Calculation of scalar condensation decay in two different approaches

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Introduction

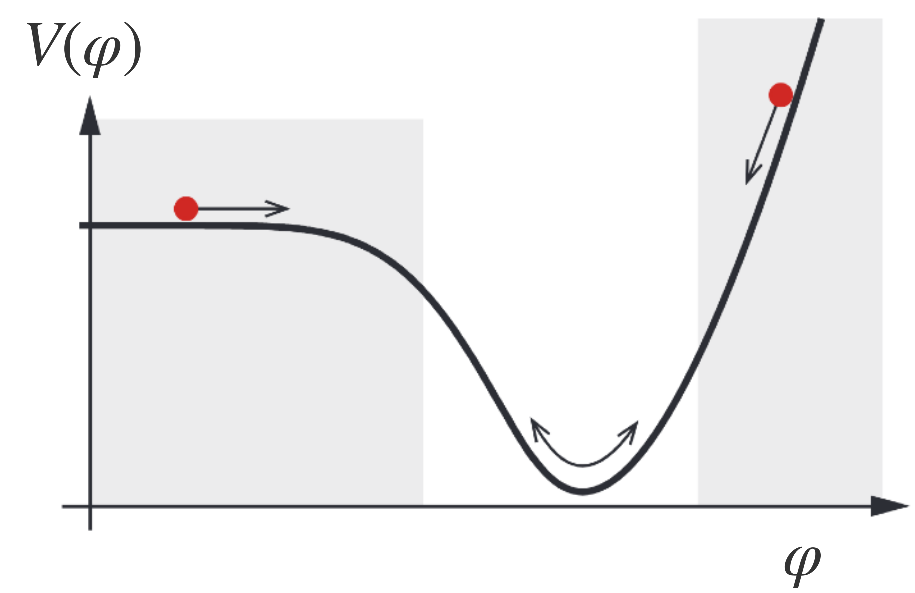
- Scalar condensation: scalar bosons exist in the same quantum state such that it behave coherently in whole spacetime.

    Example: Inflaton, Axion, $\cdots$

    Similar phenomena: Bose-Einstein condensation
    a state such that a macroscopic number of bosons condense into a grand state in low temperature.

- It is described by the macroscopic wave function.

[From D. baumann,1807.03098]



- After inflation, scalar condensation (infraton) decay plays an important role in the thermal history of the Universe.

- Due to the decay, catastrophic particle productions can happen (parametric resonance.)

Parametric resonance [1/2]

- Toy model

$$\mathcal{L} = \frac{1}{2}(\partial\varphi)^2 + \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}m_\varphi^2\varphi^2 - \frac{1}{2}m_\chi^2\chi^2 - \frac{1}{2}\mu\varphi\chi^2$$

φ: Parent particle $\quad \varphi = A_\varphi \sin(m_\varphi t)\,.$

χ: Douter particle

- Equation of motion (Matheiu equation) $\quad \chi_k$: Mode function of χ

$$\frac{d^2}{dt^2}\chi_k(t) + E_k^2(t)\chi_k(t) = 0$$

[L. Kofman, A. D. Linde, A. A. Starobinsky,PRD56, 3258(1997)]
[Yoshimura, PTP, 1995]

$$E_{\chi k}^2(t) = k^2 + m_{\chi\mathrm{eff}}^2(t) \quad , \; m_{\chi\mathrm{eff}} = m_\chi^2 + \mu A_\varphi \sin(m_\varphi t)$$

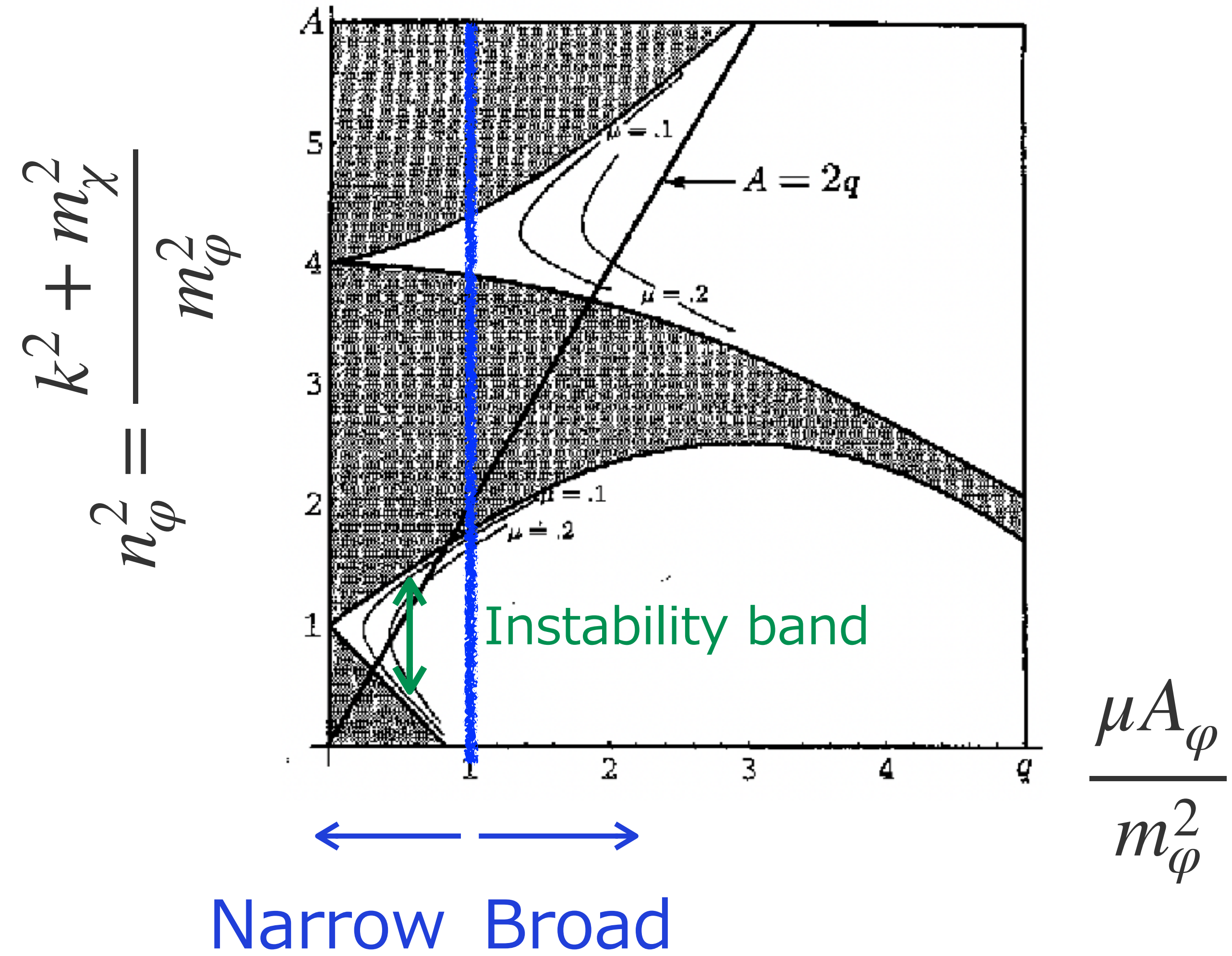
- Hubble expansion rate is neglected.
- The effective mass $m_{\chi\mathrm{eff}}$ oscillates depending on time.

Parametric resonance [2/2]

- growing mode

$$\chi_k(t) \propto \exp(\lambda(k) m_\varphi t/2)$$

[L. Kofman, A. D. Linde, A. A. Starobinsky,PRD56, 3258(1997)]



- Instability bound

$$n_\varphi^2 = 1 : \varphi \to \chi\chi$$

$$n_\varphi^2 = 4 : \varphi\varphi \to \chi\chi$$

$$\vdots$$

- Narrow resonance regime: ← Our focus

$$\mu A_\varphi/(m_\varphi^2) \lesssim 1$$

We evaluate the scalar condensation decay rate :

$$\Gamma_\varphi = \Gamma_\varphi^{n_\varphi=1} + \Gamma_\varphi^{n_\varphi=2} + \dots$$

Open question

There are distinct ways to evaluate the scalar condensation decay.

- 1.Feynman diagramatic approach — Evaluation of decay amplitude of scalar condendation
- 2. Parametric resonance approach — Track the evolution of the mode function of the daughter particle

Do they give the same result?

- Whether these approaches are consistent or not is nontrivial.
- Considering the model with a trilinear interaction, we clarified it.

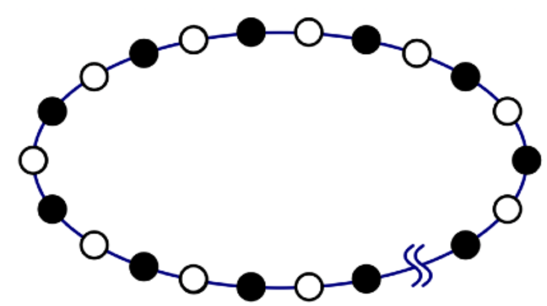
1. Feynman diagramatic approach

[T. Moroi, S. Matsumoto,PRD77 (2008)]
[A. Kamada, KS, JHEP 04 (2026) 003]

- Vacuum-to-vacuum transition amplitude can be evaluated the effective action:

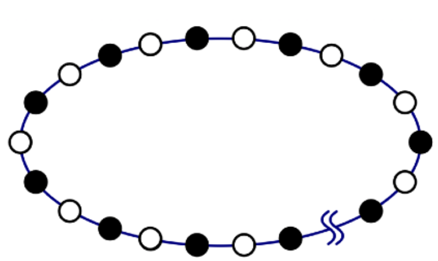
$$ {}_{\mathrm{out}}\langle 0|0\rangle_{\mathrm{in}} = \exp\left(iS_{\mathrm{eff}}[\varphi]\right) = \mathcal{D}\chi \exp\left(i\int S[\varphi,\chi]\right) $$

- S_{eff} is given by the sum of the bubble diagrams:

$\exp\left(iS_{\mathrm{eff}}[\varphi]\right) \ni \cdots +$  $+ \cdots$

$\varphi^-(x)$: ingoing φ
$\varphi^+(x)$: outgoing φ

- Im(bubble diagrams) gives the decay rate of φ.

$\Gamma_\varphi \sim$ Im[$\cdots +$  $+ \cdots$]

$$ \left|{}_{\mathrm{out}}\langle 0|0\rangle_{\mathrm{in}}\right|^2 \propto \exp\left(-\Gamma L^3 T\right), $$

2. Parametric resonance approach

[Yoshimura, PTP, 1995]

- Vacumme to vaccum transtion amplitude:

$$|{}_{\rm out}\langle 0|0\rangle_{\rm in}|^2 = \prod_k \frac{1}{\sqrt{1+n_k}}$$

$$n_k \sim \exp(\lambda m_\varphi t)$$

$$\Rightarrow \quad |{}_{\rm out}\langle 0|0\rangle_{\rm in}|^2 \simeq \exp\left(-\frac{1}{2}\sum_k \log n_k\right)$$

$$\propto \exp\left(-\frac{m_\varphi}{2}\int\frac{dk^3}{(2\pi)^3}\lambda V t\right) \equiv \exp\left(-\Gamma_\varphi V t\right)$$

$$\Rightarrow \quad \Gamma_\varphi = \frac{m_\varphi}{2}\int\frac{dk^3}{(2\pi)^3}\lambda_k$$

Expanstions parameters

- The decay rate can be schematically given by

$$\Gamma \ni A\beta^{1-2q}\theta^{2p} \qquad q, p : \text{positive integer}, p \geq 1, q \geq 0,$$

$$\theta = \frac{\mu A_\varphi}{m_\varphi^2}$$ Small amplitude: $\theta \ll 1$

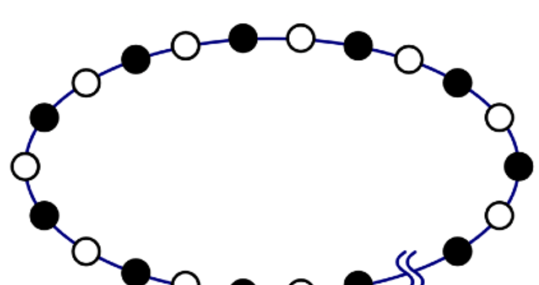
$$\beta = \sqrt{1 - 4m_\chi^2/(n_\varphi^2 m_\varphi^2)}$$ Small velocity: $\beta \ll 1$

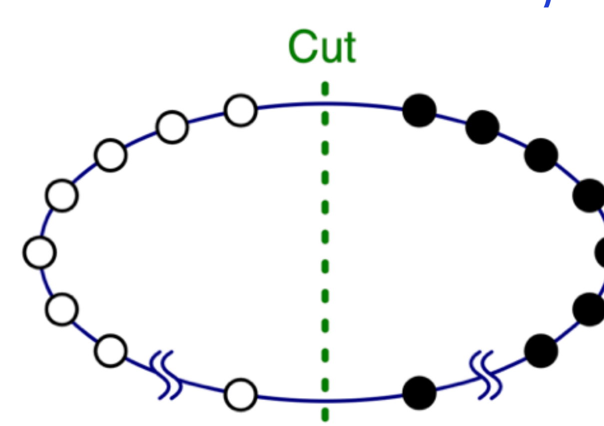
- For given p, the maximum of q is fixed.

$$\Gamma = \ldots + \beta\theta^{2p}\left[\underbrace{\frac{A_{q_{\max}}}{\beta^{2q_{\max}}}} + \frac{A_{q_{\max}-1}}{\beta^{2(q_{\max}-1)}} + \ldots + \underbrace{A_0}\right] + \ldots$$

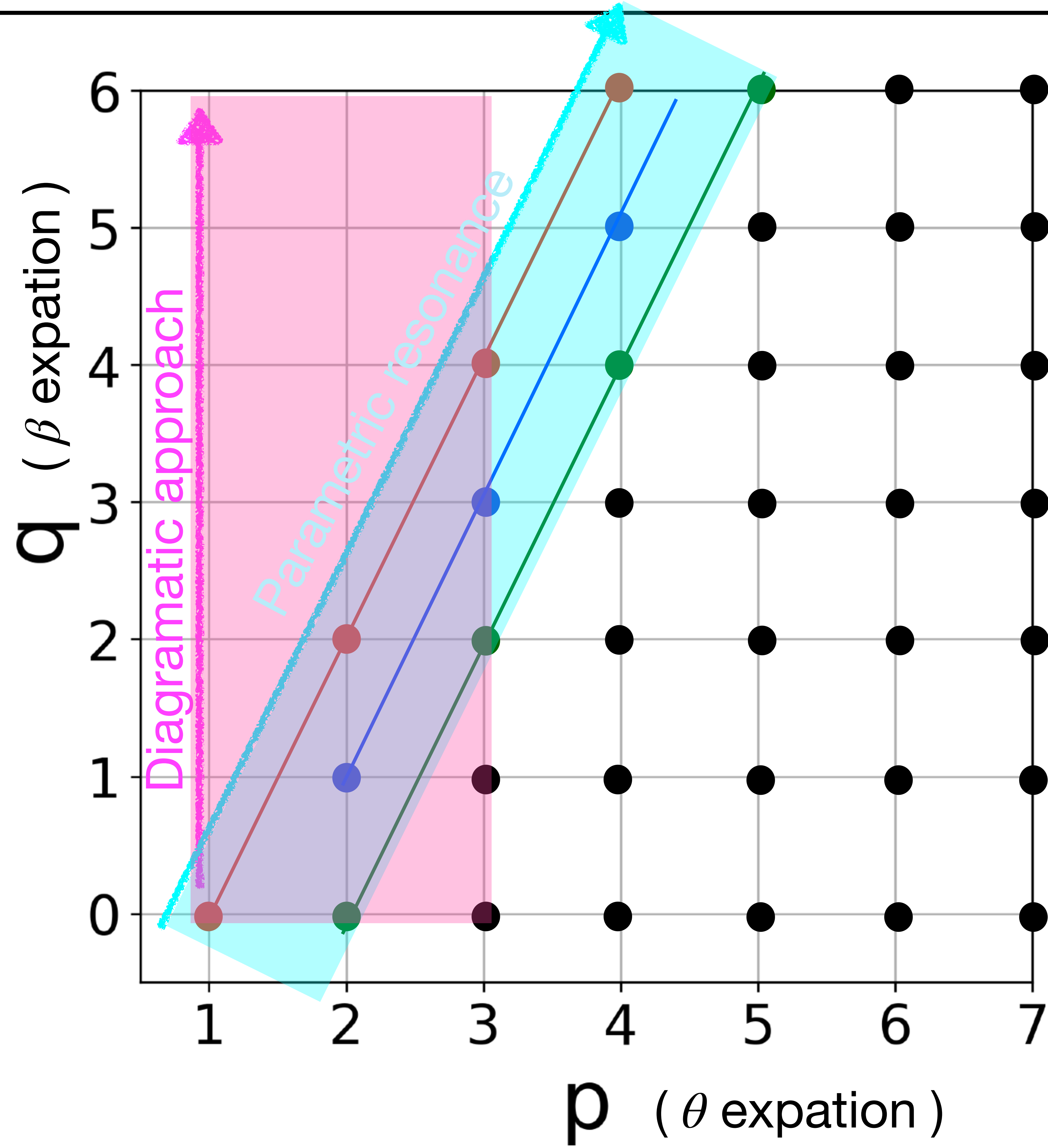
Dominant contributions in β expansion

Higheset order contributions in β expansion

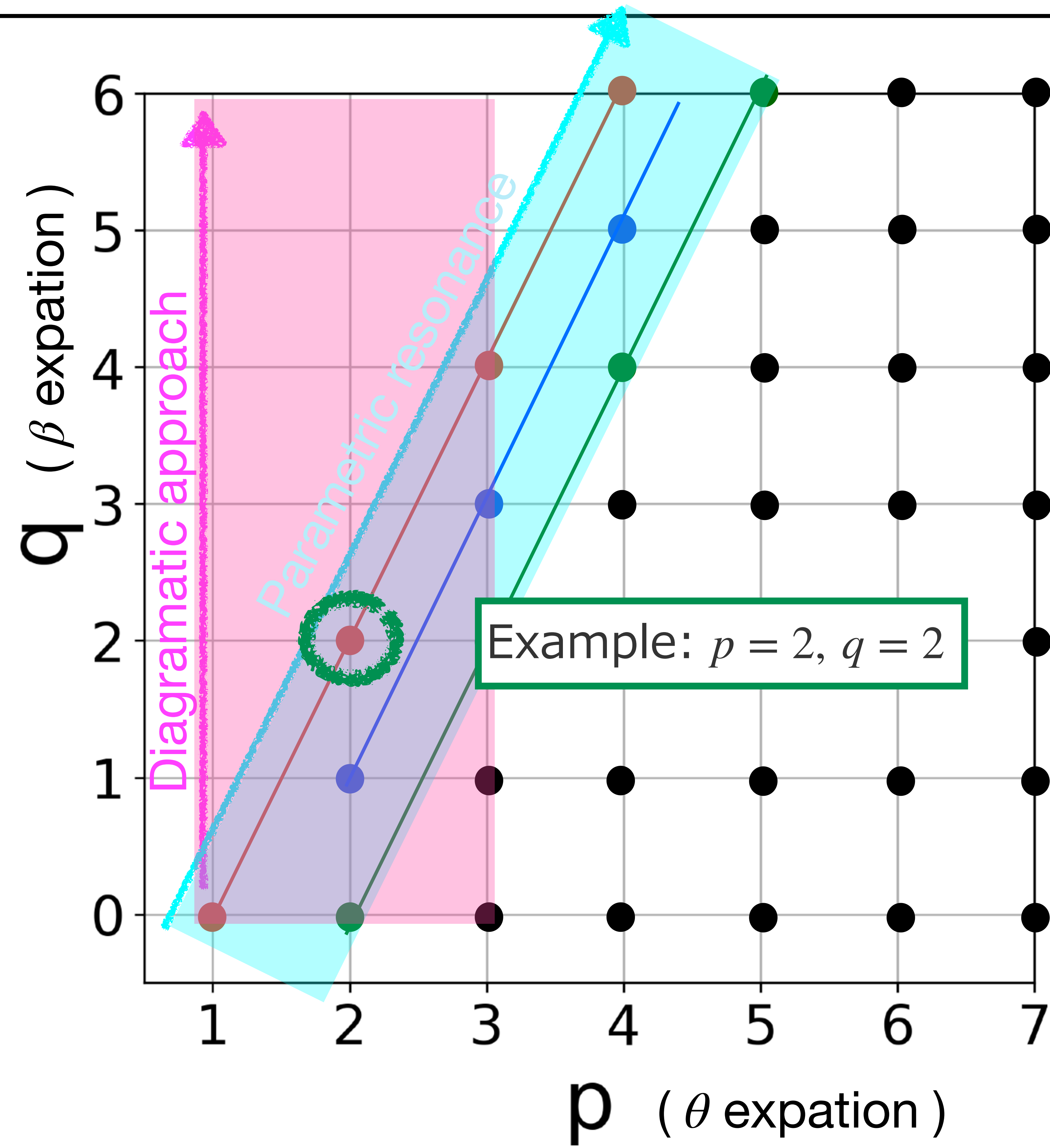




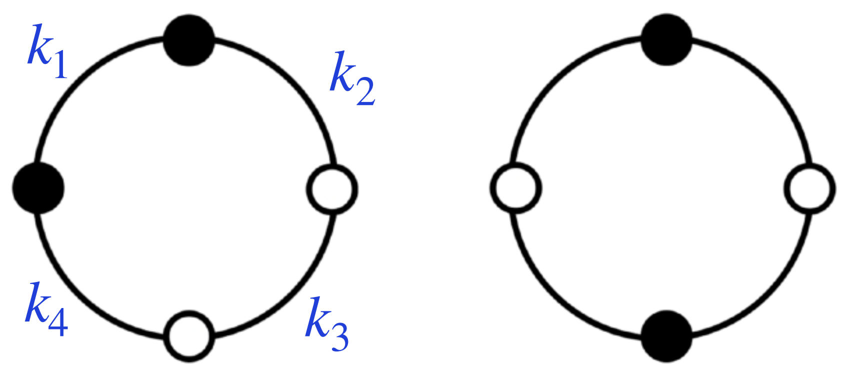
Schematic picture of expanstions in each way



Schematic picture of expanstions in each way



Example:p=2, $n_\varphi = 1$ [Feynman diagramatic approach]

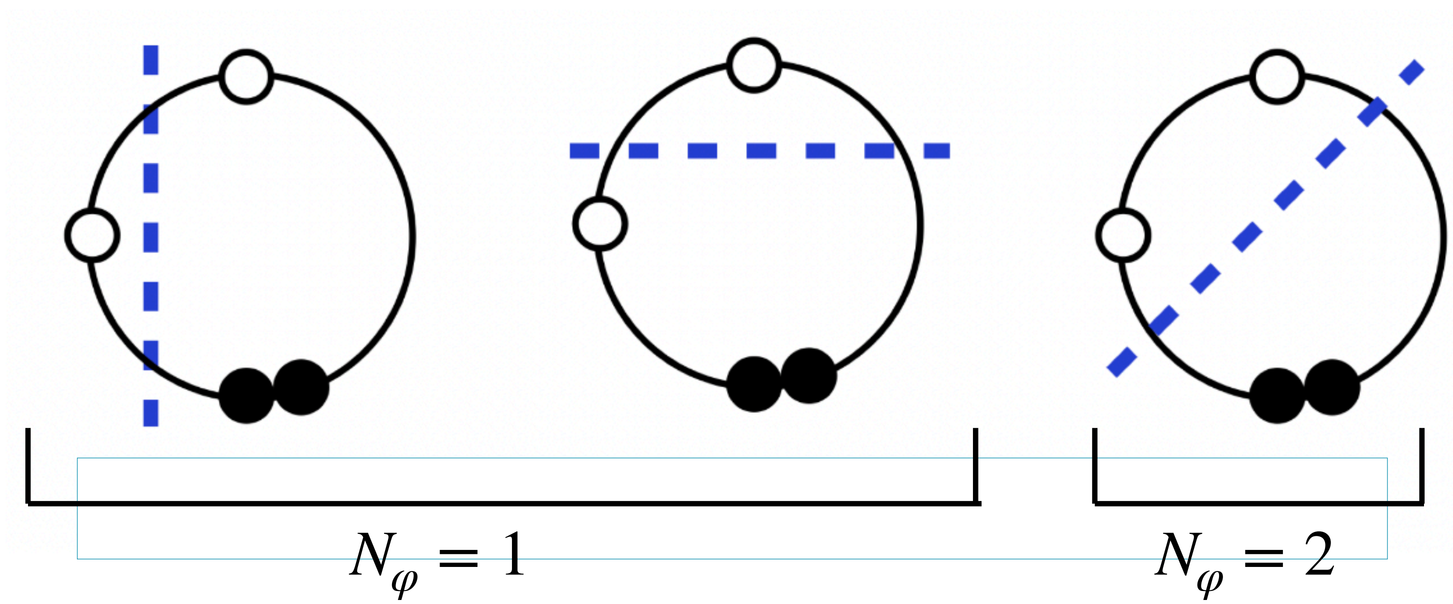


(1) (2)

For diagram1: $k_1 = k_3$

For diagram2: $k_1 = k_3 \quad k_2 = k_4$

- Procedure to evaluate bubble diagrams



①"Pinch" diagrams: $(k^2 - m_\chi^2)^{-2} \to -\lim_{\xi \to m_\chi^2} \frac{\partial}{\partial \xi}(k^2 - \xi)^{-1}$

②Apply cutting rules: $(k'^2 - m_\chi^2)^{-1} \to \delta(k'^2 - m_\chi^2)$

$$\Gamma_\varphi = -\frac{m_\varphi^4}{16\pi}\left(\frac{\mu A_\varphi}{2m_\varphi^2}\right)^4 \times \left[\frac{1}{2\beta_1^3} + \frac{3}{2\beta_1} + \beta_1\right]$$

Example:p=2, $n_\varphi = 1$ [Parametric resonance apprroach]

$$\Gamma^{N_\varphi=1}_{\beta_1\to 0} = \frac{m_\varphi^4}{128\pi^2}\int_{-\theta}^{\theta} \underbrace{d\epsilon}_{\sim\mathcal{O}(\theta)} \underbrace{\sqrt{(\beta_1^2+\epsilon)}}_{} \underbrace{\sqrt{(\theta^2-\epsilon^2)}}_{\sim\mathcal{O}(\theta)}$$

$$\lambda = \frac{1}{2}\sqrt{\theta^2-\epsilon^2}$$

$$-\theta < \epsilon < \theta$$

$$= \beta_1\left[1 - \frac{1}{8}\left(\frac{\epsilon}{\beta^2}\right)^2 - \frac{5}{128}\left(\frac{\epsilon}{\beta^2}\right)^4 \ldots\right]$$

The order of β and that of θ is related:

$q = 2(p-2)$ [for $\beta^{1-2q}\theta^{2p}$]

$$= -\frac{m_\varphi^4}{16\pi}\sum_{p=1}^{\infty}\beta^{5-4p}\left(\frac{\mu A_\varphi}{2m_\varphi^2}\right)^{2p}\frac{(4p-7)!!}{(p-1)!p!}$$

$$= -\frac{m_\varphi^4}{16\pi}\left[-\beta_1\left(\frac{\mu A_\varphi}{2m_\varphi^2}\right)^2 + \frac{1}{2\beta_1^3}\left(\frac{\mu A_\varphi}{2m_\varphi^2}\right)^4 + \frac{5}{4\beta_1^7}\left(\frac{\mu A_\varphi}{2m_\varphi^2}\right)^6 + \ldots\right]$$

$\Leftrightarrow$ (QFT result): $\Gamma^{(N_\varphi=1)} = -\frac{m_\varphi^4}{16\pi}\left(\frac{\mu A_\varphi}{2m_\varphi^2}\right)^4\left[\frac{1}{2\beta_1^3} + \frac{3}{2\beta_1} + \beta_1\right]$

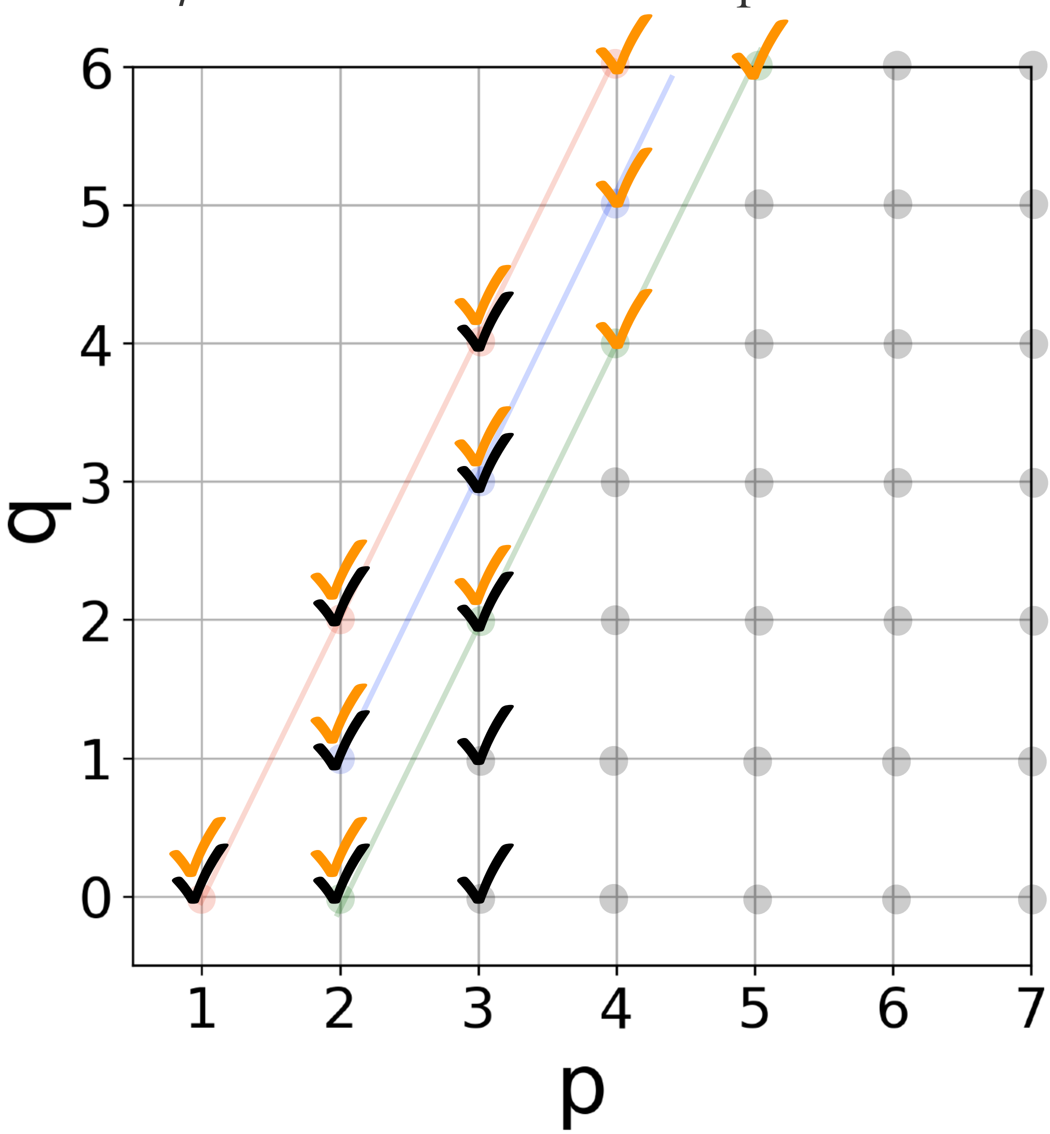
Equivarent!

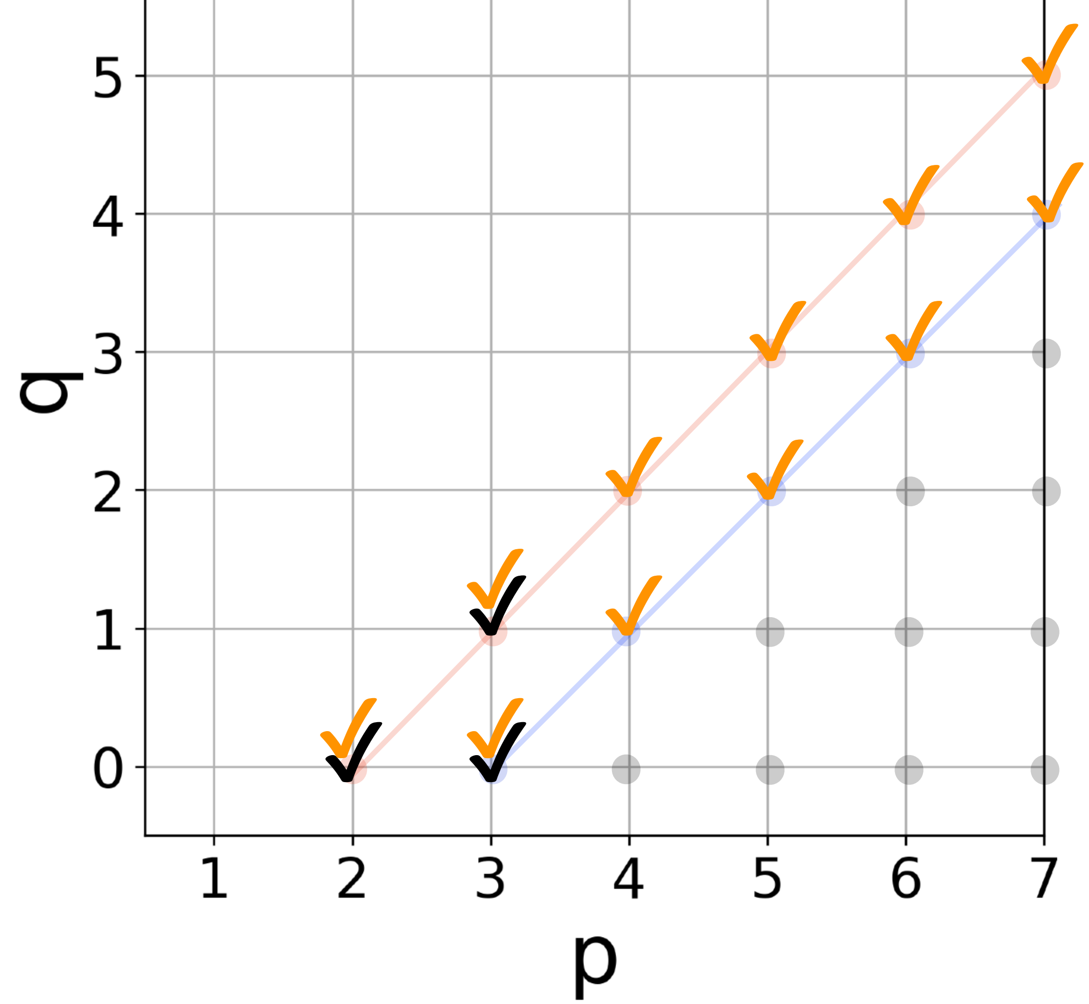
Consistency between two approaches

✓: computed by QFT approach
✓: computed by parametric resonance

$n_\varphi = 1$ case $(\Gamma \ni \beta_1^{1-2q}\theta^{2p})$

$n_\varphi = 2$ case $(\Gamma \ni \beta_2^{1-2q}\theta^{2p})$





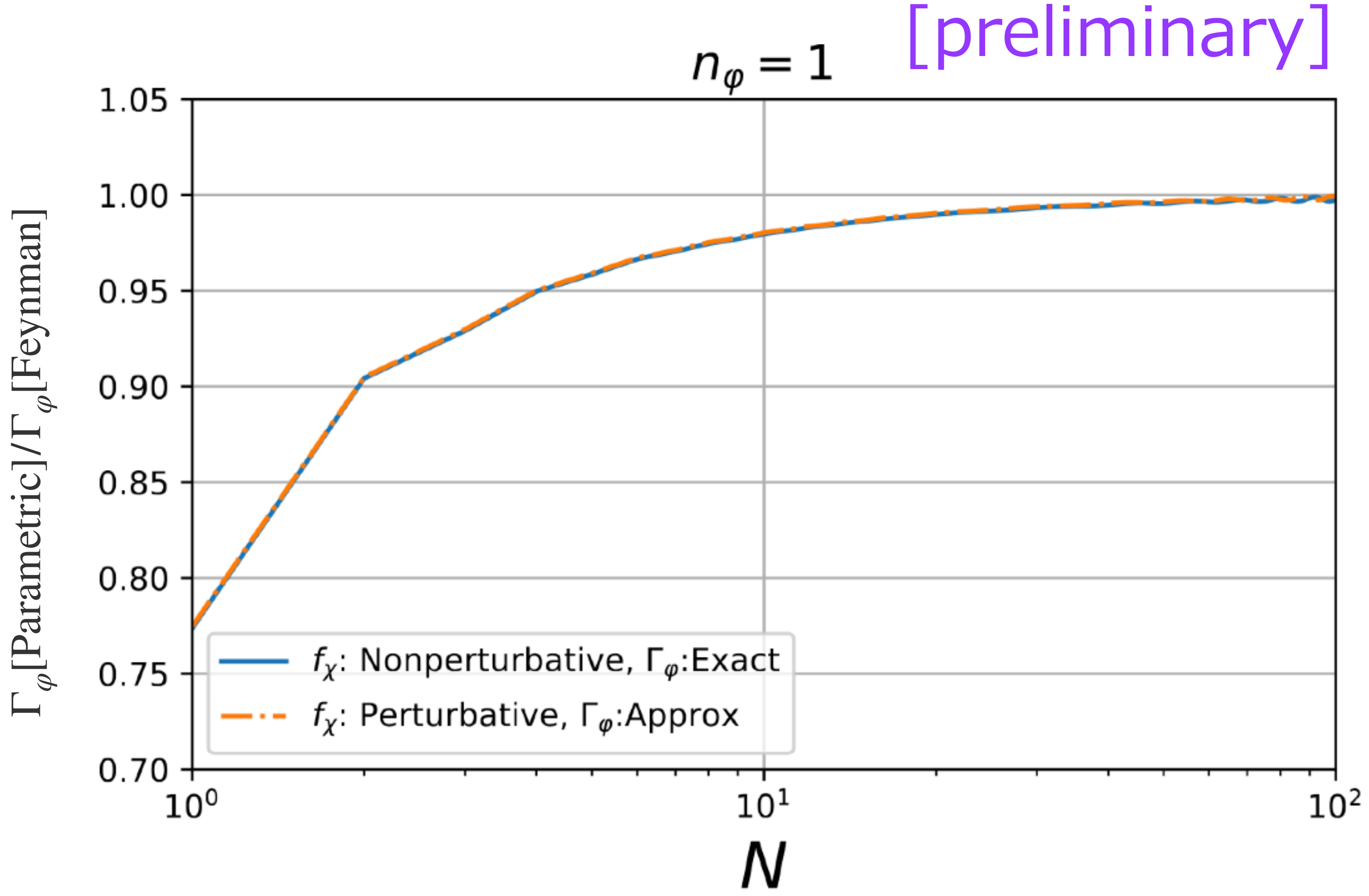
We checked consistenny in between these two appaoaches for overlapped points.

Time evolution of scalar condensation decay

- **Feynman-diagrammatic:** initial and final states are specified as asymptotic states, and no explicit time evolution is involved in the evaluation of the scattering amplitude.

- **Parametric resonance:** we do not use a approximation of the growing mode. we follow time evolution of the distribution function in both perturbative and nonperturbative ways.

[preliminary]



- At N~100, the distribution function is much populated.

- At N~2, the distribution function is smaller than one.

Summary

- We discussed the evaluation of scalar condensation decays in two different ways.

- Calculations in the Feynman diagrammatic approach coincide with the Parametric resonance approaches. This is checked by including lower orders in the (p,q) plane.

- When N>2, two approaches are consistent within the error of ~10%.

Back up

Approach 1:Scalar condensation decays in QFT [1/3]

Set up

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\varphi)(\partial^\mu\varphi) + \frac{1}{2}(\partial_\mu\chi)(\partial^\mu\chi) - \frac{1}{2}m_\varphi^2\varphi^2 - \frac{1}{2}\mu\varphi\chi^2$$

φ : scalar condenstaion, χ : scalar field

Bacis ideas

- We assume that φ can be expressed as a coherent state.

$$|\varphi\rangle \equiv e^{-|C_\varphi|^2/2} e^{C_\varphi \hat{a}_{\mathbf{0}}^\dagger}|0\rangle,$$

Solution of clacical EOM of φ is produced:

$$\langle\varphi\,|\,\hat{\varphi}\,|\,\varphi\rangle = A\cos(m_\varphi t)$$

- From the unitarity of S-matrix, the decay rate of any particle can be calcurated by Im(T-matrix):

$$\mathrm{Prob}(|\varphi\rangle \to \mathrm{all}) = \sum_f \left|\langle f|\hat{\mathcal{T}}|\varphi\rangle\right|^2 = 2\Im\left[\langle\varphi|\hat{\mathcal{T}}|\varphi\rangle\right].$$

→From them, scalar condendation decay rate can be systematically computed.

Approach 1:Scalar condensation decays in QFT [2/3]

[T. Moroi, S. Matsumoto,PRD77 (2008)]

$$\langle \varphi | \hat{\mathcal{T}} | \varphi \rangle \equiv \sum_{p=1}^{\infty} \sum_{\mathcal{F}^{(2p)}} \mathcal{T}^{\mathcal{F}^{(2p)}} \longleftarrow$$

Feynman diagrams with 2p coupling insertions at 1 loop level.

$$\sum_{\mathcal{F}^{(2p)}} \mathcal{T}^{\mathcal{F}^{(2p)}} = \sum_{p=1}^{\infty} \frac{-i}{(2p)!} \left(\frac{i\mu}{2} \right)^{2p} \left\langle 0 \middle| T \left[\int d^4x \, \underbrace{\{\varphi_-(x) + \varphi_+(x)\}} \, \hat{\chi}(x) \hat{\chi}(x) \right]^{2p} \middle| 0 \right\rangle,$$

$$\langle \varphi | \hat{\varphi}(x) | \varphi \rangle = \varphi_-(x) + \varphi_+(x)$$

$\varphi^-(x)$: ingoing φ

$\varphi^+(x)$: outgoing φ

Approach 1:Scalar condensation decays in QFT [3/3]

[T. Moroi, S. Matsumoto,PRD77 (2008)]

$$\langle\varphi|\hat{\mathcal{T}}|\varphi\rangle \equiv \sum_{p=1}^{\infty}\sum_{\mathcal{F}^{(2p)}} \mathcal{T}^{\mathcal{F}^{(2p)}}$$

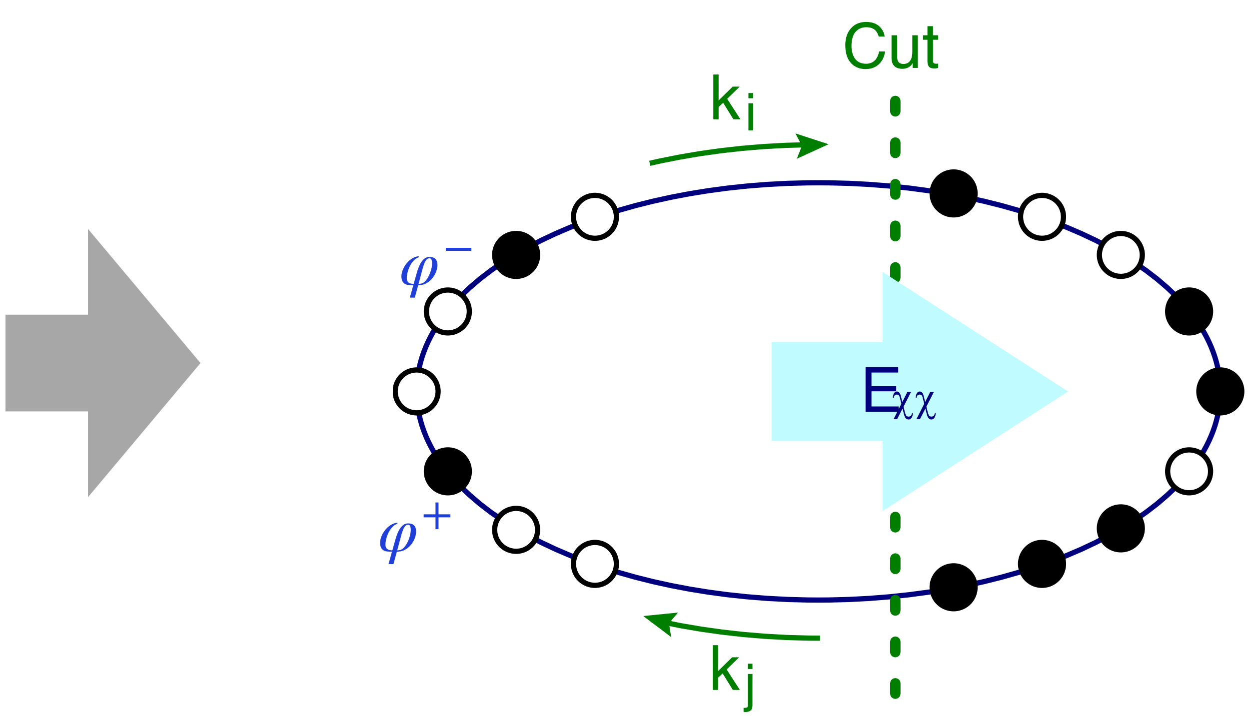
Feynman diagrams with 2p coupling insertions at 1 loop level.

$$\sum_{\mathcal{F}^{(2p)}} \mathcal{T}^{\mathcal{F}^{(2p)}} = \sum_{p=1}^{\infty} \frac{-i}{(2p)!}\left(\frac{i\mu}{2}\right)^{2p} \left\langle 0 \middle| T\left[\int d^4x \underbrace{\{\varphi_-(x)+\varphi_+(x)\}}\hat{\chi}(x)\hat{\chi}(x)\right]^{2p} \middle| 0\right\rangle,$$

$$\langle\varphi|\hat{\varphi}(x)|\varphi\rangle = \varphi_-(x)+\varphi_+(x)$$

$\varphi^-(x)$: ingoing φ

$\varphi^+(x)$: outgoing φ



Matrix element:

$$L^3 T S_{\mathcal{F}} \left|\frac{\mu A_\varphi}{2}\right|^{2p} \int \frac{d^4\tilde{k}}{(2\pi)^4} \prod_{I=1}^{2p} (k_I^2 - m_\chi^2 + i0^+)^{-2},$$

A_φ : amplitude of φ

The imaginary part is computed by applying cuts.

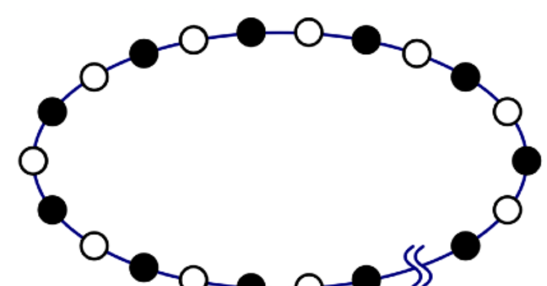
Effective action

[T. Moroi, S. Matsumoto,PRD77 (2008)]

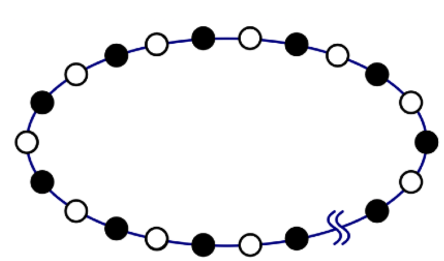
- Another way for the decay of the condensation is to evaluate the effective action:

$$\exp\left(iS_{\text{eff}}[\varphi]\right) = \exp\left(i\int \mathcal{D}\chi S_{\text{eff}}[\varphi,\chi]\right)$$

- S_{eff} is given by the sum of the bubble diagrams:

$\exp\left(iS_{\text{eff}}[\varphi]\right) \ni \cdots +$  $+ \cdots$

- Im(bubble diagrams) gives the decay rate of φ.

$\Gamma_\varphi \sim$ Im[$\cdots +$  $+ \cdots$]

Approach 2: Formulation of the parametric resonance [1/2]

- Schrödinger equation of wave functional Ψ :

$$i\frac{\partial}{\partial t}\Psi(q_k,t) = H\Psi(q_k,t), \qquad H(t) = \sum_k = \frac{1}{2}\pi_k^2 + \frac{1}{2}E_{\chi.k,\mathrm{eff}}^2(t)q_k^2$$

  - q_k corresponds to the special Fourier components of $\chi(\mathbf{x},t)$.

  - Ψ can be decomposed into the products of each mode (space translation invariance):

$$\Psi(q_k,t) = \prod_k \psi_k(q_k,t)$$

$$i\frac{\partial}{\partial t}\psi(q_k,t) = -\frac{1}{2}\psi(q_k,t) + \frac{1}{2}E_{\chi,\mathrm{eff}}^2\psi(q_k,t)$$

Approach 2: Formulation of the parametric resonance [2/2]

- Gaussian ansatz:

$$\psi(q_k, t) = \frac{1}{u_k} \exp\left(\frac{i}{2}\frac{\dot{u}_k}{u_k} q_k^2\right), \qquad \ddot{u}_k(t) + E_{\chi,\mathrm{k,eff}}(t)^2 u_k(t) = 0$$

- Initial conditions: $u_k(0) = \left(\frac{\pi}{E_{\chi,k,\mathrm{eff}}}\right)^{1/2}, \quad \frac{\dot{u}_k(0)}{u_k(0)} = iE_{\chi,k,\mathrm{eff}}(0)$ ➡ Gaussian solution for $\psi(q_k,0)$

- Decay amplitude: $|\langle \psi(q_k,0) | \psi(q_k, t)\rangle|^2 = \frac{1}{\sqrt{1 + n_k}}$ $\qquad n_k \sim \exp(\lambda m_\varphi t)$

➡ $$|\langle \Psi(q_k,0) | \Psi(q_k, t)\rangle|^2 \simeq \exp\left(-\frac{1}{2}\sum_k \log n_k\right)$$

$$\propto \exp\left(-\frac{m_\varphi}{2}\int \Pi_k \lambda V t\right) \equiv \exp(-\Gamma V t)$$

Approach 2: Parametric resonance [1/3]

Matheieu equation: ($z \equiv m_\varphi t/2$)

$$\frac{d^2 u_{\mathbf{k}}}{dz^2} + \{h - \theta \exp(-2iz) - \theta \exp(2iz)\} u_{\mathbf{k}} = 0$$

$$\hat{\chi} = \frac{1}{(2\pi)^{3/2}} \int dk^3 \left(u^*_{\mathbf{k}} \hat{a}_{\mathbf{k}} \exp(i\mathbf{k}\cdot\mathbf{x}) + u_{\mathbf{k}} \hat{a}_{\mathbf{k}} \exp(-i\mathbf{k}\cdot\mathbf{x}) \right)$$

$$h = 4\frac{k^2 + m_\chi^2}{m_\varphi^2}, \qquad \theta = \frac{\mu A_\varphi}{m_\varphi^2}$$

Floquet solution : $u = \exp(\lambda z) P(z)$

λ: growth factor

$P(Z)$:periodic function

- Parametric resonacne for Re(λ) >0
- Bloch wave for Re(λ) =0

The decay rate of φ can be also obtained by integrateging groth factor in the phase space.

Approach 2: Parametric resonance [2/3]

[M.Yoshimura,Prog. Theor. Phys.94 (1995)]

Calcuration of growth factor:

The periodic function $P(z)$ can be expressed by Fourier series

$$u = \exp(\lambda z) P(z),\ \ P(Z) = \sum_{k=-\infty}^{\infty} C_k(z) \exp(in + 2k) z$$

The coefficent C_k satisfies the recurrence relation,

$$\gamma_k C_k - \theta_+ C_{k+1} - \theta_- C_{k+1} = 0$$

$$\gamma_k = \left[\begin{array}{ll} -4k(n+k) + \mathcal{O}(\lambda, \epsilon) & (k \neq 0, -1) \\ \epsilon \pm 2in\lambda + \mathcal{O}(\lambda^2) & (k = 0, -1) \end{array} \right.$$

- We have applied small amplitude limit ($\theta \ll 1$).
- The leading order term vanishes in the case of $k = 0, -1$.

Approach 2: Parametric resonance [3/3]

[M.Yoshimura,Prog. Theor. Phys.94 (1995)]

λ can be obtained from the relations for k=0,-1.

$$\left[\begin{array}{l} \gamma_0 C_0 - \theta_+ C_1 - \theta_- C_{-1} = 0 \\ \gamma_{-n} C_{-n} - \theta_+ C_{-n+1} - \theta_- C_{-n-1} = 0 \end{array}\right.$$

Relation between C_k and $C_{k\pm1}$ are calcurated by solving the tridiagonal matrix:

$$\begin{pmatrix} \gamma_N & -\theta_- & 0 & \cdots & & & & & & \\ -\theta_+ & \gamma_{N-1} & -\theta_- & \cdots & & & & & & \\ 0 & -\theta_+ & \cdots & \cdots & & & & & & \\ \cdots & \cdots & \cdots & \cdots & -\theta_- & \cdots & \cdots & & & \\ \cdots & \cdots & \cdots & -\theta_+ & \gamma_0 & \cdots & & & & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & -\theta_- & \cdots & \cdots & \\ \cdots & \cdots & \cdots & \cdots & \cdots & -\theta_+ & \gamma_{-n} & & & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & -\theta_- \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & -\theta_+ & \gamma_{-N} \end{pmatrix} \begin{pmatrix} c_N \\ c_{N-1} \\ \vdots \\ c_0 \\ \vdots \\ c_{-n} \\ \vdots \\ C_{-N} \end{pmatrix} = 0$$

Concreate expressions of λ for $n = 1$ at LO

$$\lambda = \frac{1}{2}\sqrt{\theta^2 - \epsilon^2} \quad -\theta < \epsilon < \theta$$

Analytical expression of λ can be obtained by itterative approximation.