Boosted Dark Matter Heating of Compact Stars by Particle Beams

Jaime Hoefken Zink, Shihwen Hor, and Maura E. Ramirez-Quezada,
JHEP 08 (2026) 036, arXiv: 2602.06111 [hep-ph].

Shihwen Hor
Tsung-Dao Lee Institute,
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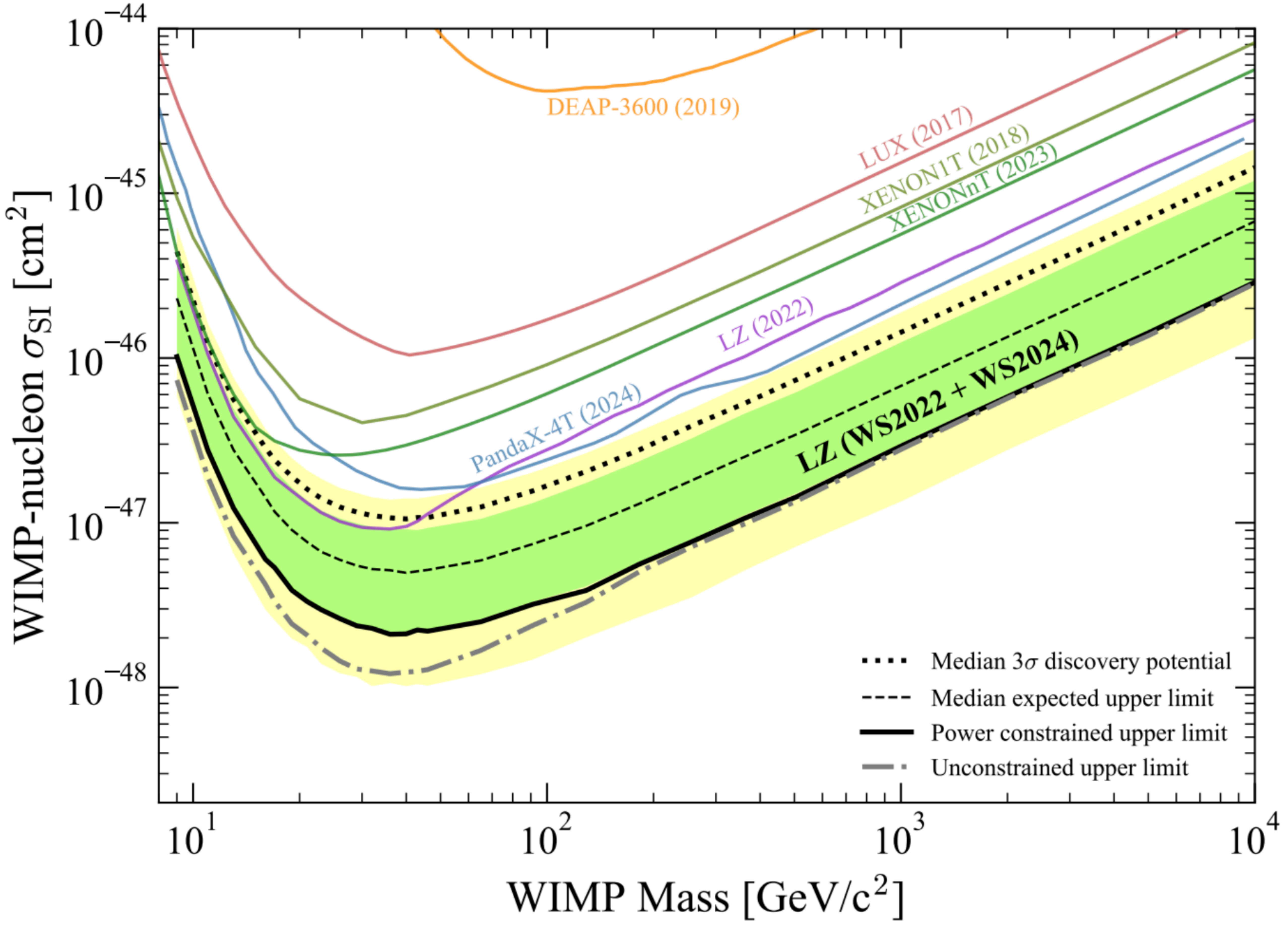
@ TeVPA 2026

Outlines

- ⋆ 1. Introduction
- ⋆ 2. Boosted dark matter
- ⋆ 3. Particle beams in compact objects
- ⋆ 4. Blazar boosted dark matter in white dwarf
- ⋆ 5. Summary

1. Introduction

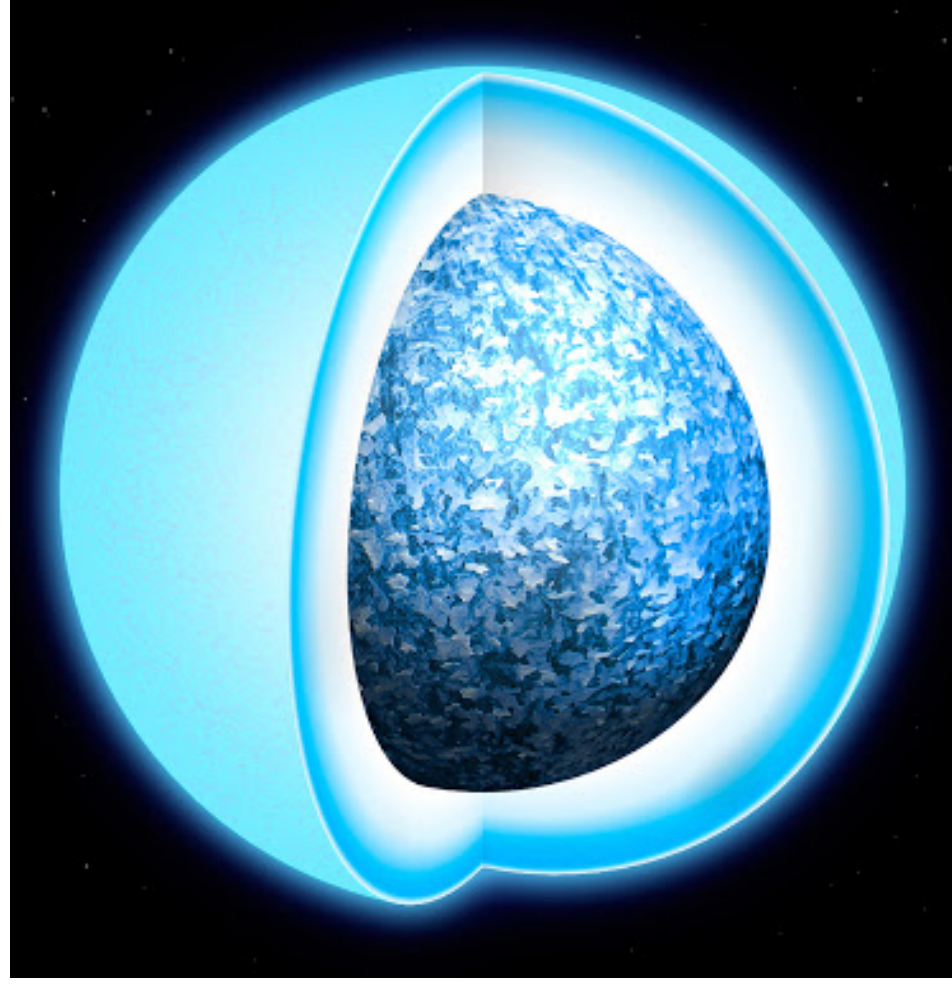
Dark matter detections



[The LUX-ZEPLIN Collaboration,
Phys. Rev. Lett. 135, 011802 (2025).]

Compact stars

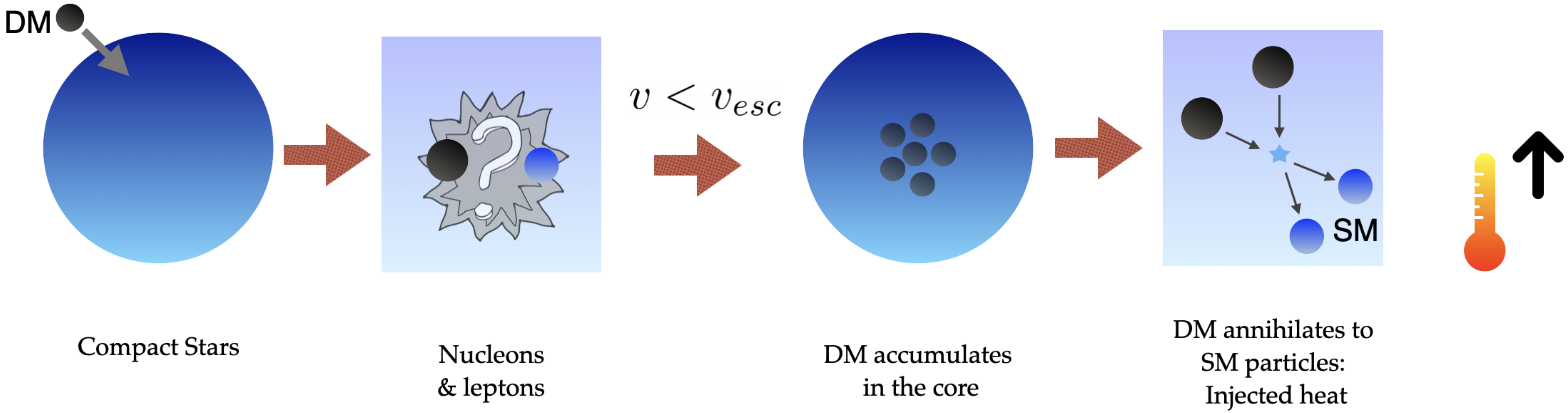
Benchmark	$T_{\rm eff}$ [K]	$T_\star$ [K]	Log(g) [cgs]	$M_\star$ [$M_\odot$]	$R_\star$	More
WD	3048	3050	8.195	0.69	1.2 $R_\oplus$	Spectral Type: DZQH
NS	$5.8 - 11.6 \times 10^5$	7.3×10^5	14.4	1.4	12.1 km	Name: RX J1856.5-3754



- Compact stellar remnants: strong gravitational force
- The Tolman-Oppenheimer-Volkoff (TOV) equations for EOS
- White dwarfs
  - Final state of massive stars (below $M_\star \sim 8M_\odot - 10M_\odot$) after collapsing gravitationally
  - Composed of C and O and possesses an atmosphere either H or He dominated
  - No fusion: the only support against gravitational collapse is the electron degeneracy pressure
  - DM-nucleon scattering can probe the sup-GeV regime (beyond direct detection)
- Neutron stars
  - Composed of neutrons, protons, electrons, and muons
  - Fermi-Dirac distribution (degenerate matter)

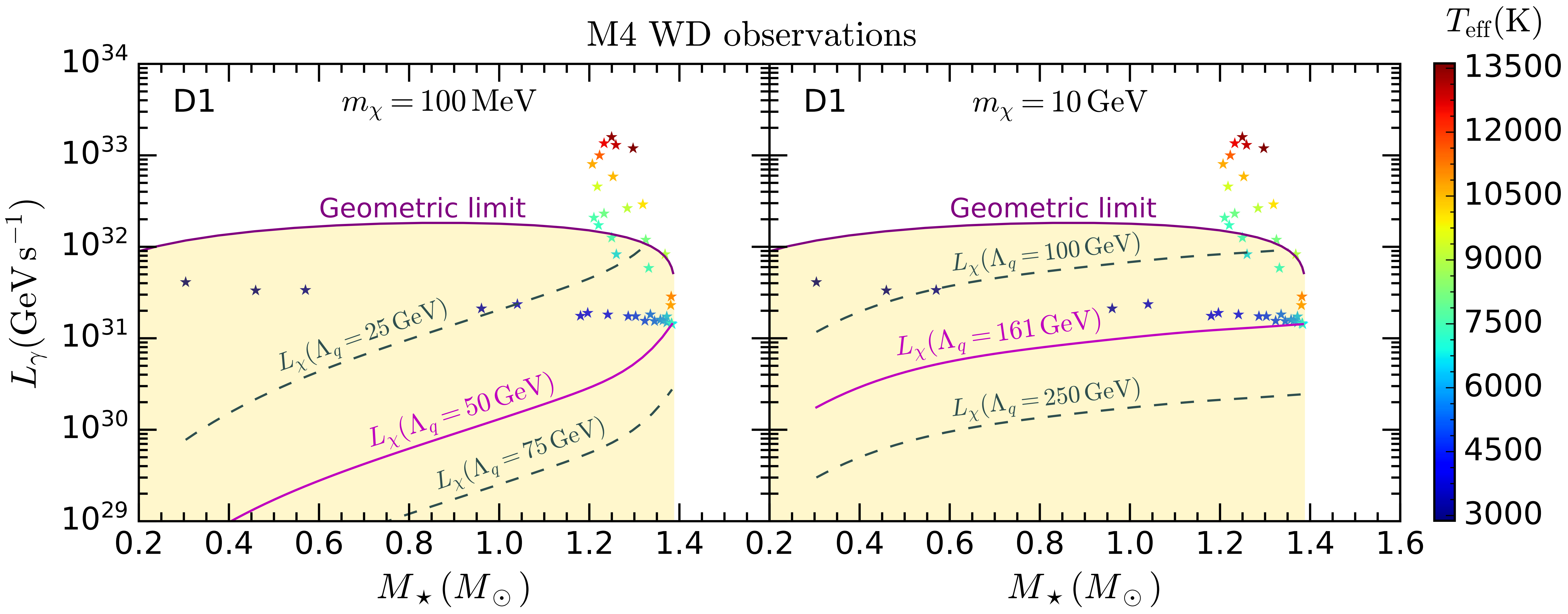
Dark matter capture by compact stars

- Heating of compact objects: DM can accumulate in the core of compact stars, heat them up and hence generate an observable signal



White dwarfs capture rate

- Assuming DM capture and annihilation are in equilibrium, the star luminosity due to DM is $L_\chi = m_\chi C(m_\chi)$ $\quad$ ($C(m_\chi)$: capture rate)
- The observed WD luminosity $L_\gamma \geq L_\chi$



[N. Bell, G. Busoni, S. Robles, M. E. Ramirez-Quezada, M. Viriato, JACP 10 (2021), 083.]

Boosted DM

- Local DM: low DM density —> challenging

[J. W. Wang, A. Graneli, and P. Ulio, Phys. Rev. Lett. 128 (2022) 221104.]

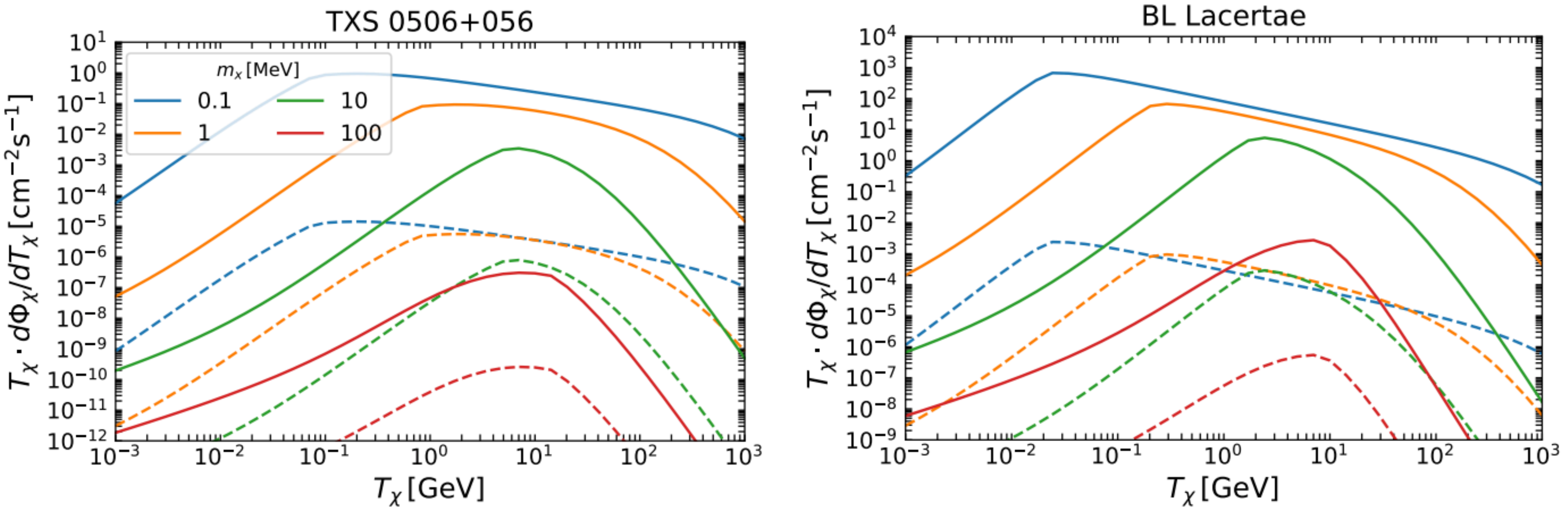
- Boosted DM (high-density flux): improve the bounds for light DM candidates in direct detection experiments
  - Boosting source: blazers, cosmic rays, diffuse supernova neutron background
  - Relativistic
- **A multi-energy approach to the WD interaction and capture**
  - Elastic scattering with nuclei, nucleon, resonance, deep inelastic scattering (DIS)

[Hoefken Zink, Hor, and Ramirez-Quezada, *JHEP* 05 (2025) 160]

Boosted DM flux

★ Blazar boosted DM

$$\sigma_{\chi p} = 10^{-30}\ \text{cm}^2$$



[J. Wang, A. Granelli, and P. Ullio, Phys. Rev. Lett. 128, 221104 (2022).]

2. Boosted Dark Matter

Blazar boosted DM

- The CGRABS catalogue (a uniform all-sky sample)
- Particle emission jets
  - $$\frac{d\Gamma_i}{dT_i d\Omega} = \frac{k_i}{4\pi}\left(1 + \frac{T_i}{m_i}\right)^{-\alpha_i} \frac{\beta_i (1-\beta_i\beta_B\mu)^{-\alpha_i}\Gamma_B^{-\alpha_i} e^{-(m_i+T_i)/(m_i+T_i^{\max})}}{\sqrt{(1-\beta_i\beta_B\mu)^2 - (1-\beta_i^2)(1-\beta_B^2)}},$$
  - with $\Gamma_B = (1-\beta_B^2)^{-1/2}$

Quantity	Value
Number of sources	324
z (min, median, max)	(0.02, 1.02, 3.41)
d_L [Mpc] (min, median, max)	(85.9, 4.83e+03, 1.47e+04)
Ranking proxy	$k_p/(4\pi d_L^2)$

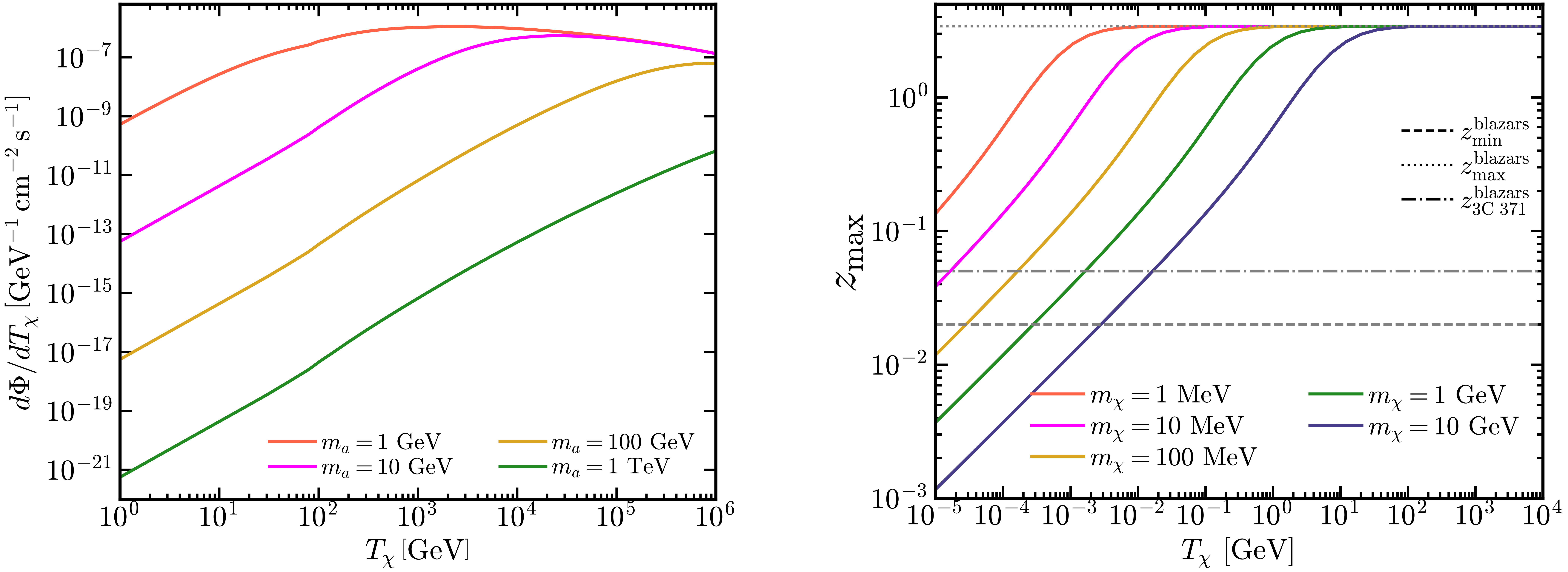
Rank	Source	z	d_L [Mpc]	$\Gamma_{\rm B}$	$\theta_{\rm LOS}$ [deg]	L_p [erg/s]	k_p [s^{-1} sr^{-1}]	% of stack
1	Mkn 501	0.03	129	5.8	9.88	5.01e+46	9.91e+41	32.5
2	PKS 2155-304	0.12	527	4	14.3	1.58e+47	6.59e+42	13
3	TXS 0214+083	0.09	393	7.4	7.74	3.16e+47	3.05e+42	10.8
4	3C 371	0.05	216	9.2	6.23	1e+47	7.86e+41	9.19
5	Mkn 180	0.04	173	6.2	9.24	2e+46	3.45e+41	6.34

Blazar boosted DM

- BBDM flux
  - $$\frac{d\Phi_\chi}{dT_\chi} = \frac{\Sigma_{\rm DM}}{2\pi m_\chi d_L^2} \sum_i \int_0^{2\pi} d\phi \int_{T_i^{\rm min}(T_\chi)}^{T_i^{\rm max}} dT_i \frac{d\Gamma_i}{dT_i d\Omega} \frac{d\sigma_{\chi i}}{dT_\chi} \, ,$$
  - The column density: $\Sigma_{\rm DM} = \int_{R_{\rm min}}^{r} \rho_\chi(r') dr' \, ,$
- Cross section (**elastic** & **DIS**):
  - $$\frac{d\sigma_{\chi N}}{dT_\chi} = \underbrace{\frac{dQ^2}{dT_\chi}\frac{d\sigma_{\rm el}}{dQ^2}}_{\rm EL} + \underbrace{\int_{\nu_{\rm min}}^{\nu_{\rm max}} d\nu \frac{d\sigma^2_{\rm DIS}}{dxdy} \left| \frac{\partial(x,y)}{\partial(T_\chi,\nu)} \right|}_{\rm DIS} \, ,$$
- Redshift: DM fermion is redshifted as it reaches our galaxy
  - $$\frac{d\Phi_\chi}{dT_\chi^{(\infty)}} = \frac{d\Phi_\chi}{dT_\chi} \times \frac{dT_\chi}{T_\chi^{(\infty)}} \, , \qquad \text{with} \quad \frac{dT_\chi}{dT_\chi^{(\infty)}} = \frac{(1+z)^2 \left(m_\chi + T_\chi^{(\infty)}\right)}{\sqrt{m_\chi^2 + (1+z)^2 T_\chi^{(\infty)} \left(2m_\chi + T_\chi^{(\infty)}\right)}} \, .$$

Blazar boosted DM

★ Fermionic DM & ALP mediator ($\mathscr{L}_a = \sum_q g_{qa}\partial_\mu a \bar{q}\gamma^\mu\gamma^5 q + g_D \partial_\mu a \bar{\chi}\gamma^\mu\gamma^5\chi\,,$)

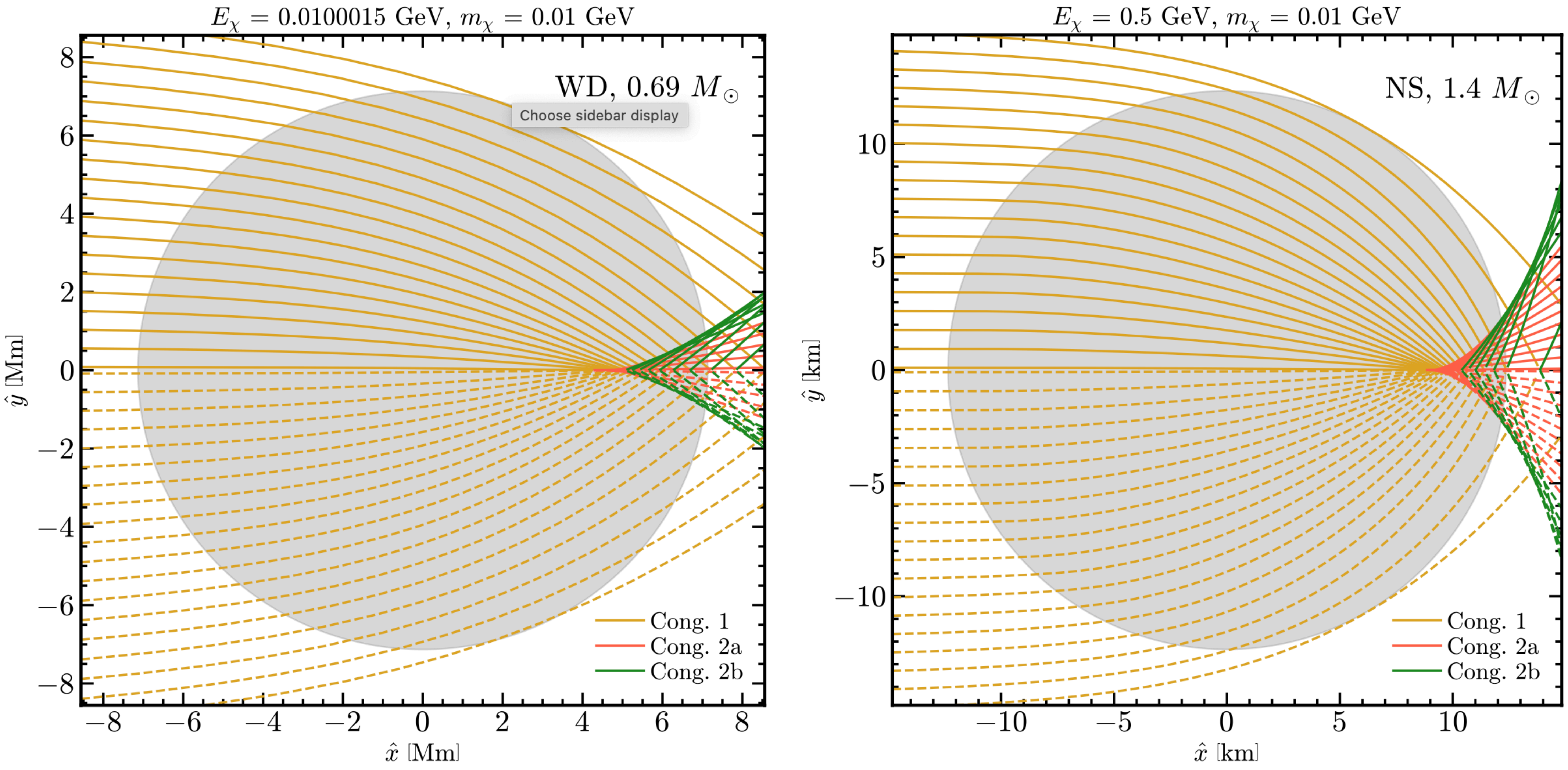


3. Particle Beams in Compact Objects

General formalism

- A incident beam of particles with a preferred direction
- Geodesic propagation
- The metric of a non-rotating compact star
  - $ds^2 = -\, g_{tt}(r)\, dt^2 + g_{rr}(r)\, dr^2 + r^2\, d\Omega^2\,,$
- The geodesic equations for a particle starting far from the star
  - $\dfrac{dt}{d\lambda} = \dfrac{\mathscr{E}}{g_{tt}}\,, \qquad \dfrac{dr}{d\lambda} = \pm\sqrt{\dfrac{\mathscr{E}^2}{g_{tt}g_{rr}} - \dfrac{1}{g_{rr}}\left(1 + \dfrac{\mathscr{L}^2}{r^2}\right)}\,,$
  - $\dfrac{d\phi}{d\lambda} = \dfrac{\mathscr{L}}{r^2}\,, \qquad \dfrac{d\theta}{d\lambda} = 0\,,$
  - with $\mathscr{E} \equiv E_\chi/m_\chi$ and $\mathscr{L} \equiv -\, b\, p_\chi/m_\chi$

General formalism



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General formalism

★ **Capture rate**

$$C = \int dn_\chi(T_\chi^\infty, r, \phi)\, \Omega_{v_e}^-\big(\omega(T_\chi^\infty, r)\big)\, dV = \int dV \int dT_\chi^\infty \frac{d\Phi}{dT_\chi^\infty} \frac{dV_0}{dV_r} \Omega_{v_e}^-\big(\omega(T_\chi^\infty, r)\big)\,,$$

$$\frac{dV_0}{dV_r} = \frac{b\, r_0 \, \left| r_t \phi_{\tilde{b}} - r_{\tilde{b}} \phi_t \right|_0}{r^2 \, \sin\phi \sqrt{g_{rr}} \, \left| r_t \phi_{\tilde{b}} - r_{\tilde{b}} \phi_t \right|}\,,$$

★ **Energy deposition**

$$\dot{E}_\chi = 2\pi \int d\rho\, dz\, dT_\chi^\infty\, d\Pi\, \rho \sqrt{\frac{g_{rr}(\rho, z)}{g_{tt}^s}}\, X(\rho, z) \frac{d\Phi}{dT_\chi^\infty} \frac{d\Omega^-}{d\Pi} E_k(\Pi, v_e)$$

$$= 2\pi \int d\rho\, dz\, dT_\chi^\infty\, d\Pi\, \rho \sqrt{\frac{g_{rr}(\rho, z)\, g_{tt}(\rho, z)}{g_{tt}^s}}\, n_T(\rho, z)\, \omega \left(1 + \frac{T_\chi^\infty}{m_\chi}\right) X_V(\rho, z) \frac{d\Phi}{dT_\chi^\infty} \frac{d\sigma}{d\Pi} E_k(\Pi, v_e)\,,$$

Interaction roof and geometric limit

- **Interaction roof (IR)**: all particles interact with the star and deposit energy
  - Mean energy transferred per interaction: $\langle q_0 \rangle = \dfrac{m_\chi + T_\chi^\infty}{\sqrt{g_{tt}(R_*)}} \dfrac{\int d\Pi \, y \, \frac{d\sigma}{d\Pi}}{\sigma_{\rm tot}(T_\chi^\infty)}$,
  - $\langle \dot{E}_\chi \rangle_{\rm IR} = \int dT_\chi^\infty \dfrac{d\Phi}{dT_\chi^\infty} \left(\pi b_{\rm max}^2\right) \dfrac{\langle q_0 \rangle}{\sqrt{g_{tt}(R_*)}}$.
- **Geometric limit (GL)**: all particles traversing the star are captured
  - $\langle \dot{E}_\chi \rangle_{\rm GL} = \int dT_\chi^\infty \dfrac{d\Phi}{dT_\chi^\infty} \left(\pi b_{\rm max}^2\right) \dfrac{\left(1 - \sqrt{g_{tt}(R_*)}\right) m_\chi + T_\chi^\infty}{g_{tt}(R_*)}$.

Optical factor

- As particles propagate through the star, sufficiently strong interactions can reduce the number of particles reaching a given point inside the star
- **Optical factor:** $\eta = \dfrac{N_\chi}{N_\chi^{(0)}} = \exp\left(- \int dt^{\rm loc} \, \dfrac{\Omega^-}{\sqrt{g_{tt}(r)}} \right) ,$
- $$\int dt \, \Omega^- = \frac{m_\chi + T_\chi^\infty}{J(b, T_\chi^\infty, m_\chi)} \int_{\phi^*}^{\phi(r)} d\phi' \, \left[r'(\phi') \right]^2 \int d\Pi \, \frac{d\sigma}{d\Pi} \, \omega \, n_T\big(r'(\phi') \big) \ ,$$
- The optical factor must be included in the volume amplification.

4. Blazar Boosted Dark Matter in White Dwarfs

The model

- A **fermonic DM** interacting through an **axion-like-particle (ALP) mediator**
- Lagrangian: $\mathscr{L}_a = \sum_q g_{qa} \partial_\mu a \bar{q} \gamma^\mu \gamma^5 q + g_D \partial_\mu a \bar{\chi} \gamma^\mu \gamma^5 \chi \,,$
  - Elastic interactions $\chi(k_1) + N(p_1) \to \chi(k_2) + N(p_2)$

$$i\mathscr{M} = i g_D q_\mu q_\nu \bar{u}(k_2)\gamma^\mu \gamma^5 u(k_1) \frac{i}{Q^2 + m_a^2} \frac{2\, m_N}{m_u + m_d} \sum_q g_{qa} G^q_{5,N}(Q^2) \bar{u}(p_2) \gamma^\nu \gamma^5 u(p_1) \,,$$

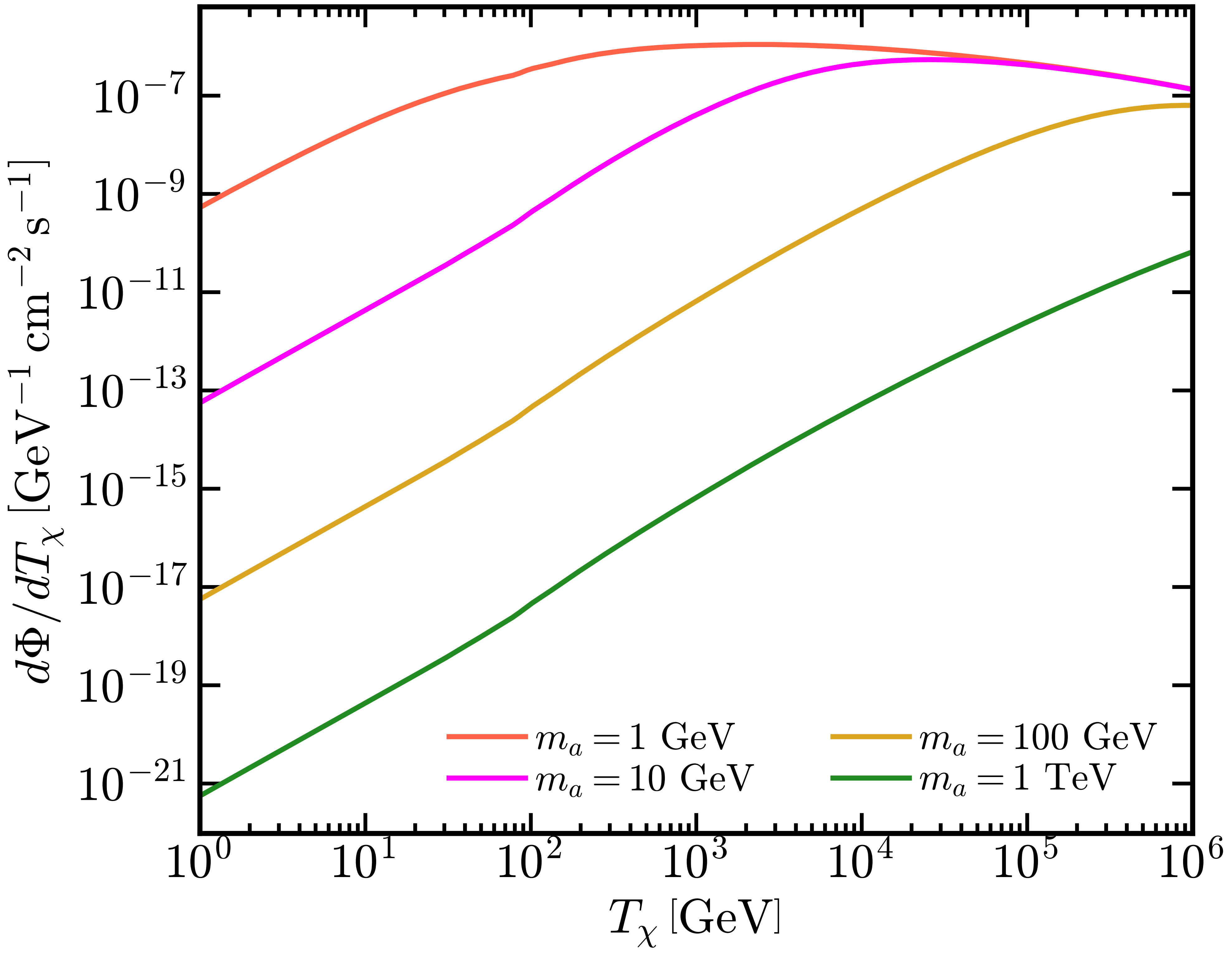
  - **DIS** $\chi(k_1) + N(p_1) \to \chi(k_2) + X(p_2)$

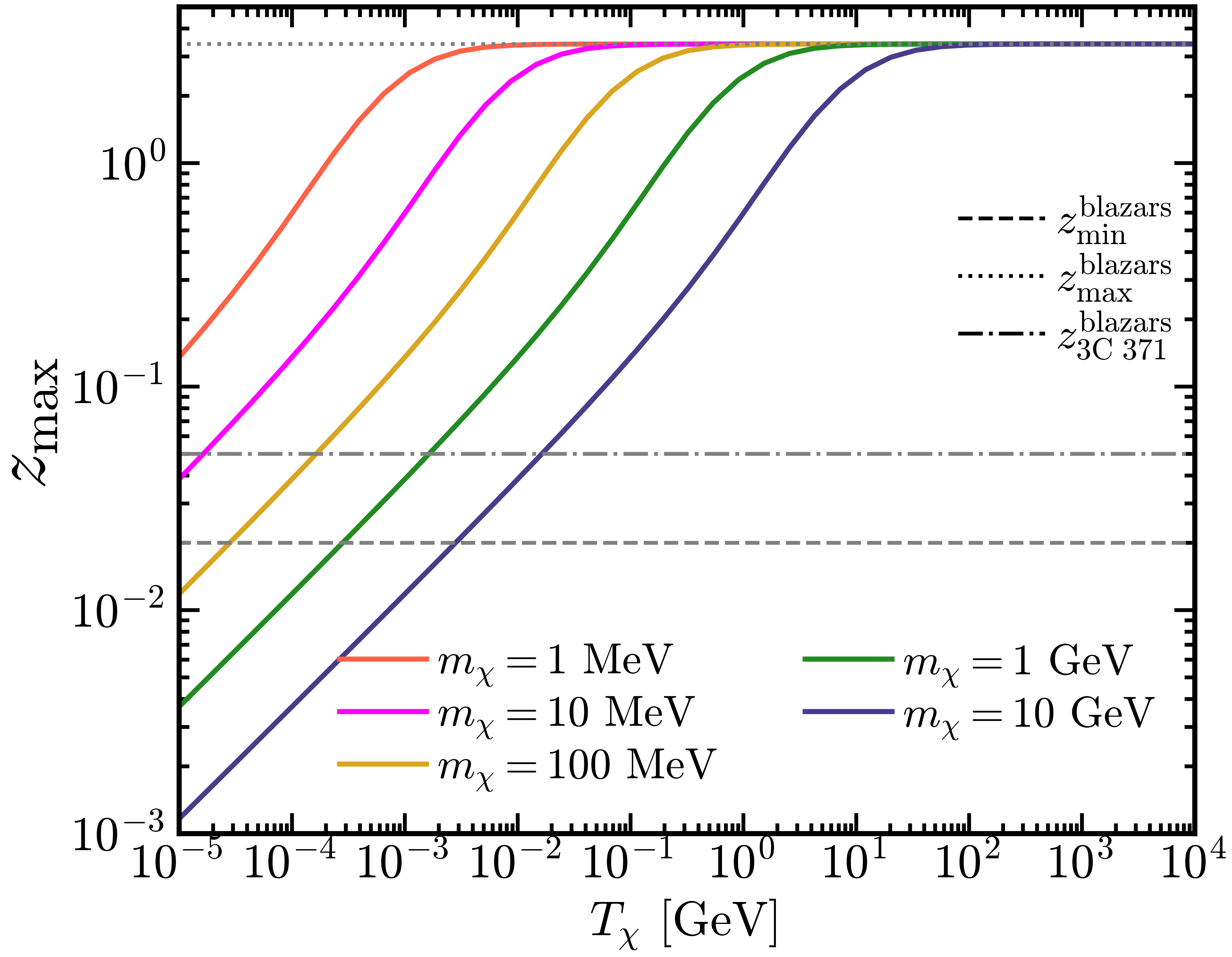
$$\left| \overline{\mathscr{M}^{\rm DIS}_{\chi q}} \right|^2 = 2 g_D^2 g_{qa}^2 \frac{\left(Q^2\right)^2}{y\,\left(Q^2 + m_a^2\right)^2} \left(Q^2 + 2m_\chi^2\right) \left(\frac{\left(Q^2\right)^2}{2E_\chi^2} + Q^2\left(3y - 1\right) \right) , \quad \text{with}$$

$$\frac{d^2\sigma}{dxdy} = \frac{y}{16\pi} \frac{Q^2 \sum_q f_q(x, Q^2) \left| \overline{\mathscr{M}^{\rm DIS}_{\chi q}} \right|^2}{\left(Q^2\right)^2 - 4 m_N^2 m_\chi^2 x^2 y^2} \,,$$

Blazar boosted DM

★ Fermonic DM & ALP mediator ($\mathscr{L}_a = \sum_q g_{qa}\partial_\mu a\bar{q}\gamma^\mu\gamma^5 q + g_D\partial_\mu a\bar{\chi}\gamma^\mu\gamma^5\chi\,,$)





Heating of white dwarf

Table 1. Physical properties of selected compact objects.

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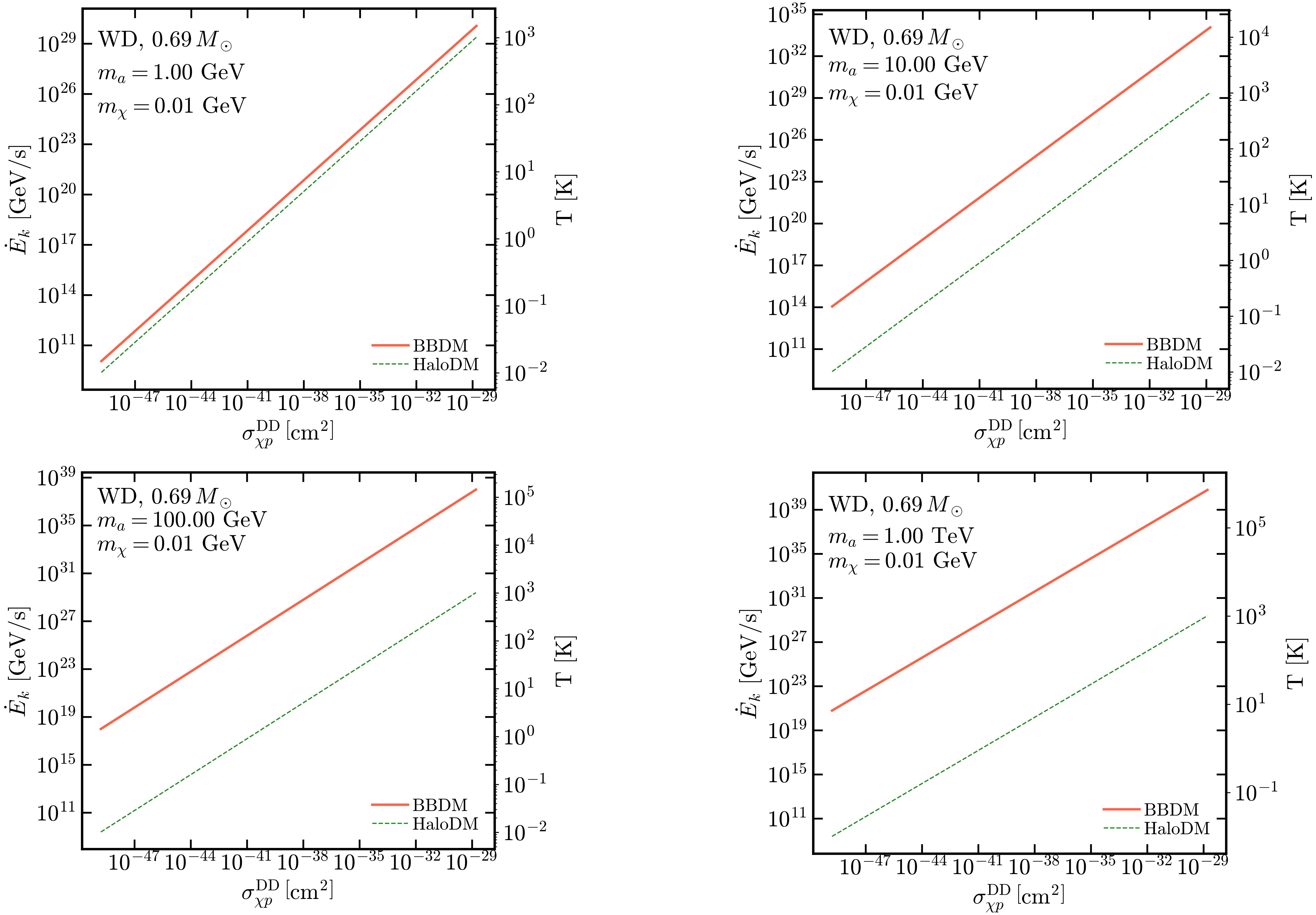

$$\sigma_{\chi p}^{\rm DD} \propto g_D^2 g_{aN}^2 .$$

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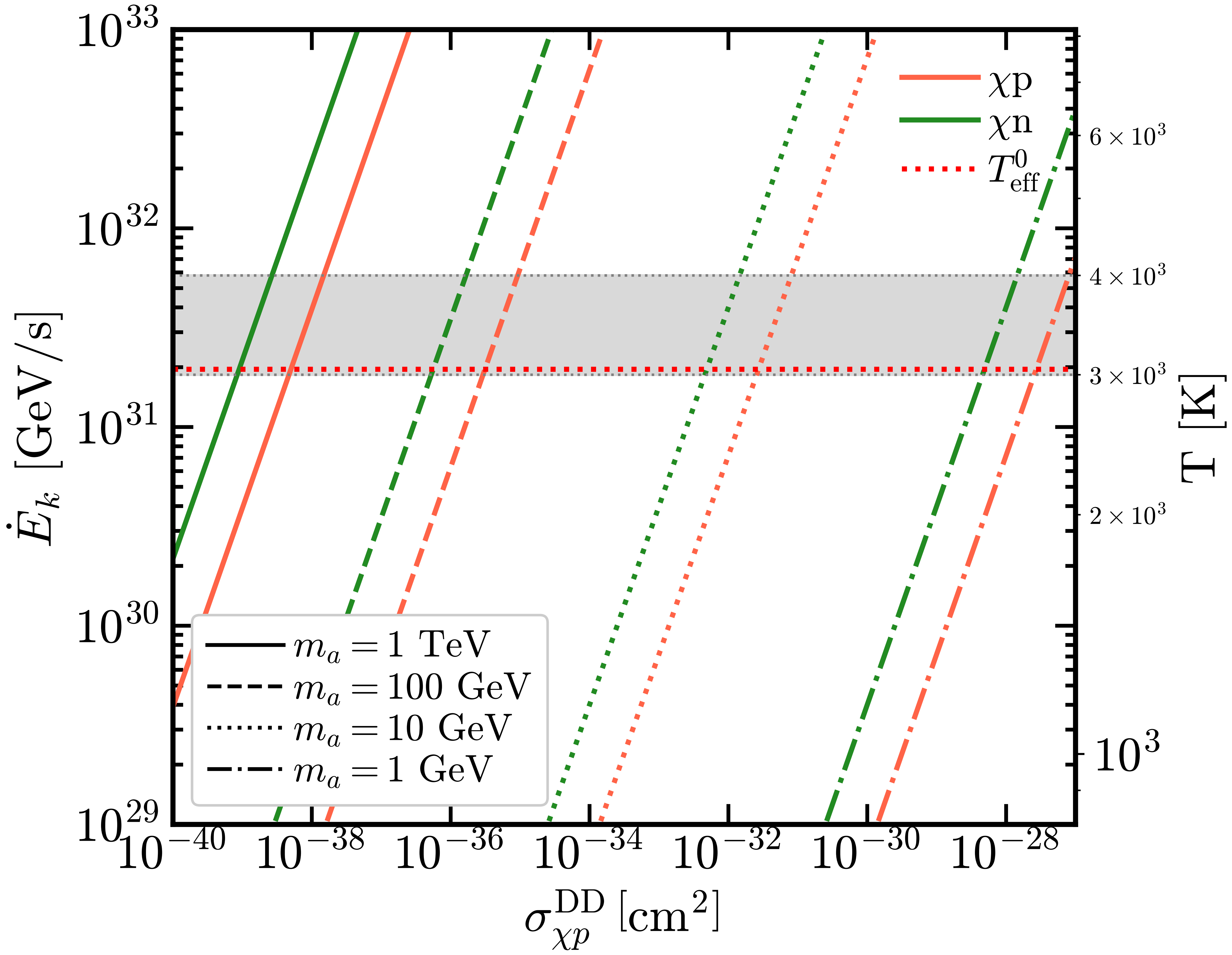


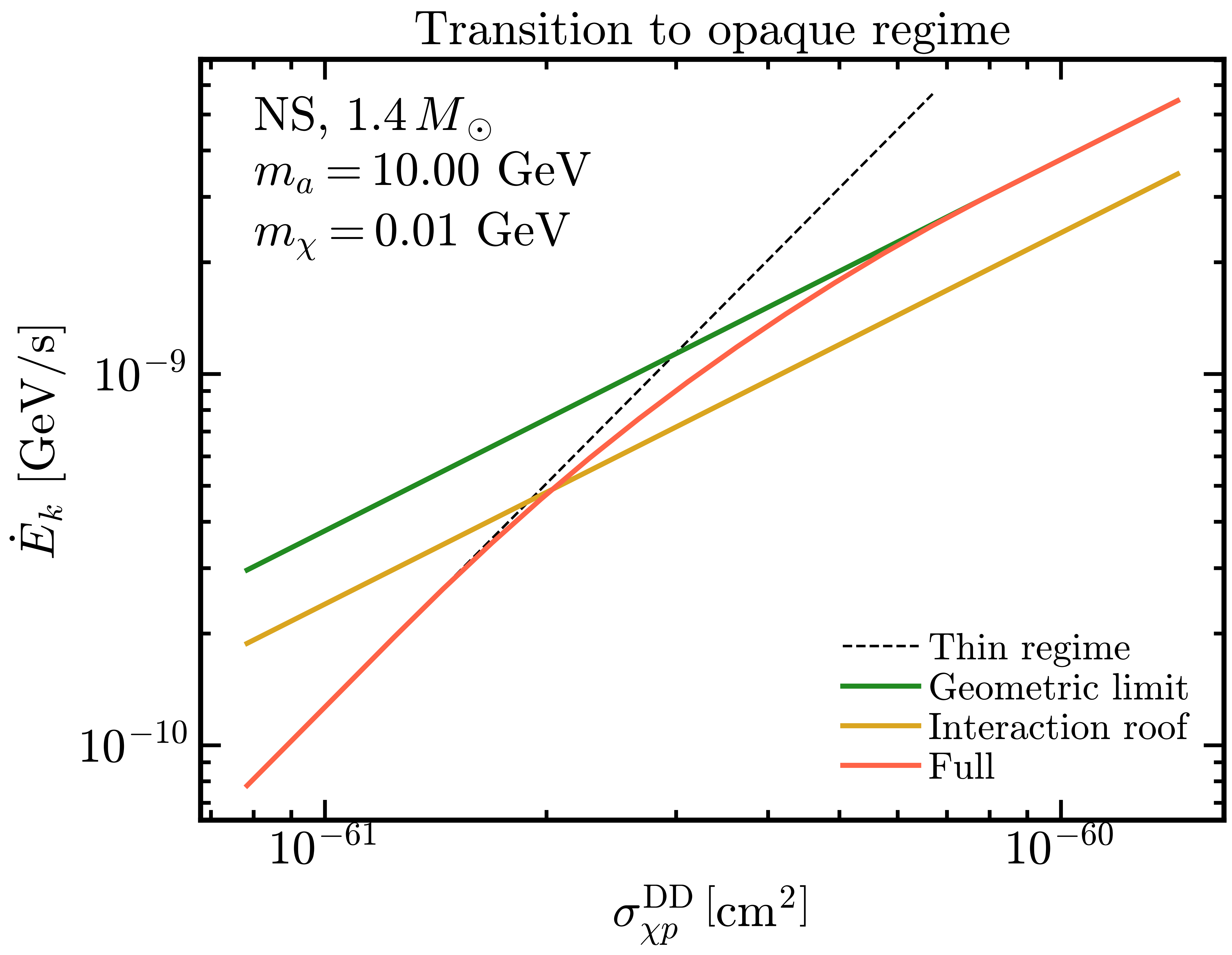
5. Summary

Summary

- ★ A general framework to study the heating of compact stars induced by **DM jets**
- ★ Can be extended to isotropic fluxes
- ★ **A fully relativistic description of energy deposition** by DM in compact stars
  - ★ Gravitational focusing & optical-depth effects
  - ★ Elastic scattering & DIS
- ★ ALP mediated fermionic DM ($m_\chi = 10\,\mathrm{MeV}$) boosted by 324 blazer
  - ★ BBDM heating of white dwarfs can be competitive with (for $m_a = 1\,\mathrm{GeV}$) or significantly exceed (for $m_a > 1\,\mathrm{GeV}$) the standard halo DM heating contribution

Backup Slides

Heating of neutron star



$$\sigma_{\chi p}^{\mathrm{DD}} \propto g_D^2 g_{aN}^2.$$

NR elastic cross sections

- To compare with the direct detection, we transfer the couplings into NR elastic cross sections

$$\mathcal{L}^{q}_{\text{eff}} = -\frac{g_D}{m_a^2} \sum_q g_{qa} \left[\overline{\chi}\gamma^\mu\gamma_5\chi\right] \left[\overline{q}\gamma^\nu\gamma_5 q\right] q_\mu q_\nu \,.$$

$$\mathcal{L}^{\text{NR}}_{\text{eff}} = -\frac{16 m_\chi\, m_N\, g_D\, g_{Na}\, G_{qN}}{m_a^2} \times \mathcal{O}_6^{\text{NR}} \,.$$

$$\sigma^{\text{DD}}_{\chi N} = \frac{16\left(g_D\, g_{Na}\, G_{qN}\right)^2}{\pi} \times \frac{m_\chi^4\, m_N^4}{\left(m_\chi + m_N\right)^2\, m_a^4} \,.$$

where $G_{qN} \equiv \Delta_u^N + \Delta_d^N + \Delta_s^N - 3\,\left(m_u^{-1} + m_d^{-1} + m_s^{-1}\right)^{-1}\,\left(\Delta_u^N/m_u + \Delta_d^N/m_d + \Delta_s^N/m_s\right)$.

Nuclei NR operators

$$\langle \Psi_f | \, H_T \, |\Psi_i\rangle = (2\pi)^3 \delta^{(3)}(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4) i\mathcal{M}_T^{NR}.$$

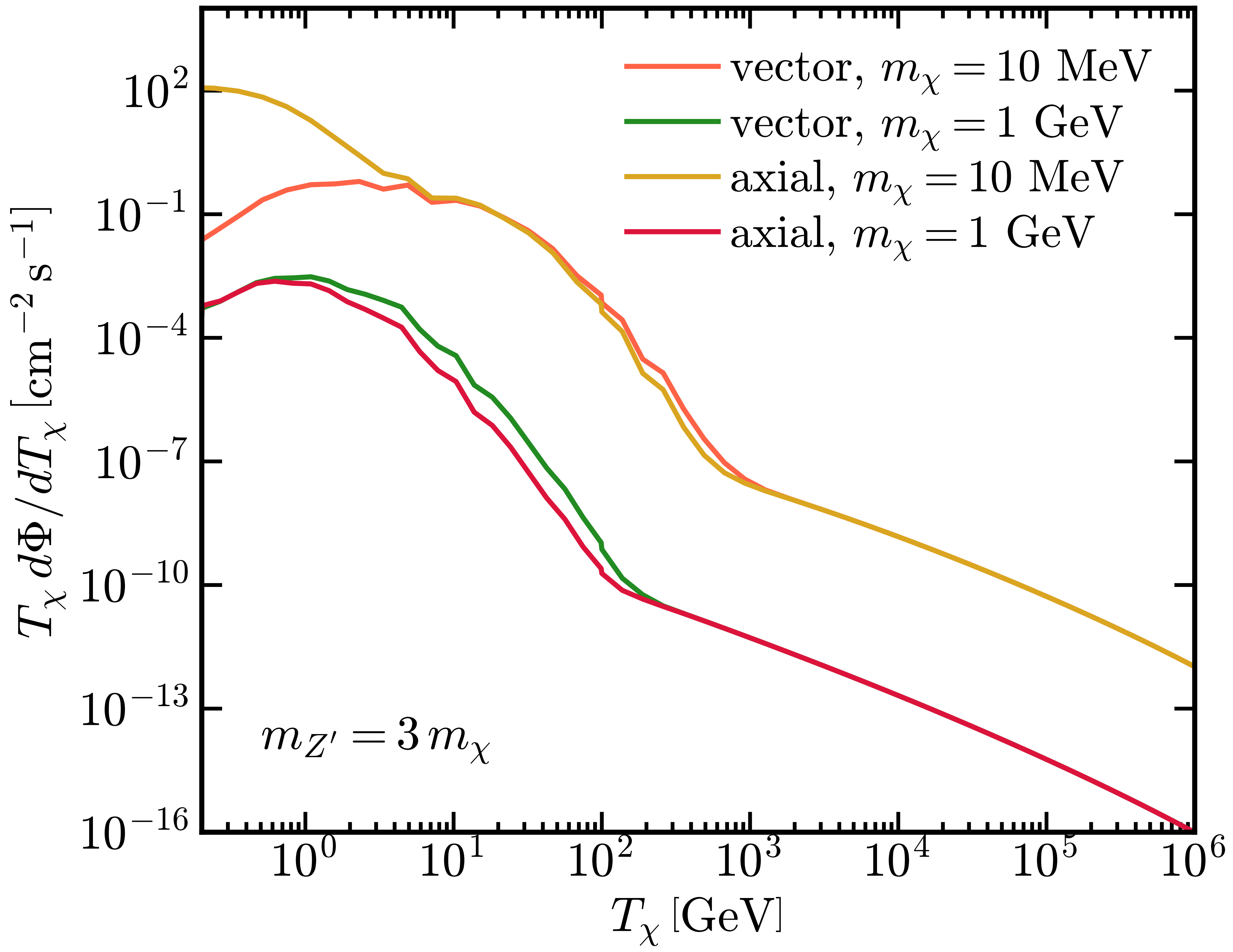
$$\mathcal{H}_T(\vec{r}) = \sum_{i=1}^{A} \sum_{j=0,1} \sum_{k=1}^{15} c_k^j \mathcal{O}_k^{(i)}(\vec{r}) t^j(i),$$

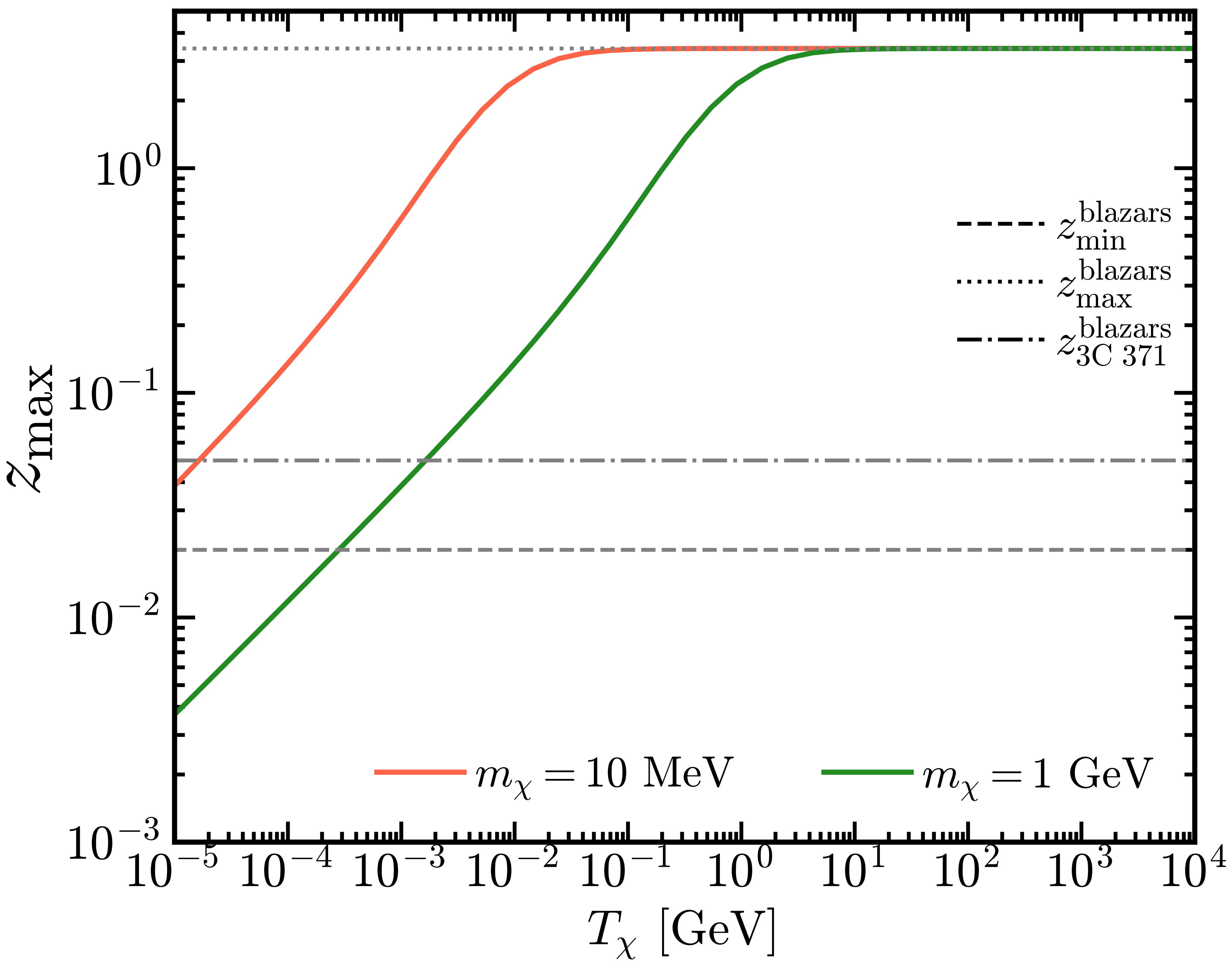
$$\frac{1}{N_i}\sum_{i,j} |\mathcal{M}_T^{NR}|^2 = \frac{m_T^2}{m_N^2} \sum_{i,j}^{15} \sum_{\alpha,\beta=0,1} c_i^\alpha c_j^\beta F_{ij}^{\alpha\beta}(v^2, q^2, y). \qquad \frac{d\sigma_T^{NR}}{d\cos\theta} = \frac{1}{32\pi(m_\chi + m_T)^2} \frac{1}{N_i} \sum_{i,j} |\mathcal{M}_T^{NR}|^2.$$

$\hat{\mathcal{O}}_1 = \mathbb{1}_{\chi N}$	$\hat{\mathcal{O}}_9 = i\hat{\mathbf{S}}_\chi \cdot \left(\hat{\mathbf{S}}_N \times \frac{\hat{\mathbf{q}}}{m_N}\right)$
$\hat{\mathcal{O}}_3 = i\hat{\mathbf{S}}_N \cdot \left(\frac{\hat{\mathbf{q}}}{m_N} \times \hat{\mathbf{v}}^\perp\right)$	$\hat{\mathcal{O}}_{10} = i\hat{\mathbf{S}}_N \cdot \frac{\hat{\mathbf{q}}}{m_N}$
$\hat{\mathcal{O}}_4 = \hat{\mathbf{S}}_\chi \cdot \hat{\mathbf{S}}_N$	$\hat{\mathcal{O}}_{11} = i\hat{\mathbf{S}}_\chi \cdot \frac{\hat{\mathbf{q}}}{m_N}$
$\hat{\mathcal{O}}_5 = i\hat{\mathbf{S}}_\chi \cdot \left(\frac{\hat{\mathbf{q}}}{m_N} \times \hat{\mathbf{v}}^\perp\right)$	$\hat{\mathcal{O}}_{12} = \hat{\mathbf{S}}_\chi \cdot \left(\hat{\mathbf{S}}_N \times \hat{\mathbf{v}}^\perp\right)$
$\hat{\mathcal{O}}_6 = \left(\hat{\mathbf{S}}_\chi \cdot \frac{\hat{\mathbf{q}}}{m_N}\right)\left(\hat{\mathbf{S}}_N \cdot \frac{\hat{\mathbf{q}}}{m_N}\right)$	$\hat{\mathcal{O}}_{13} = i\left(\hat{\mathbf{S}}_\chi \cdot \hat{\mathbf{v}}^\perp\right)\left(\hat{\mathbf{S}}_N \cdot \frac{\hat{\mathbf{q}}}{m_N}\right)$
$\hat{\mathcal{O}}_7 = \hat{\mathbf{S}}_N \cdot \hat{\mathbf{v}}^\perp$	$\hat{\mathcal{O}}_{14} = i\left(\hat{\mathbf{S}}_\chi \cdot \frac{\hat{\mathbf{q}}}{m_N}\right)\left(\hat{\mathbf{S}}_N \cdot \hat{\mathbf{v}}^\perp\right)$
$\hat{\mathcal{O}}_8 = \hat{\mathbf{S}}_\chi \cdot \hat{\mathbf{v}}^\perp$	$\hat{\mathcal{O}}_{15} = -\left(\hat{\mathbf{S}}_\chi \cdot \frac{\hat{\mathbf{q}}}{m_N}\right)\left[\left(\hat{\mathbf{S}}_N \times \hat{\mathbf{v}}^\perp\right) \cdot \frac{\hat{\mathbf{q}}}{m_N}\right]$

Blazar boosted DM

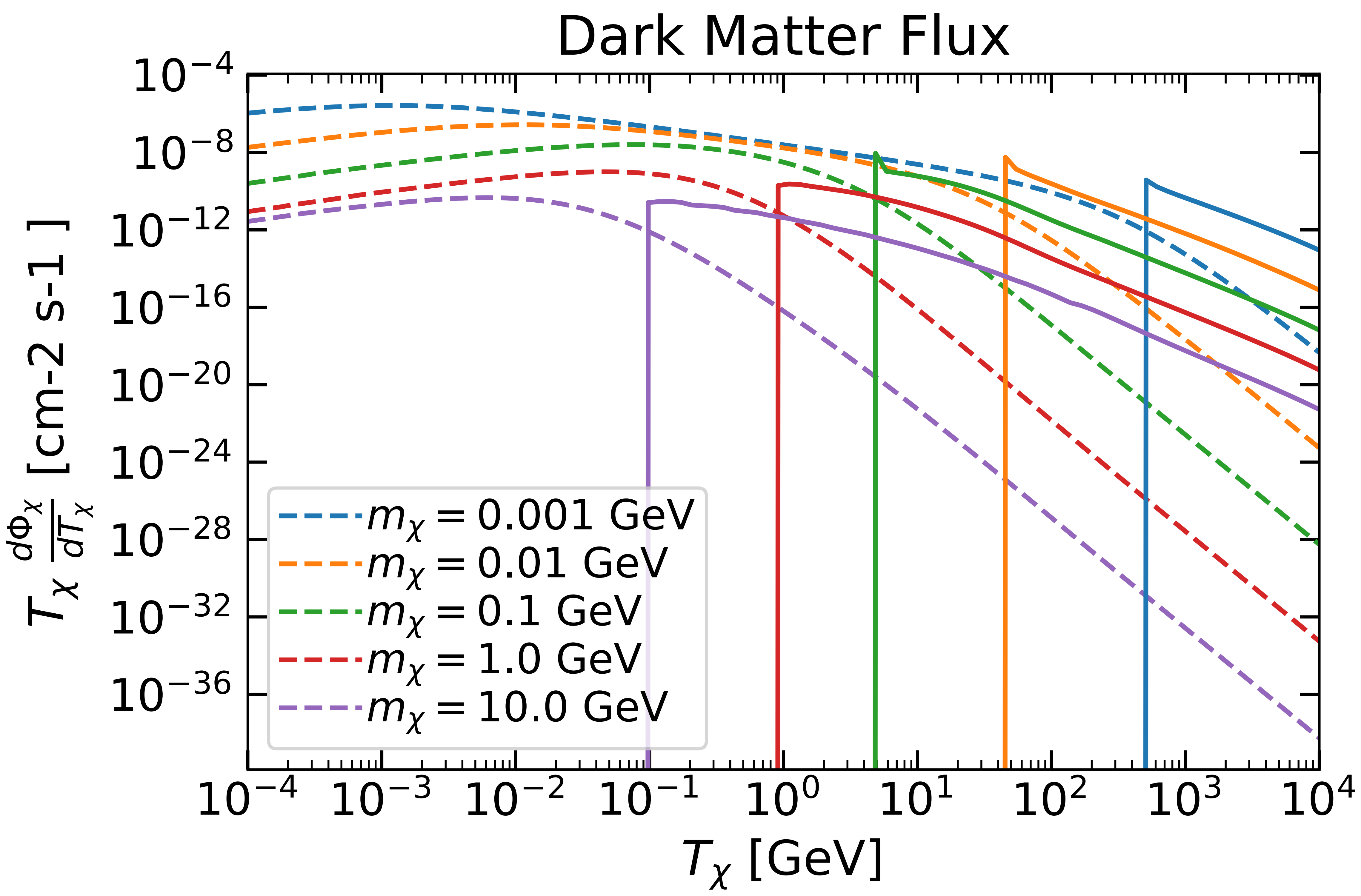
- ⋆ Fermonic DM & vector mediator

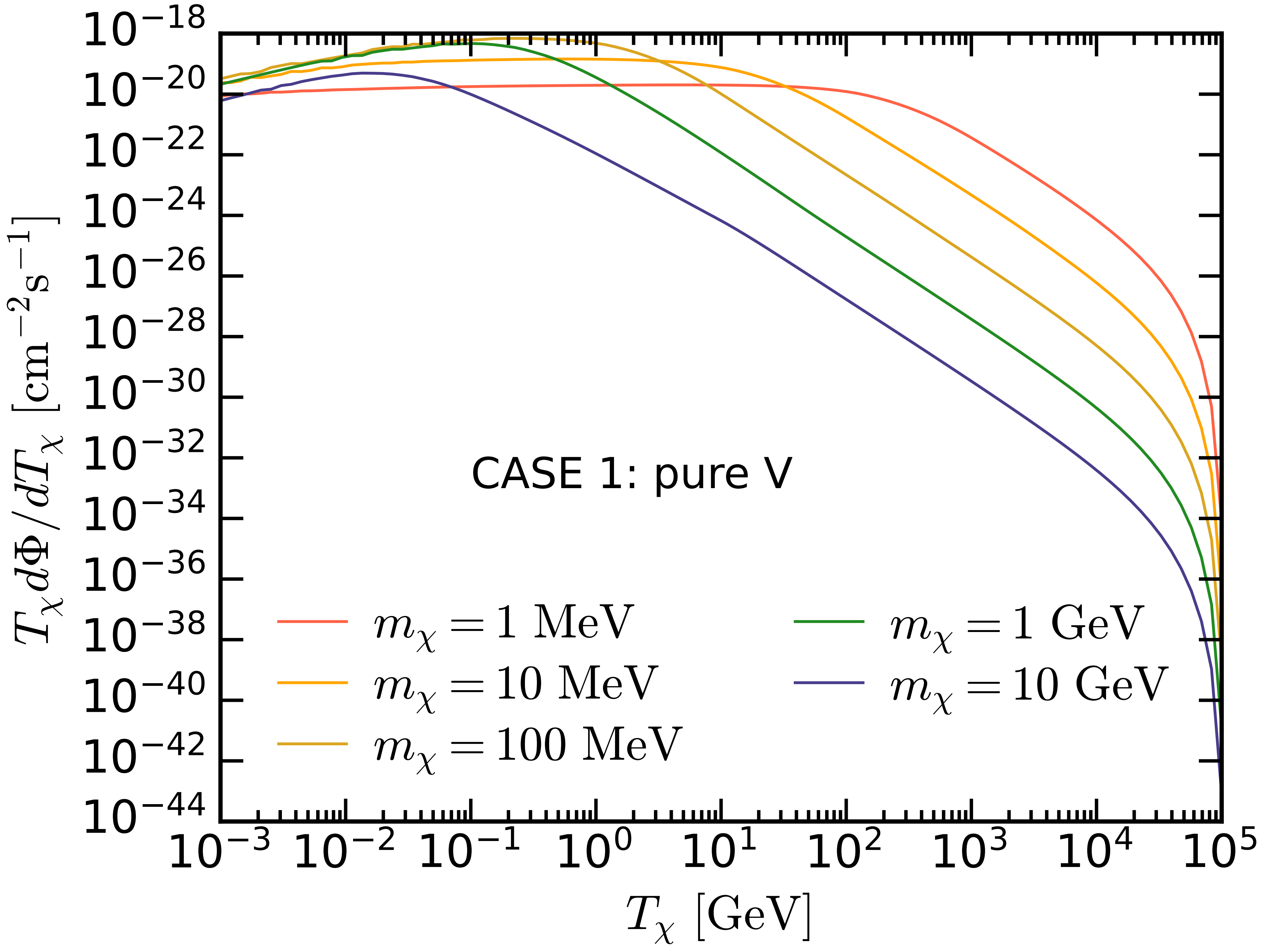




CR Boosted DM

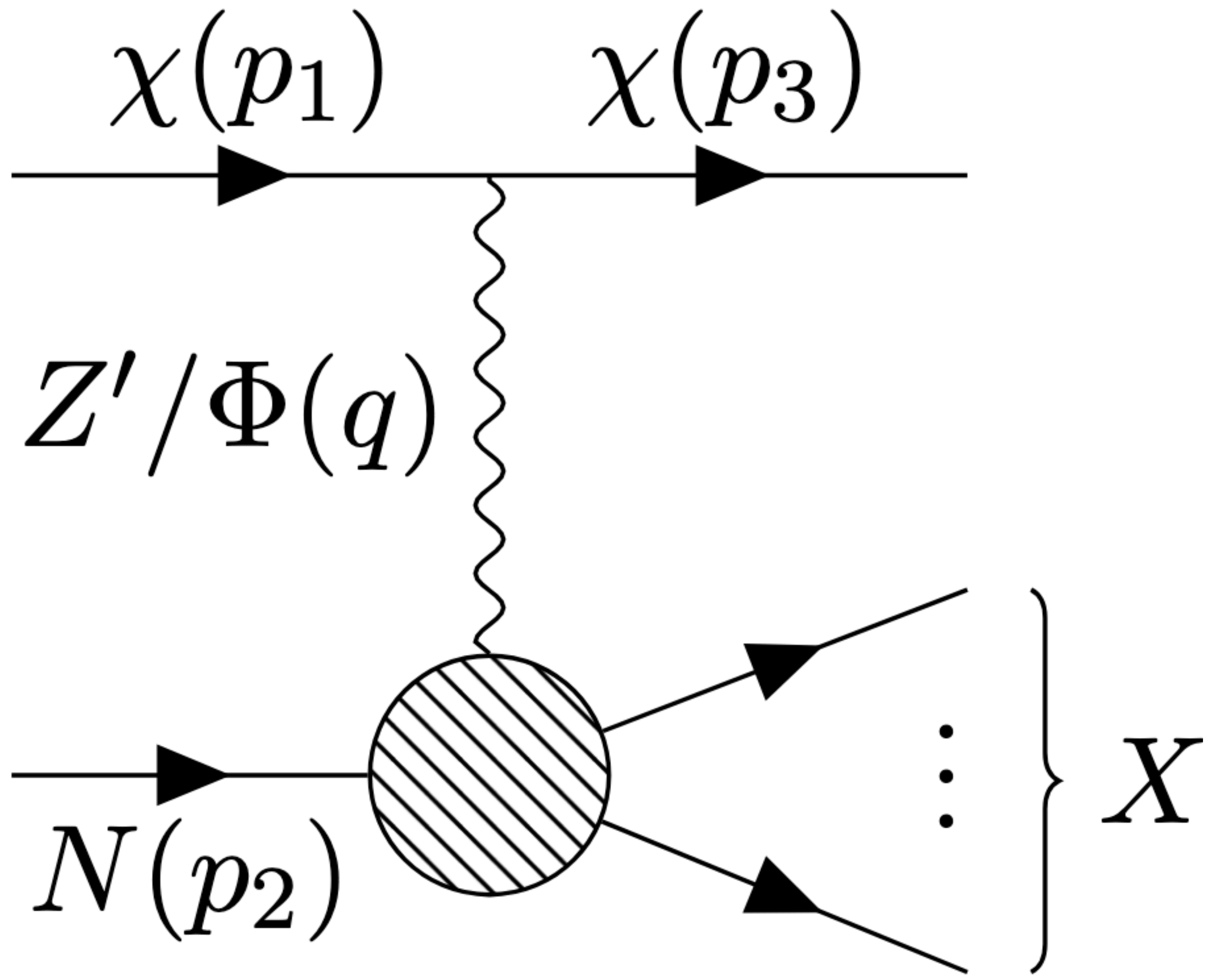
- Cosmic ray boosted DM





Deep inelastic scattering

- DM: **high incoming-energy** beyond the mass of the nucleons
- Deep inelastic scattering (**DIS**)
  - The valence quarks and the sea quarks become visible
  - Partons: carry a fraction of the total momentum of the nucleon $f_q(\xi)$
  - A hadronic shower
  - $\chi q \to \chi X$



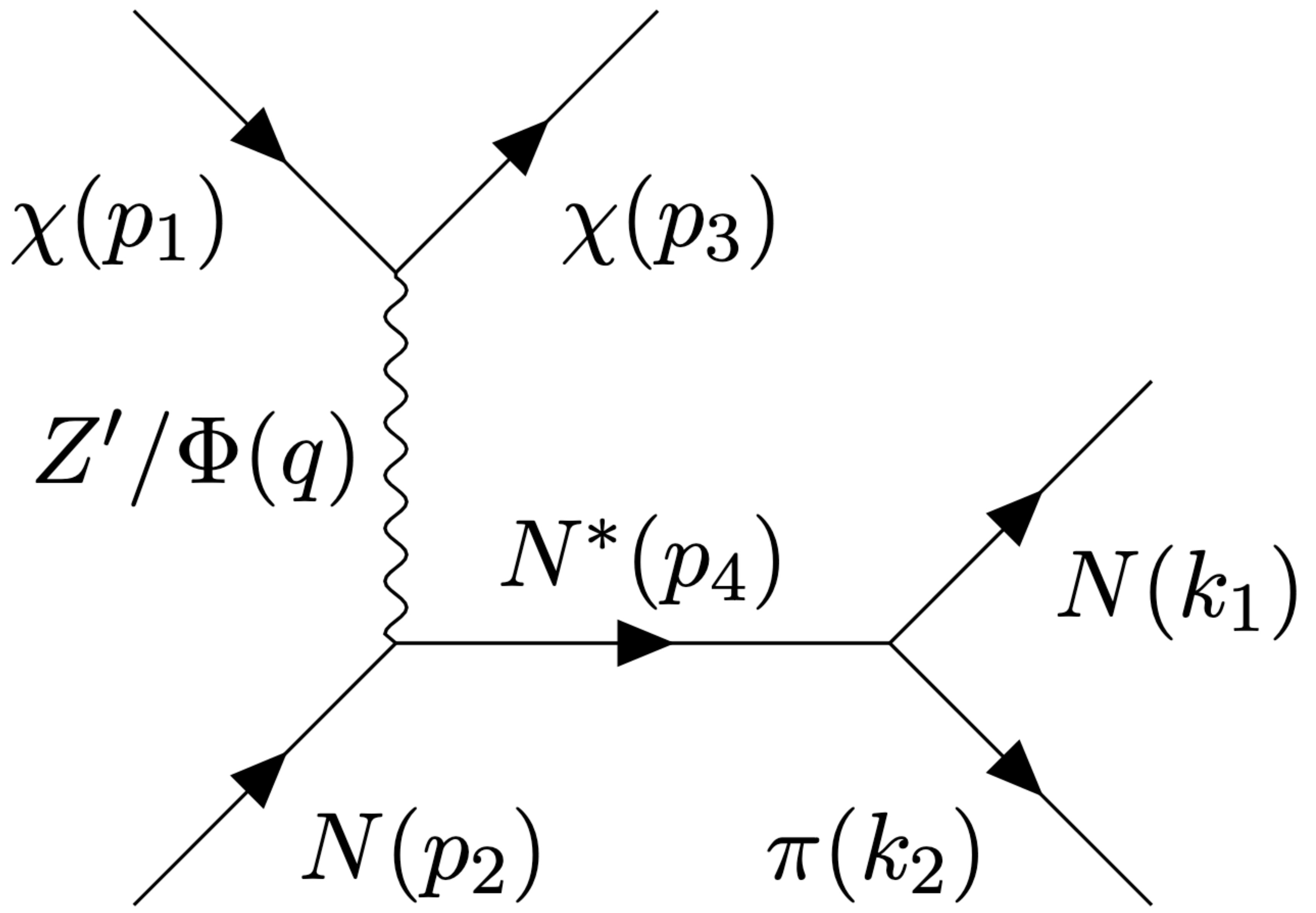
- Cross section: $\frac{d^2\sigma}{dxdy} = \frac{g_D^2}{4\pi m_{Z'}^4} \frac{E_\chi^2 m_N x}{(1+Q^2/m_{Z'}^2)^2} \frac{\sqrt{E_\chi^2(1-y)^2 - m_\chi^2}}{(1-y)(E_\chi^2 - m_\chi^2)} \times \sum_q g_{Z'q}^2 \left((A_q A_\chi + C_q C_\chi) y^2 - 2(A_q A_\chi - C_q C_\chi) y - \frac{A_q m_\chi^2}{E_\chi m_N x} B_\chi y + 2 A_q A_\chi \right) f_q(x)\,.$

Resonant scattering

- An inelastic interaction with a nucleon that produces a resonance that further decays into a nucleon and pion
- $\chi + N \to \chi + N^* \to \chi + N + \pi$.
- Neutral mediators (dark photon or scalar) have four possible channels:
  - $\chi + p \to \chi + p + \pi^0$
  - $\chi + p \to \chi + n + \pi^+$
  - $\chi + n \to \chi + n + \pi^0$
  - $\chi + n \to \chi + p + \pi^-$
- $\mathscr{M}(\chi(p_1, \lambda_1)N(p_2) \to \chi(p_3, \lambda_2)N^*(p_4))$

$$= \frac{g_D g_{Z'N}}{q^2 - m_{Z'}^2}\left[\bar{u}_{p_3\lambda_2}\gamma_\mu\left(g_V - g_A\gamma^5\right)u_{p_1\lambda_1}\right]\left(g^{\mu\nu} - q^\mu q^\nu/m_{Z'}^2\right) < N^* | J_\nu^+(0) | N >$$

$$= 2M\frac{g_D g_{Z'N}}{q^2 - m_{Z'}^2}V^\mu_{\lambda_1\lambda_2} < N^* | F_\mu^V | N > ,$$



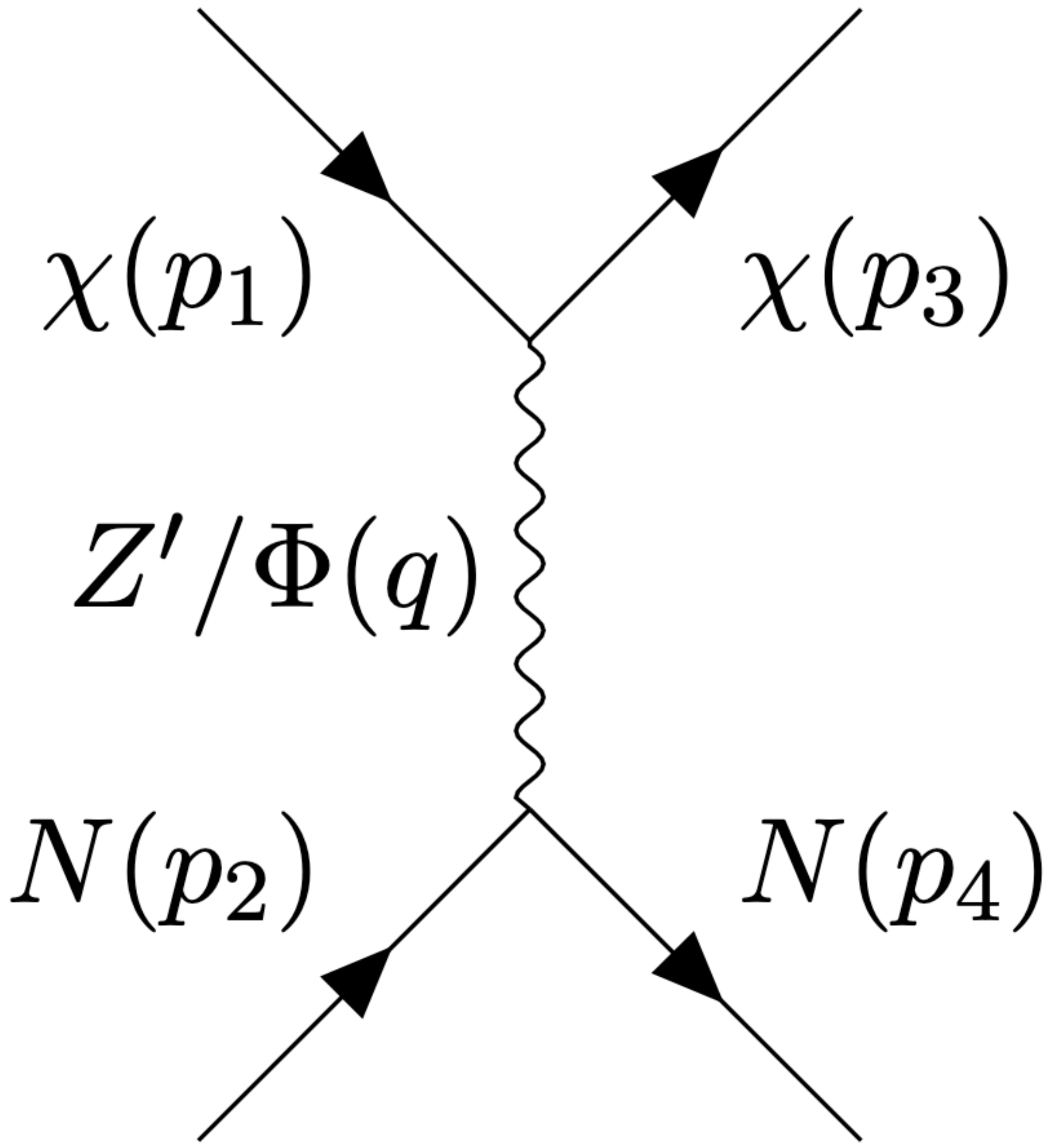
[D. Rein and L. M. Sehgal, Annals Phys. 133, 79 (1981).]

[C. Berger and L. M. Sehgal, Phys. Rev. D 76, 113004 (2007).]

[K. S. Kuzmin, V. V. Lyubushkin, and V. A. Naumov, Mod. Phys. Lett. A 19, 2815 (2004).]

Elastic scattering on nucleons

- Elastic interactions with nucleons
  - The DM incoming-energies are lower than those required for DIS
  - Form factor approach [J. D. Walecka, Vol. 16 (Cambridge University Press, 2001).]
- The amplitude



$$\mathcal{M}_N = i\frac{g_D g_{Z'N}}{q^2 - m_{Z'}^2}[\bar{u}(p_3)\gamma^\mu(g_V^\chi - g_A^\chi\gamma^5)u(p_1)]\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_{Z'}^2}\right) < N(p_4)\,|\,j^\nu_{Z'Q}(0)\,|\,N(p_2) >$$

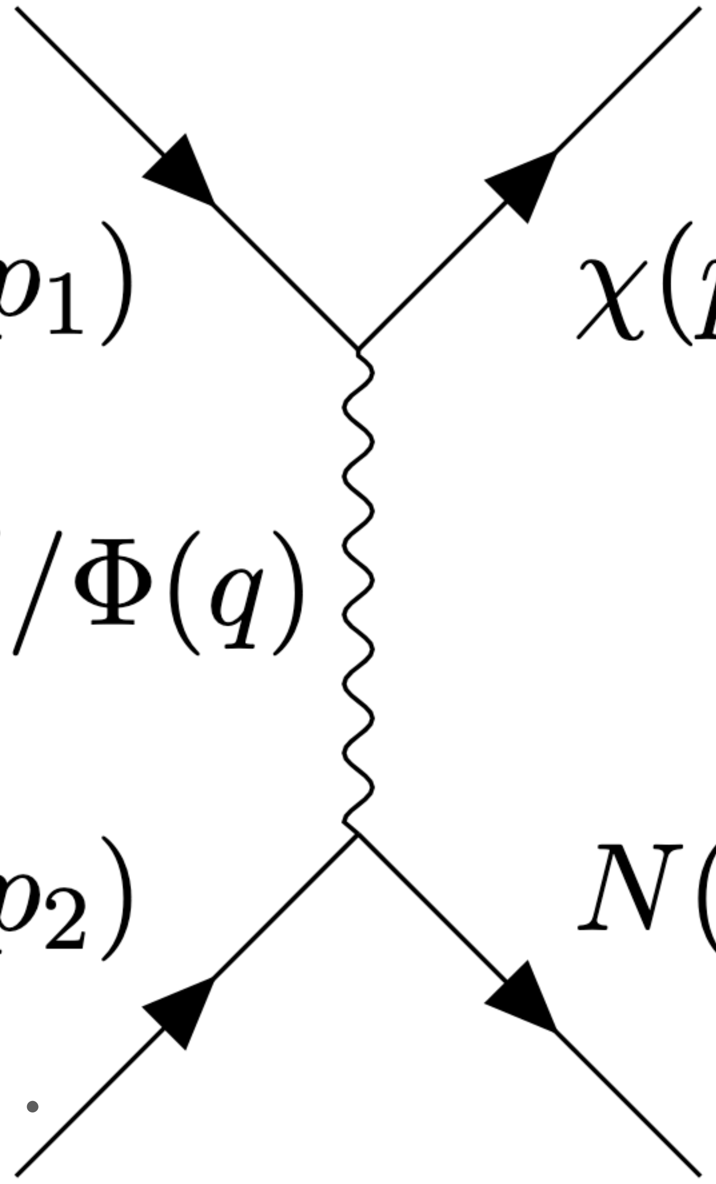
  - Hadronic current

$$< N(p_4)\,|\,v^\mu_{Z'Q}(0)\,|\,N(p_2) > = \bar{u}_N(p_4)\left[\gamma^\mu F_1^{Z'N}(Q^2) + i\frac{q_\nu}{2m_N}\sigma^{\mu\nu}F_2^{Z'N}(Q^2)\right]u_N(p_2),$$

$$< N(p_4)\,|\,a^\mu_{Z'Q}(0)\,|\,N(p_2) > = \bar{u}_N(p_4)\left[\gamma^\mu\gamma^5 G_A^{Z'N}(Q^2) + \frac{q_\mu}{m_N}\gamma^5 G_P^{Z'N}(Q^2)\right]u_N(p_2)\,.$$

Elastic scattering on nuclei

- Elastic interactions with nuclei
  - Low-energy regime [M. Gripes, G. Lalazissis, S. Massen, and C. Panos, Journal of Physics G: Nuclear and Particle Physics 17, 1093 (1991).]
  - Fermi-Symmetrized Woods-Saxon (**FS-WS**) form factor
  - A comprehensive treatment of 15 types of **non-relativistic (NR) operators** [R. Catena and B. Schwabe, JACP 04, 042 (2015).]
    - $\mathscr{H}_I^{Z'N} = \frac{g_D g_{Z'N}}{m_{Z'}^2}\left[g_V^\chi \bar{u}(p_3)\gamma_\mu u(p_1)\bar{u}(p_4)\gamma^\mu u(p_2) - g_A^\chi \bar{u}(p_3)\gamma_\mu\gamma^5 u(p_1)\bar{u}(p_4)\gamma^\mu u(p_2)\right].$



$$\bar{u}(p_3)\gamma_\mu u(p_1)\bar{u}(p_4)\gamma^\mu u(p_2) \simeq \quad 4m_\chi m_N + 2m_\chi m_N\left(i\hat{s}_N\cdot\left[\frac{\hat{q}}{m_N}\times\hat{v}^\perp\right]\right) + 2m_N^2\left(i\hat{s}_\chi\cdot\left[\frac{\hat{q}}{m_N}\times\hat{v}^\perp\right]\right)$$

$$-\frac{m_\chi}{m_N}\hat{q}^2 - 4\hat{q}^2\hat{s}_\chi\cdot\hat{s}_N + 4m_N^2\left(\hat{s}_\chi\cdot\frac{\hat{q}}{m_N}\right)\left(\hat{s}_N\cdot\frac{\hat{q}}{m_N}\right)$$

Nucleon form factors

$$\mathcal{M}_N = i\frac{g_D g_{\text{Had}}}{q^2 - m_{Z'}^2}[\overline{u}(p_3)\gamma^\mu(g_V^\chi - g_A^\chi\gamma^5)u(p_1)]\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_{Z'}^2}\right)\langle N(p_4)|\, j^\nu_{Z'Q}(0)\,|N(p_2)\rangle$$

$$j^\nu_{Z'Q} = \sum_q g_V^q \bar{q}\gamma^\nu q - \sum_q g_A^q \bar{q}\gamma^\nu\gamma^5 q.$$

$$j^\nu_{Z'Q} \equiv v^\nu_{Z'Q} - a^\nu_{Z'Q},$$

$$v^\nu_{Z'Q} = -2(g_V^u + 2g_V^d)v_3^\nu + 3(g_V^u + g_V^d)j^\nu_{AQ} + (g_V^u + g_V^d + g_V^s)v_s^\nu - [g_V^s\bar{b}\gamma^\nu b + (3g_V^u + 3g_V^d + g_V^s)(\bar{c}\gamma^\nu c + \bar{t}\gamma^\nu t)]$$

$$a^\nu_{Z'Q} = (g_A^u - g_A^d)a_3^\nu + (g_A^u + g_A^d)a_0^\nu + g_A^s a_s^\nu - \sum_{q=c,b,t}(g_A^s - g_A^q)\bar{q}\gamma^\nu\gamma^5 q.$$

$$\langle N(p_4)|\, v^\mu_{Z'Q}(0)\,|N(p_2)\rangle = \overline{u}_N(p_4)\left[\gamma^\mu F_1^{Z'N}(Q^2) + i\frac{q_\nu}{2m_N}\sigma^{\mu\nu}F_2^{Z'N}(Q^2)\right]u_N(p_2),$$

$$\langle N(p_4)|\, a^\mu_{Z'Q}(0)\,|N(p_2)\rangle = \overline{u}_N(p_4)\left[\gamma^\mu\gamma^5 G_A^{Z'N}(Q^2) + \frac{q_\mu}{m_N}\gamma^5 G_P^{Z'N}(Q^2)\right]u_N(p_2).$$

$$F_i^{Z'N} \simeq \mp(g_V^u + 2g_V^d)(F_i^p - F_i^n) + 3(g_V^u + g_V^d)F_i^N + (g_V^u + g_V^d + g_V^s)F_i^{sN}$$

$$G_k^{Z'N} \simeq \pm\frac{1}{2}(g_A^u - g_A^d)G_k + (g_A^u + g_A^d)G_k^{0N} + g_A^s G_k^{sN},$$

Fermi-symmetrized Woods-Saxon form factors

$$\frac{d\sigma^N}{dQ^2} = \frac{\sigma_0 E_\chi^2}{4\mu_N (E_\chi^2 - m_\chi^2)} F_H^2(Q^2),$$

$$F^{FS-WS}(Q) = \frac{3\pi a}{r_0^2 + \pi^2 a^2} \frac{a\pi \coth(\pi Q a) \sin(Q r_0) - r_0 \cos(Q r_0)}{Q r_0 \sinh(\pi Q a)},$$

$s \simeq 0.9$ fm, $a \simeq 0.523$ fm and $c \simeq 1.23 A^{1/3} - 0.60$ fm, for an atomic mass number A.