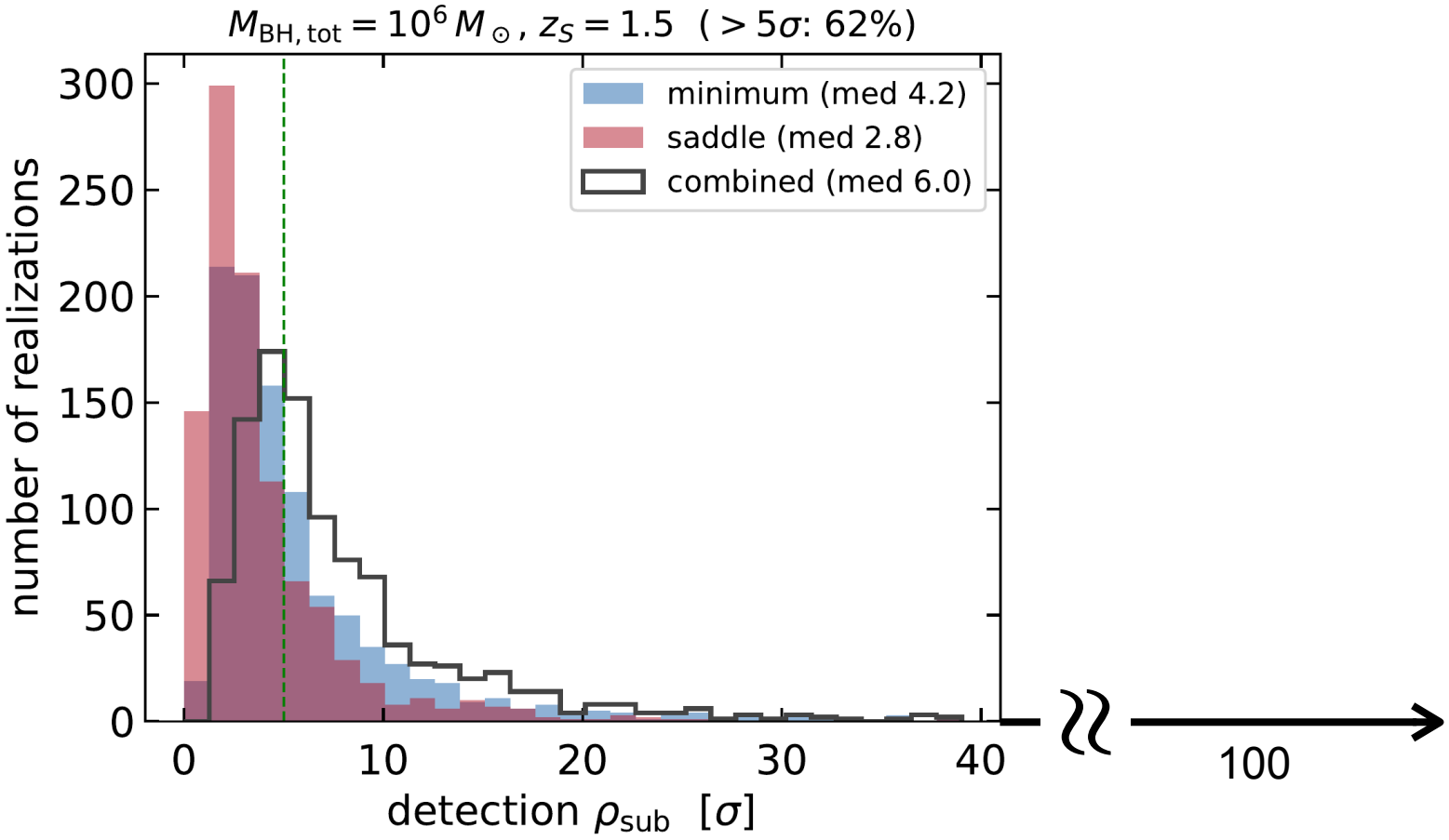
A few closest subhalos dominate the detection



- Long tail dominated by a few closest subhalos (leave-one-out: -90%)

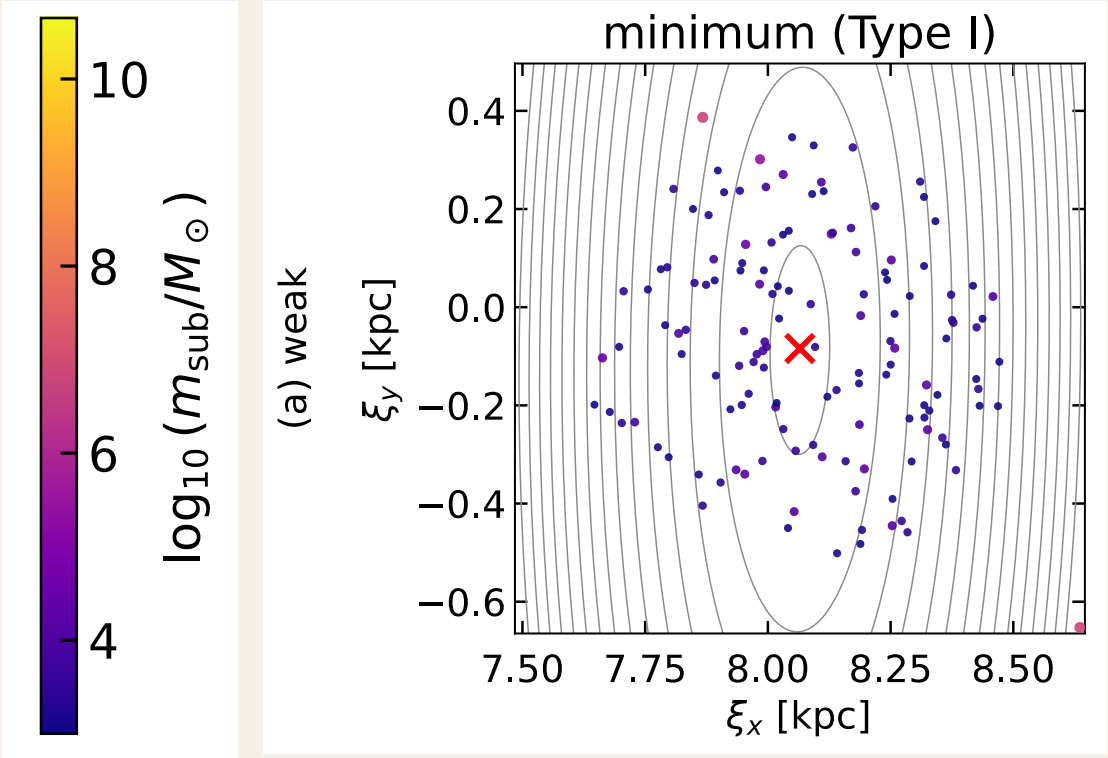
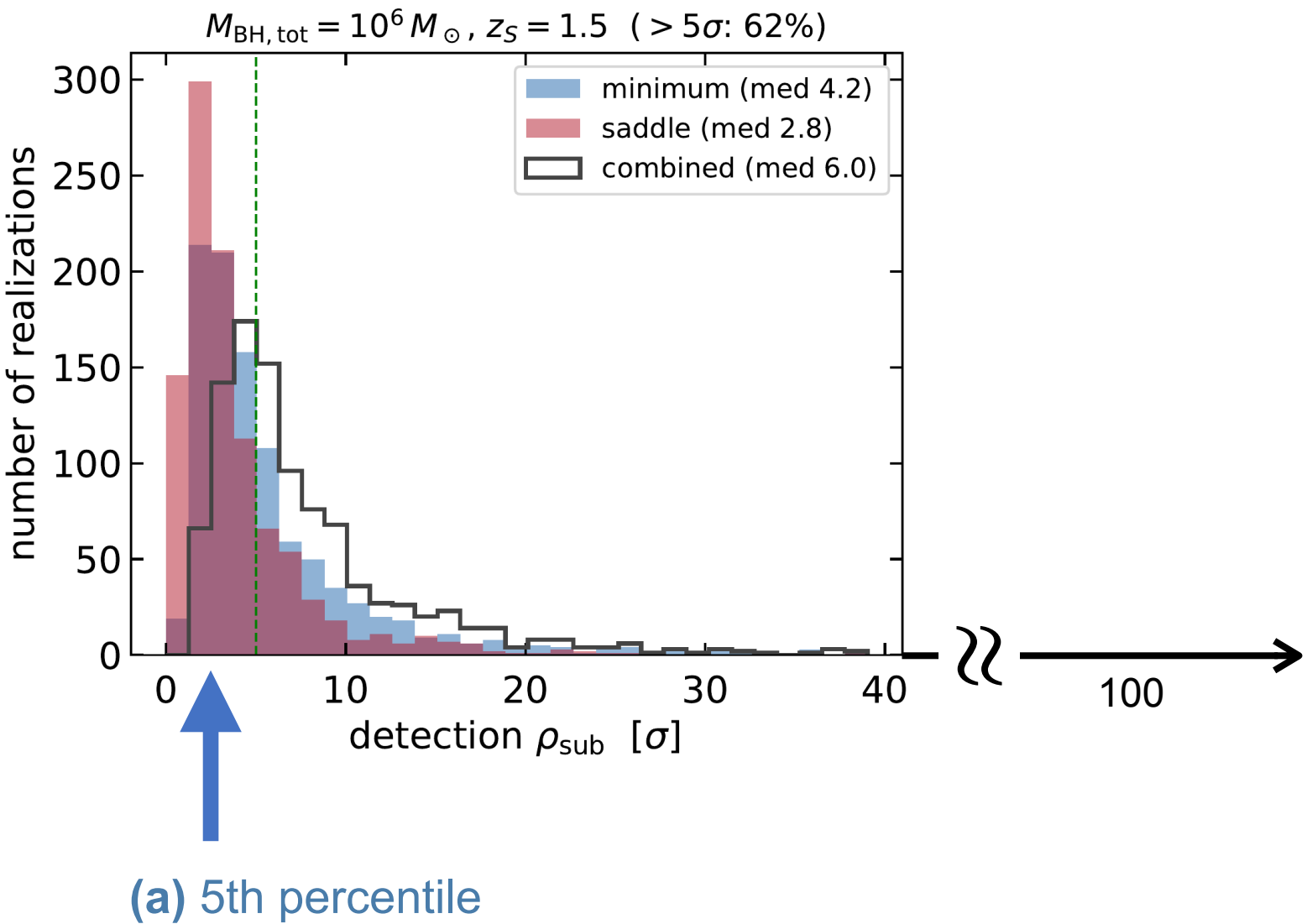
A few closest subhalos dominate the detection

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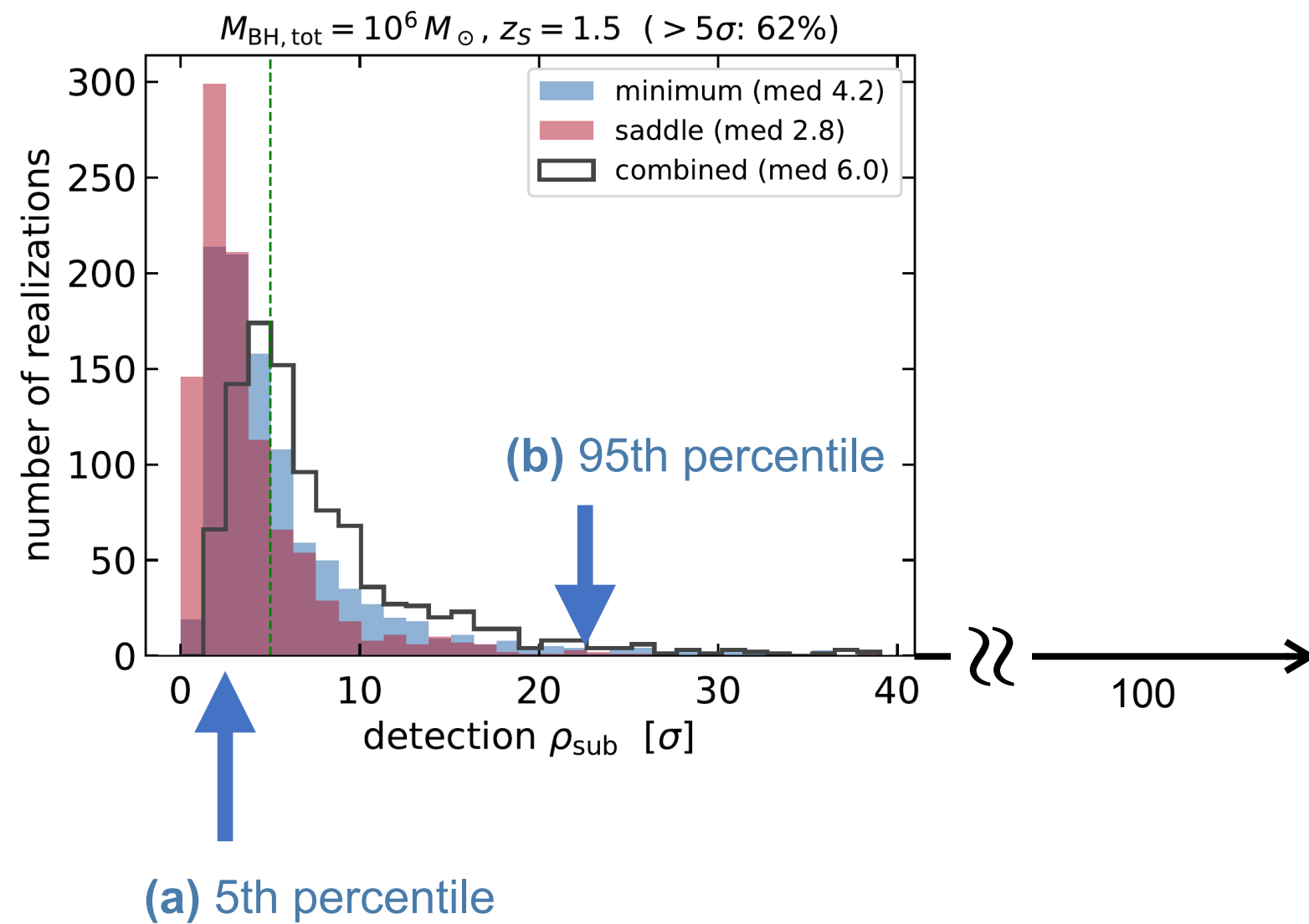


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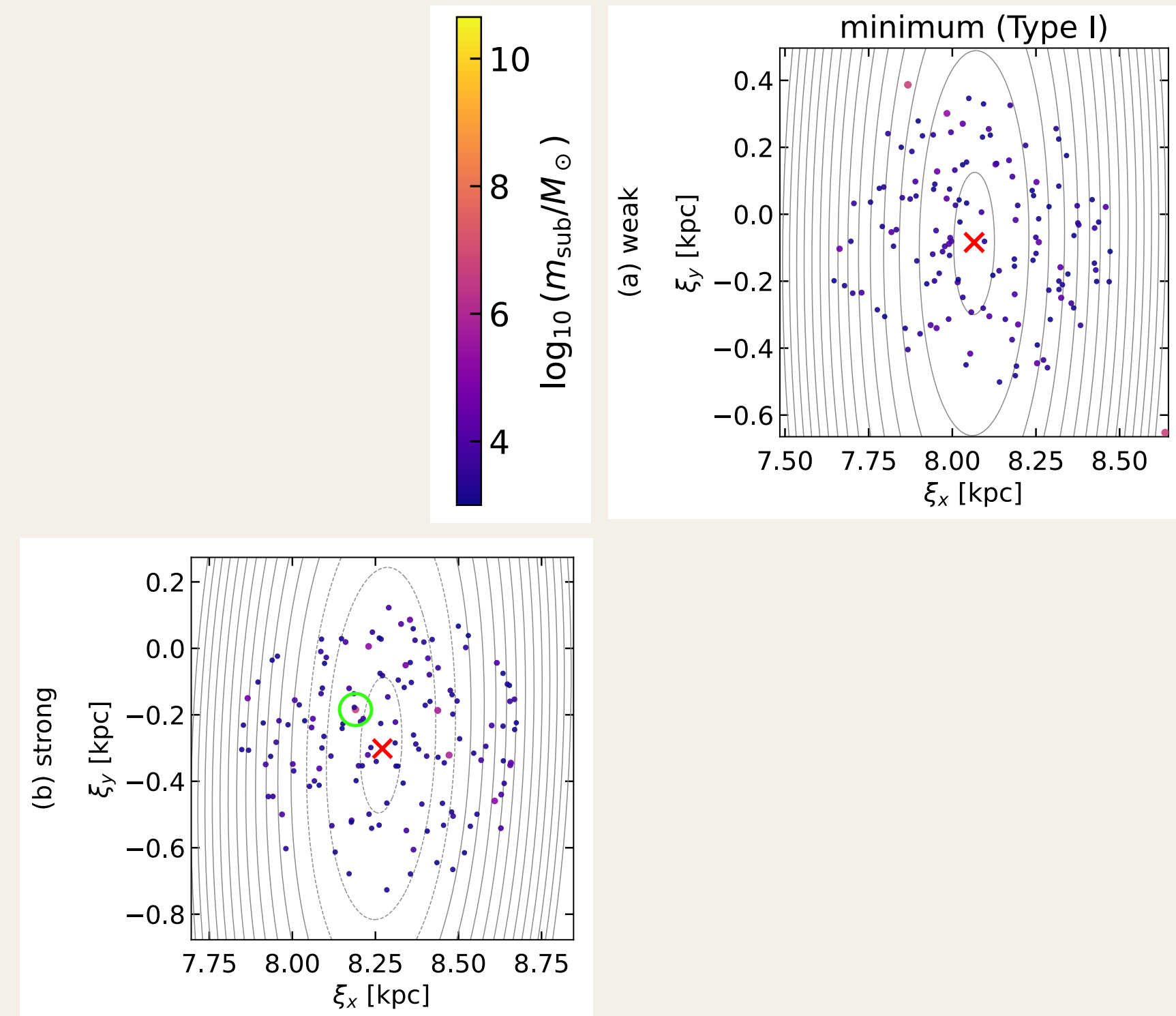
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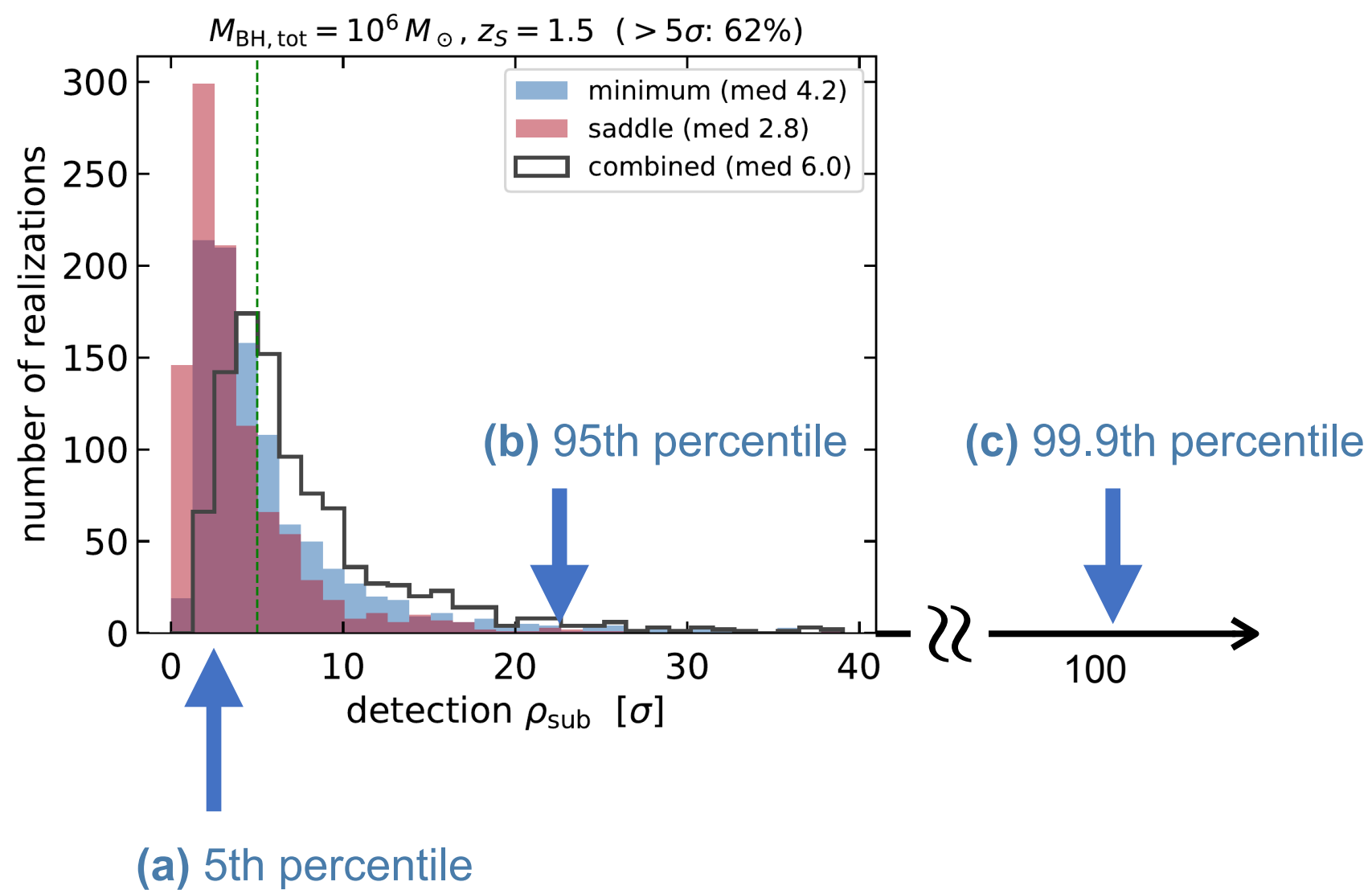
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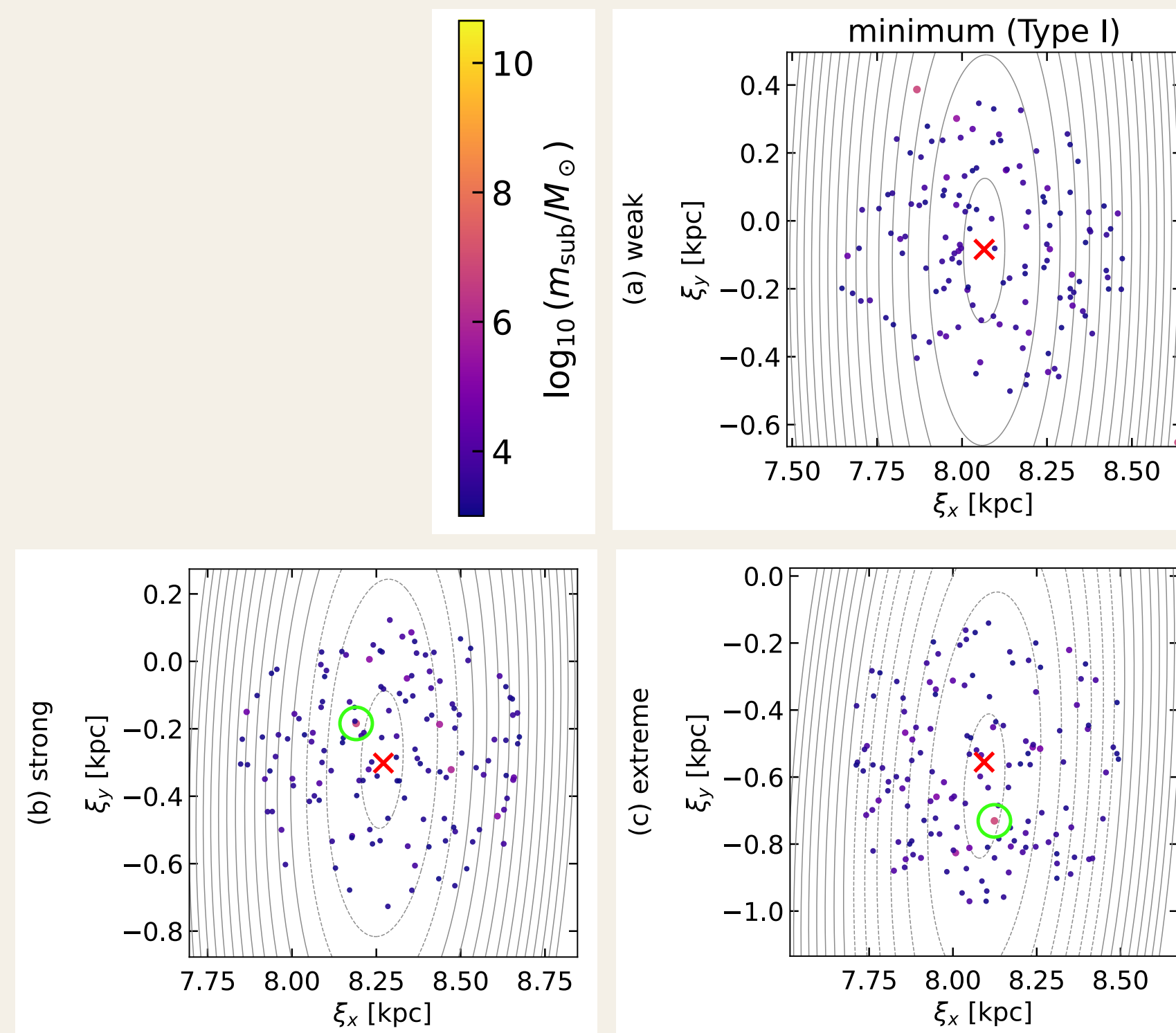
- Long tail dominated by a few closest subhalos (leave-one-out: $\sim 90\%$)



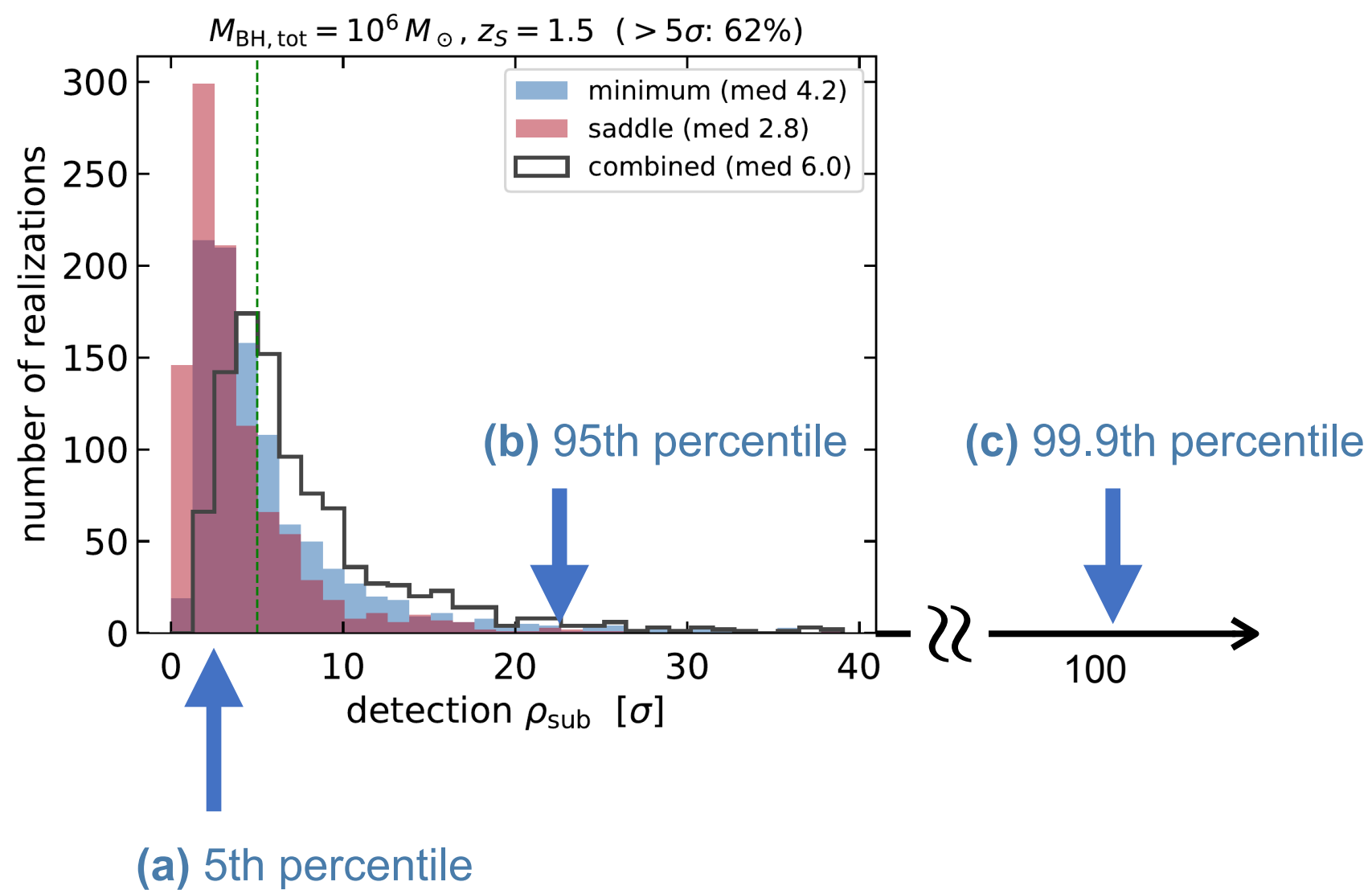
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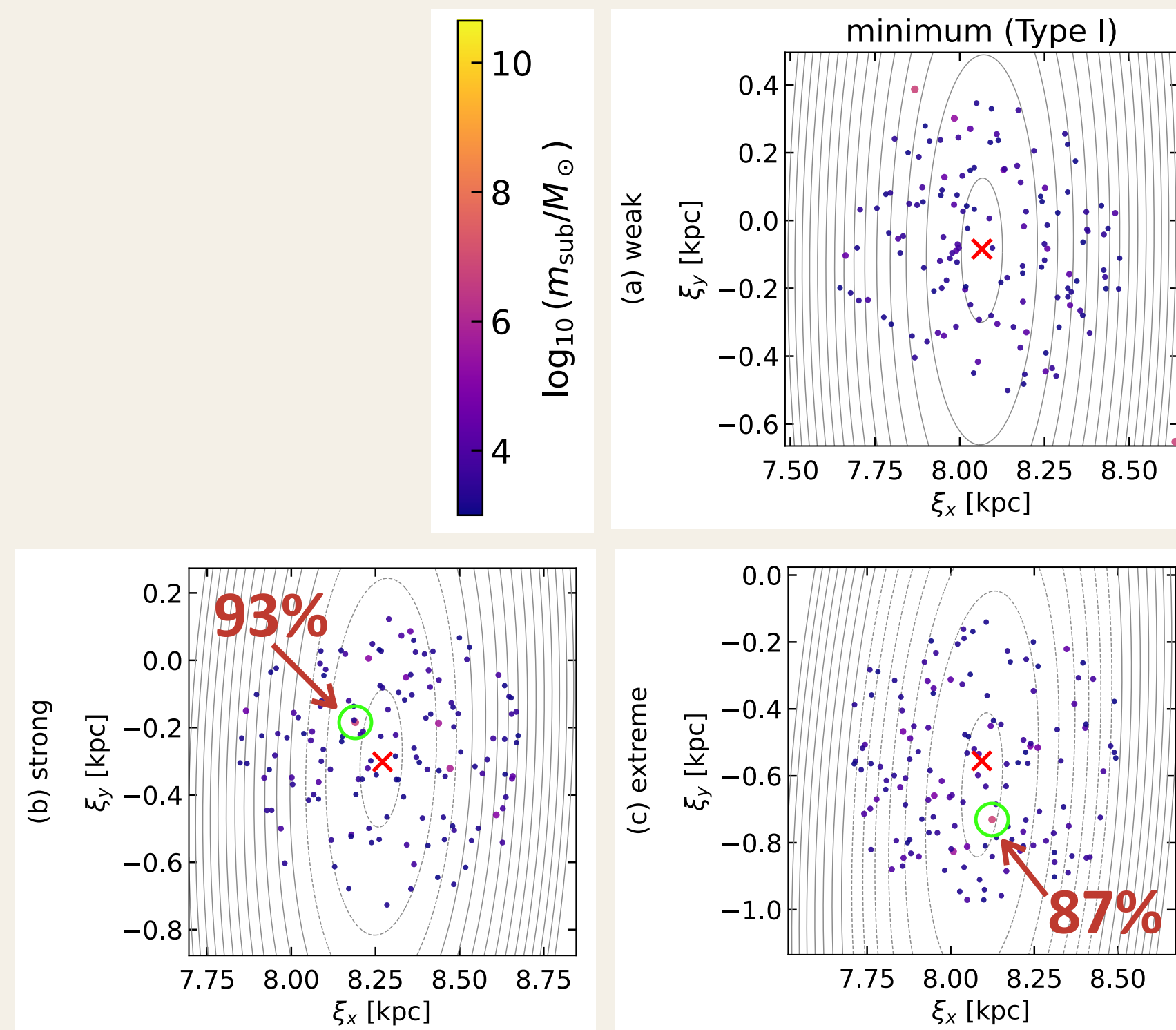
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A few closest subhalos dominate the detection

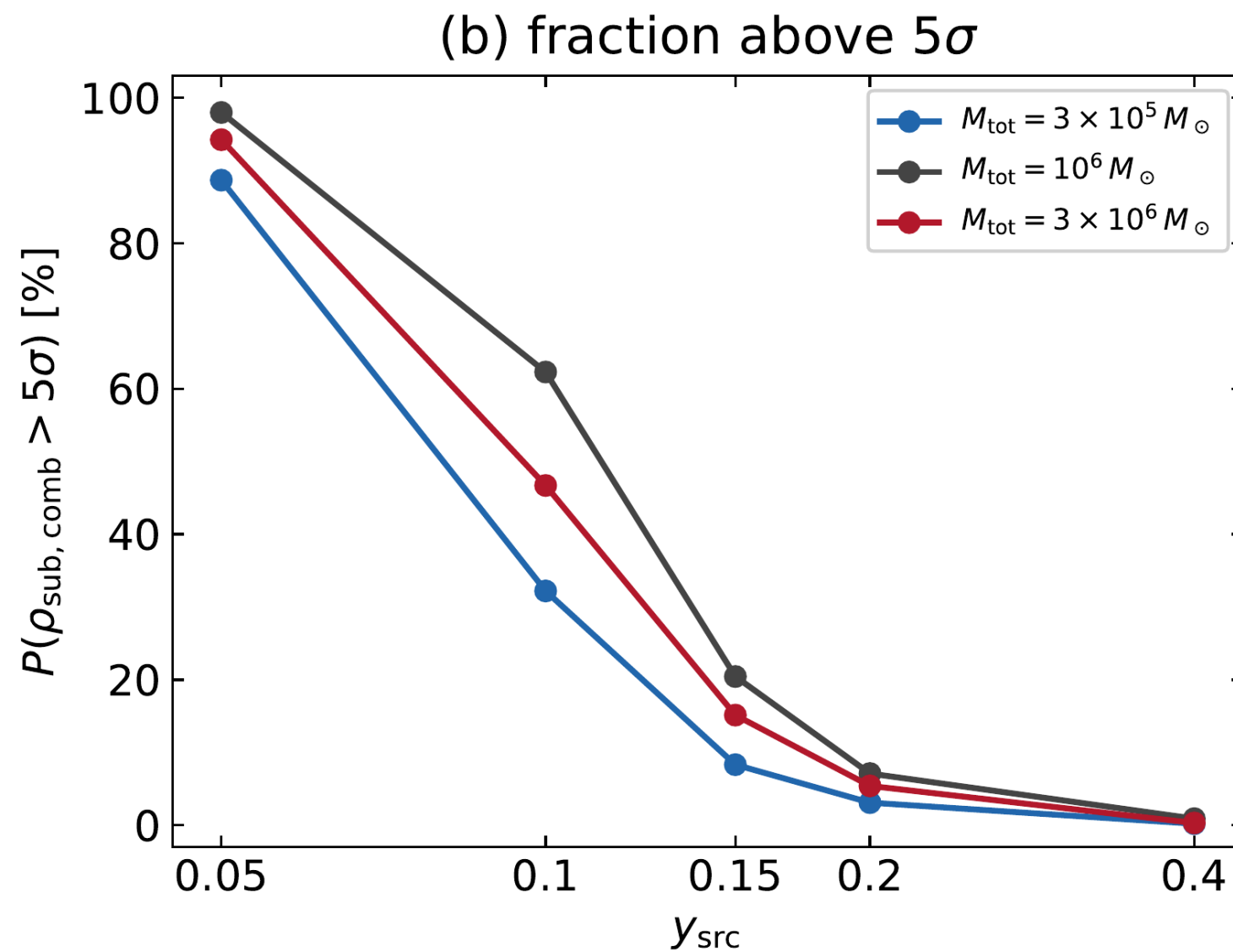
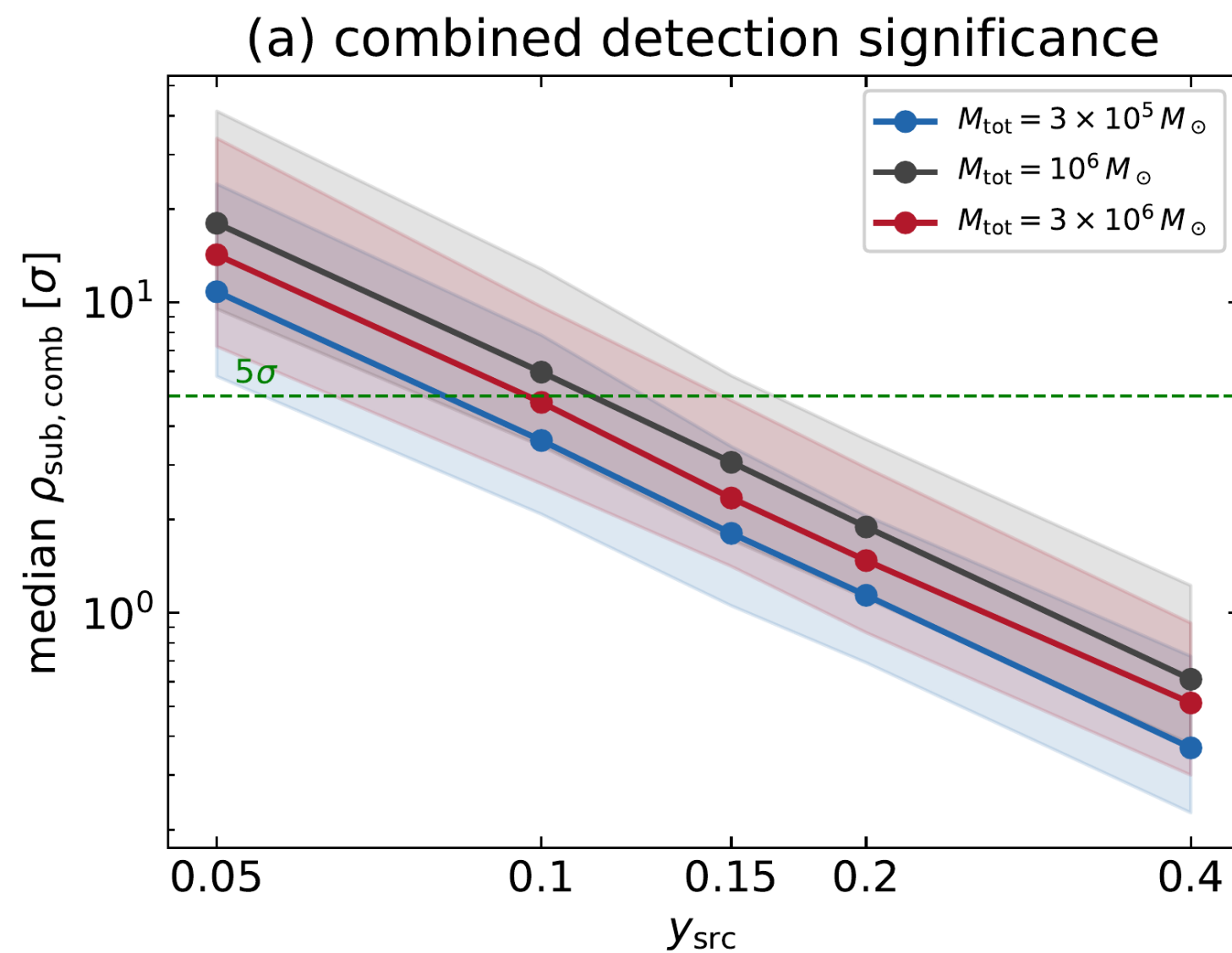


- Long tail dominated by a few closest subhalos (leave-one-out: $\sim 90\%$)

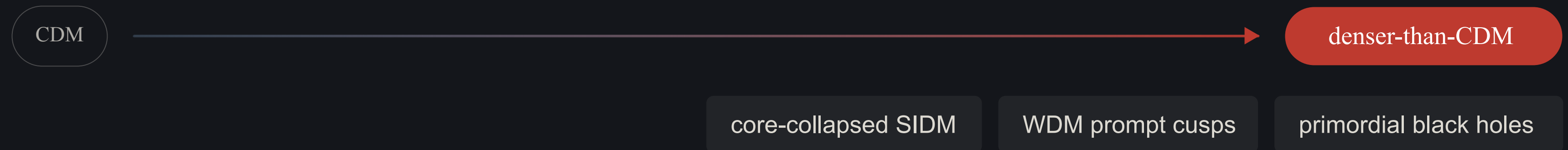


Yield off the caustic and over LISA

- Significance falls steeply off the caustic: $\rho \propto y_{\text{src}}^{-1.6}$
- Over LISA: **~10–20** (optimistic) to **<1** (conservative) detections
- Robust detection needs $y_{\text{src}} \lesssim 0.1$ (near caustic)
- Best stated **per event**: one loud near-caustic event already informs



What this probes: compactness is the dial



A new window on $10^4 - 10^7 M_\odot$
— beyond electromagnetic reach.

Outlook

01 Population forecast

rate $\times$ source distribution $\rightarrow$ expected number of detections

02 Fully marginalized Fisher analysis

03 Beyond CDM: WDM / SIDM / FDM/ PBH

SASHIMI-W and SASHIMI-SIDM ready; Thakore & Ando in prep (WDM); SASHIMI-F in preparation

- 1 GW wave-optics lensing records the gravity of **starless $10^4 - 10^7 M_\odot$ subhalos** as a frequency-dependent imprint.
- 2 Strong lensing amplifies it to **%-level on both images** — minimum & saddle; the saddle is now computable.
- 3 One loud near-caustic LISA event: **$>5\sigma$ in 62%** . Most sensitive to compact, non-CDM structure.

No model stretching: standard CDM / NFW profiles — a conservative estimate.

Could baryons do this?

stars	Stellar-mass microlenses — far too light $w_M = 8\pi G(1+z_L)Mf/c^3 = 6\times 10^{-7}$ at 1 $M_\odot$, 3 mHz $\rightarrow F - 1 \approx 4\times 10^{-7}$. Appreciable diffraction needs $w_M \gtrsim 1$, i.e. $\sim 2\times 10^6 M_\odot$.
HI clouds	High-velocity clouds — too extended $\Sigma \sim 0.1\text{--}1 M_\odot \text{ pc}^{-2}$ against $\Sigma_{\text{cr}} = 2.3\times 10^3 \rightarrow \kappa \lesssim 10^{-4}$. Spread over 1–15 kpc, two decades beyond $R_F \approx 90 \text{ pc}$: a smooth sheet, no frequency dependence.
GMCs	Giant molecular clouds — effective, but not where the image is $\kappa = 0.04\text{--}0.09$ (10–100× a subhalo), sizes 10–60 pc < R_F — they do diffract. But ~90% of strong lenses are early-type galaxies: only ~20% are CO-detected, and the gas that exists reaches ~1 kpc (at most 3.4 kpc). The image sits at 8 kpc.
GCS	Globular clusters — compact enough, but too rare $r_h \sim \text{few pc}$, $\kappa \sim O(1)$. But $\Sigma_{\text{GC}} < 0.2 \text{ kpc}^{-2}$ at the 8 kpc image radius against 3.7 kpc^{-2} for mass-matched subhalos $\rightarrow$ fewer than 0.1 clusters per image.

Caveat: **gas-rich disk deflectors** — a minority — would put molecular clouds at the image radius. The statements above are for the dominant early-type lenses.

Elliptical lenses: folds and cusps

setup

Our configuration is the axisymmetric counterpart of a fold

A circular lens maps the tangential critical curve to a single point; an elliptical lens opens it into an astroid with four cusps joined by fold arcs. Source inside → four images.

mechanism

The amplification is local, so it carries over unchanged

At a fold or a cusp, exactly one eigenvalue of A vanishes — the same local structure as our near-critical minimum. Subhalo perturbations are amplified through $\|A^{-1}\|$ exactly as they are here.

rate

Ellipticity should strengthen, not weaken, the effect

A caustic curve of finite length puts a far larger fraction of the source plane close to some caustic than a single point does. Near-critical images become **more common**.

Not covered: **very close to a fold** the two merging images sit closer than the diffractive scale and cannot be treated separately. That needs a joint diffraction integral over both — left to future work.

Robustness: three tests

sampling

Doubling the sampling radius R_{near}

Median subhalos in the diffraction integral 1177 $\rightarrow$ 4703. Distribution indistinguishable (KS D = 0.027, p = 0.98). The achromatic offset does move, 1.27% $\rightarrow$ 1.56% (D = 0.10, p = 0.005): the extra perturbers act as a smooth sheet.

macro
model

Seven one-parameter variants, 1000 realizations each

Host mass ± 0.5 dex, $\sigma_v = 200 \text{ km s}^{-1}$, $z_L = 0.3\text{--}0.8$, $z_S = 1.0\text{--}2.5$. Median of amplitude modulation stays within a factor of two across a decade in host mass.

truncation

Exact macro potential instead of the quadratic frame

Recomputed realization by realization (n = 993). The ratio of amplitude modulation between frames has median **1.00** (0.93–1.12), the shape correlation is **0.9995**, and the peak frequency is distributed indistinguishably (KS D = 0.02, p = 1.00).

The scale of the effect is a property of **near-critical strong lensing by a galaxy-scale halo**, not of one particular macro model.