

Very-high-energy emission of the Crab Pulsar and tests for Lorentz Invariance Violation

Anna Campoy-Ordaz^{1,2}, Markus Gaug^{1,2},
on behalf of the MAGIC Collaboration

¹ Institut d'Estudis Espacials de Catalunya (IEEC), Spain

² Universitat Autònoma de Barcelona (UAB), Spain



IEEC[®]
Institut d'Estudis
Espacials de Catalunya

UAB
Universitat
Autònoma
de Barcelona

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I. Lorentz Invariance Violation

- Modified photon dispersion relation

$$E^2 = p^2 c^2 \times \left[1 + \sum_{n=1}^{\infty} S_n \left(\frac{E}{E_{QG,n}} \right)^n \right]$$

$S_n = +1$, superluminal
 $S_n = -1$, subluminal

$n = 1$, linear case
 $n = 2$, quadratic case

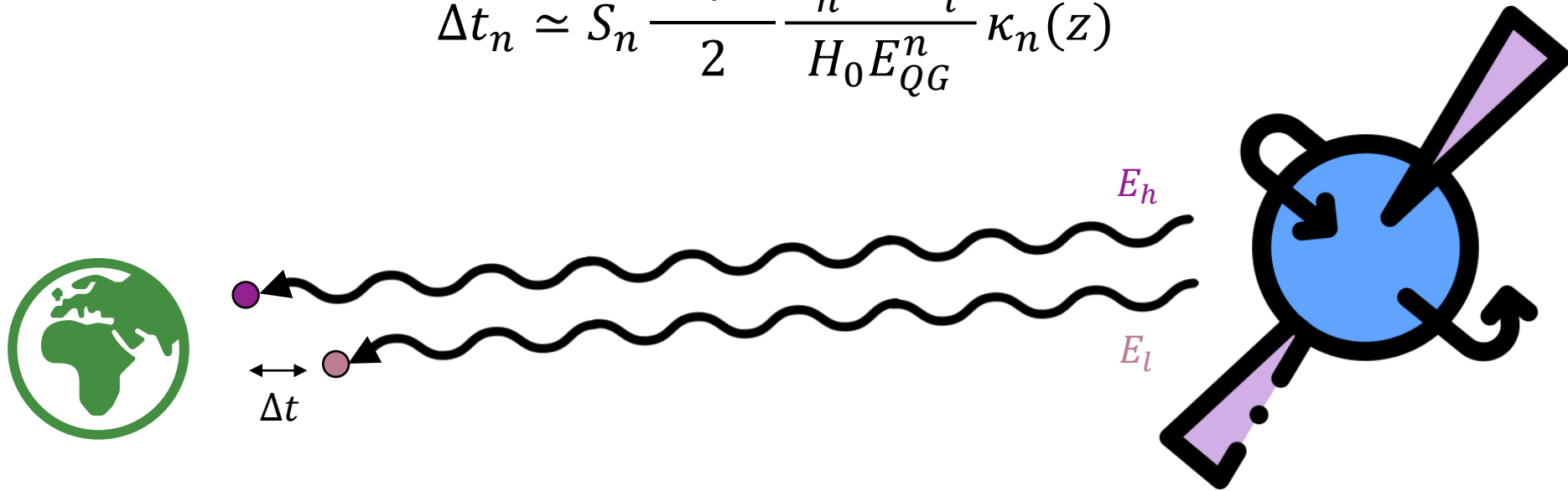
- Energy-dependent photon group velocity

$$v_\gamma = \frac{\partial E}{\partial p} \simeq c \left[1 + \sum_{n=1}^{\infty} S_n \frac{n+1}{2} \left(\frac{E}{E_{QG,n}} \right)^n \right]$$

I. Lorentz Invariance Violation

- Energy-dependent time delay

$$\Delta t_n \simeq S_n \frac{n+1}{2} \frac{E_h^n - E_l^n}{H_0 E_{QG}^n} \kappa_n(z)$$



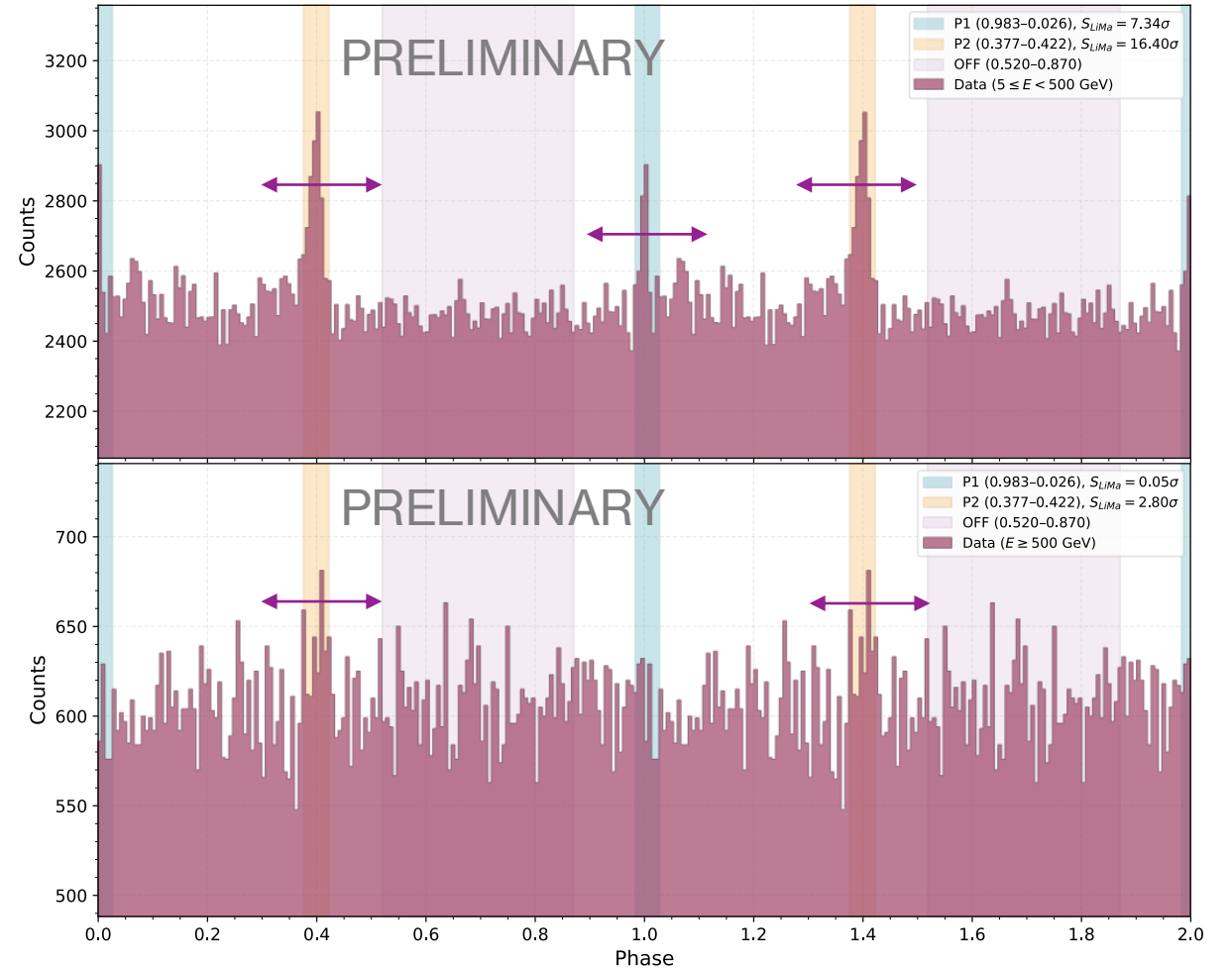
I. Lorentz Invariance Violation

- Energy-dependent phase delay

$$\Delta\phi = \frac{d_{Crab}}{cP_{Crab}} \cdot S_n \frac{n+1}{2} \frac{E_h^n - E_l^n}{E_{QG}^n}$$

- Bounds to $E_{QG,n}$

$$E_{QG,n} \gtrsim \left(S_n \frac{n+1}{2} \frac{d_{Crab}}{cP_{Crab}} \frac{E_h^n - E_l^n}{\Delta\phi} \right)^{1/n}$$



i. Competitiveness of LIV searches

- Active Galactic Nuclei (AGNs)
- Gamma-Ray Bursts (GRBs)

At cosmological distances,
BUT maximum observable
energy is limited by the
Extragalactic Background Light

- Pulsars

Independent of the
cosmological model

Source family	Distance [kpc]	Energy [GeV]	δt [s]	Expected bounds	
				$E_{QG,1}$	$E_{QG,2}$
AGN	$10^5 - 10^6$	10^4	$10^2 - 10^5$	$10^{15} - 10^{18}$	$10^9 - 10^{11}$
GRB	$10^5 - 10^7$	10^3	$10^0 - 10^2$	$10^{17} - 10^{19}$	$10^9 - 10^{10}$
Pulsar	$10^0 - 10^1$	10^2	$10^{-4} - 10^{-2}$	$10^{17} - 10^{19}$	$10^{10} - 10^{11}$

i. Competitiveness of LIV searches

- Active galactic nuclei (AGNs)
- Gamma-ray bursts (GRBs)

- Pulsars

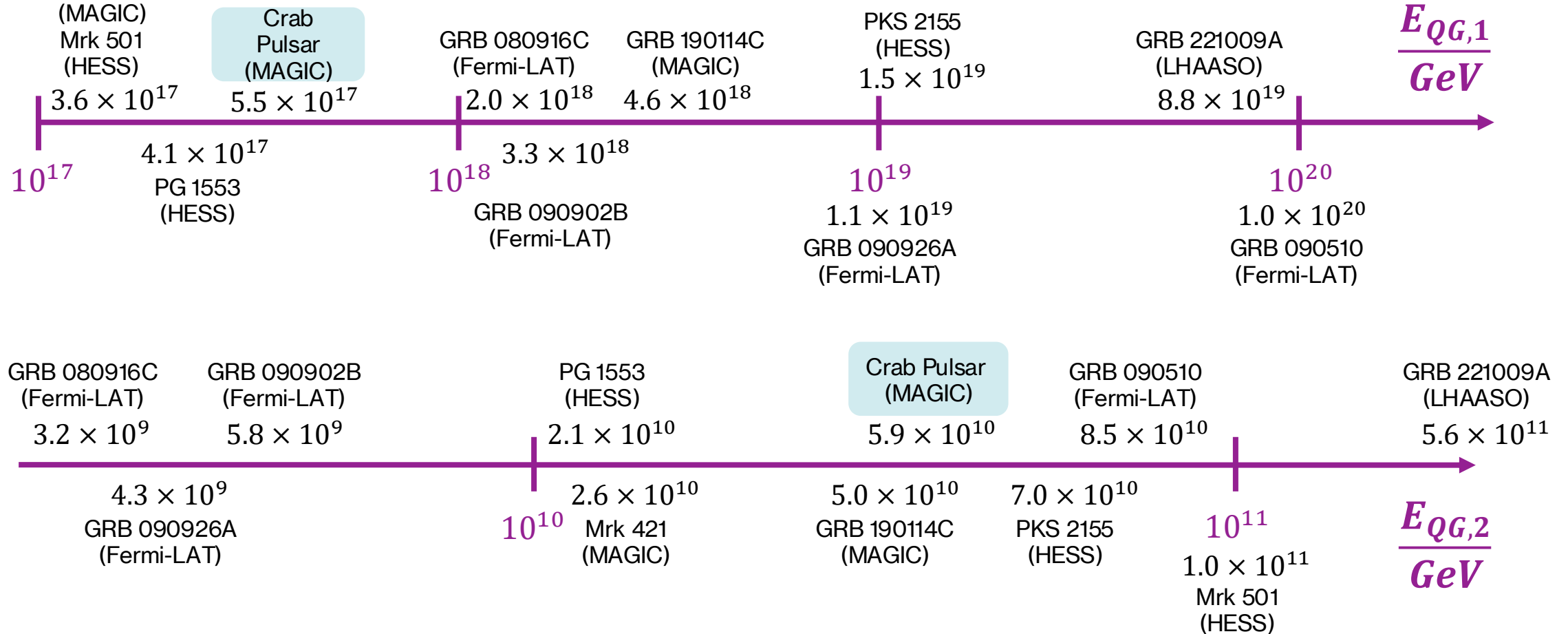


- Precisely timed flux oscillations
- Stable source
- **Sensitivity to LIV can be systematically planned**

Source family	Distance [kpc]	Energy [GeV]	δt [s]	Expected bounds	
				$E_{QG,1}$	$E_{QG,2}$
AGN	$10^5 - 10^6$	10^4	$10^2 - 10^5$	$10^{15} - 10^{18}$	$10^9 - 10^{11}$
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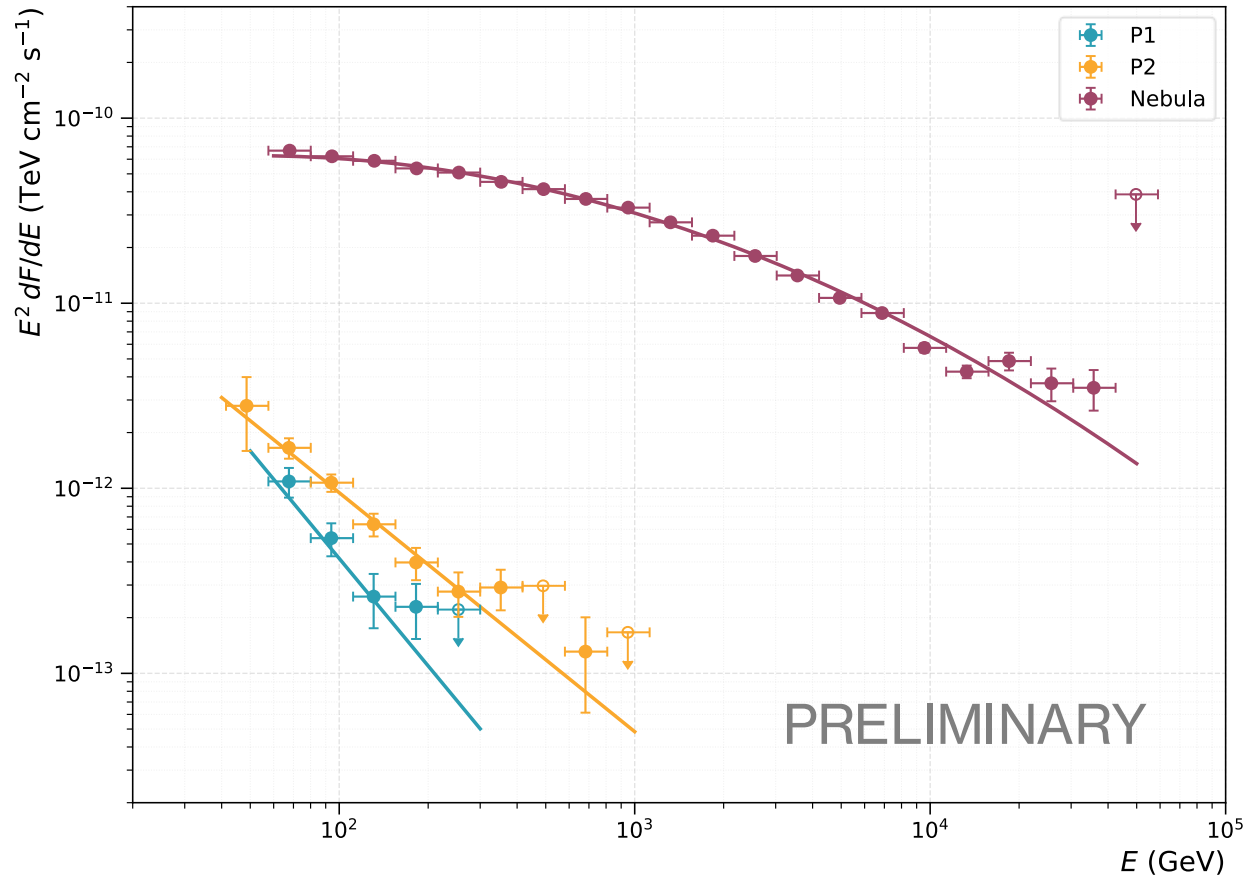
ii. Bounds on LIV from the Crab Pulsar

Guerrero, Campoy-Ordaz, Potting, Gaug (2025)



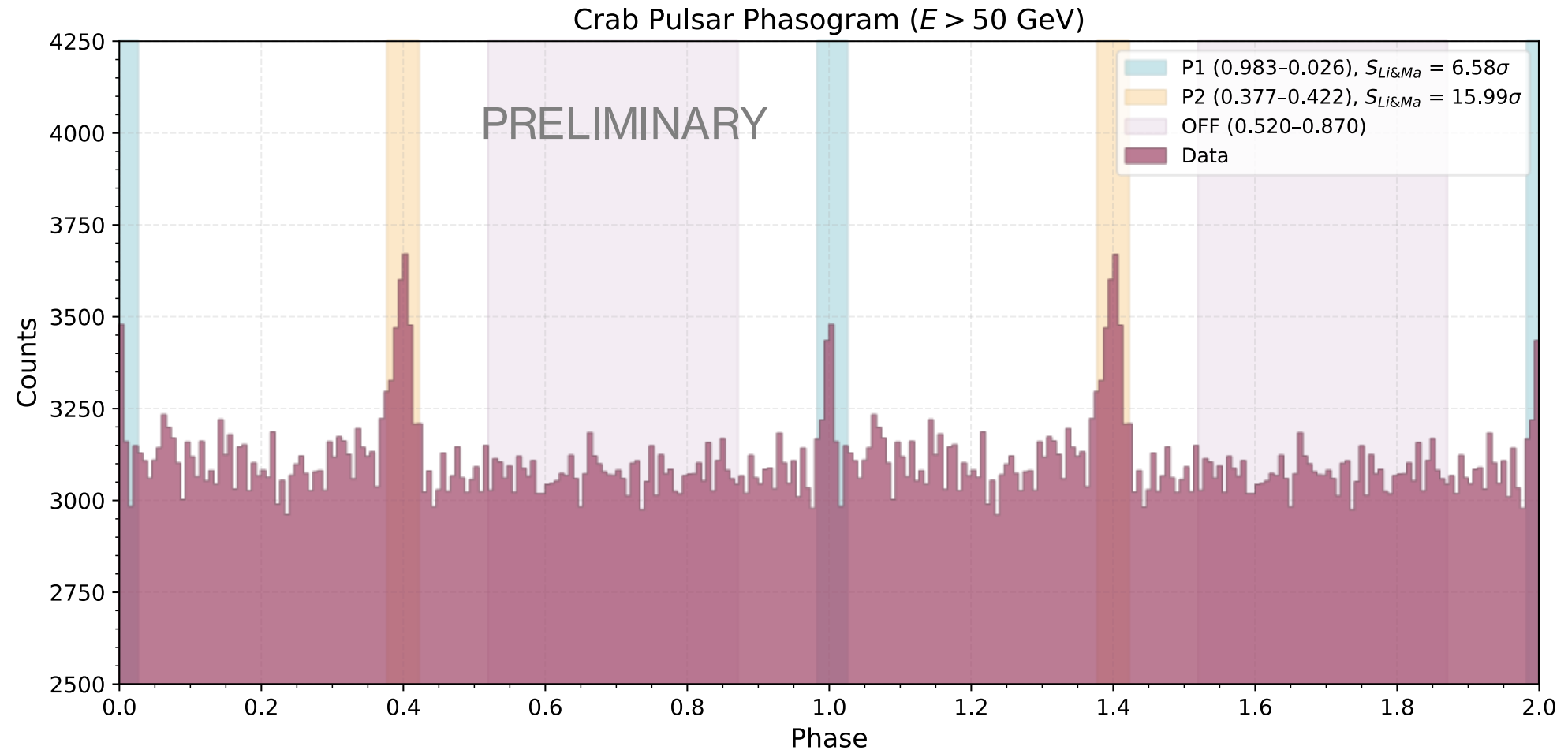
*Most stringent lower bounds based on time-of-flight studies, expressed at 95% CL.

II. Crab Pulsar Analysis



- MAGIC Telescopes
- 2009 to 2020
- ~300 hours of effective time
- Joining 11 different observation periods
- Only P2 for LIV studies
- Profile Maximum Likelihood Method

II. Crab Pulsar Analysis



II. Crab Pulsar LIV Analysis

$$\mathcal{L}(\lambda_n; \nu | X) = \mathcal{L}(\lambda_n; \underbrace{f, \alpha, \phi_{P2}, \sigma_{P2}}_{\text{Nuisance parameters}} | \underbrace{\{E'_i, \phi'_i, k_i\}}_{\text{LIV effect intensity}})$$

LIV effect intensity

$$\lambda_1 \equiv 10^{19} \text{GeV} / E_{QG_1}$$

$$\lambda_2 \equiv 10^{12} \text{GeV} / E_{QG_2}$$

Nuisance parameters

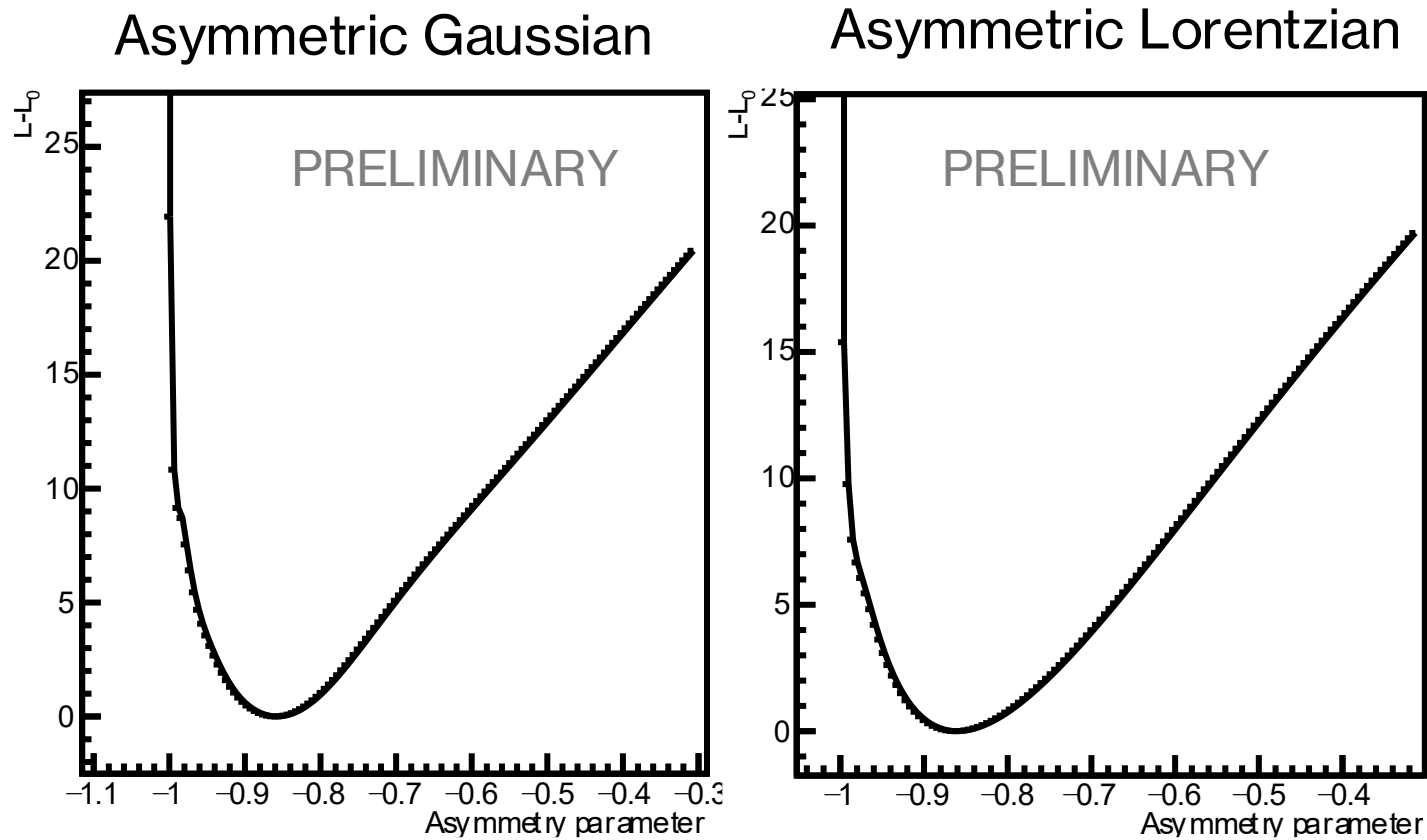
f : P2 flux normalization
 α : P2 spectral index
 ϕ_{P2} : Mean P2 pulse position
 σ_{P2} : Mean P2 width

Signal PDF $\rightarrow$ Pulsar phaseogram model for a given λ_n

- **Symmetric Gaussian**
- **Symmetric Lorentzian**
- **Asymmetric Gaussian**
- **Asymmetric Lorentzian**

a : Asymmetry parameter

II. Crab Pulsar LIV Analysis



- Asymmetric Gaussian pulse shape provides a significantly better fit than the symmetric Gaussian model (3.2σ)
- Asymmetric Lorentzian pulse shape is significantly preferred over the symmetric Lorentzian model (3.3σ)
- Asymmetric Lorentzian moderately preferred over asymmetric Gaussian ($\Delta AIC = 3.1$)

II. Crab Pulsar LIV Analysis

PRELIMINARY

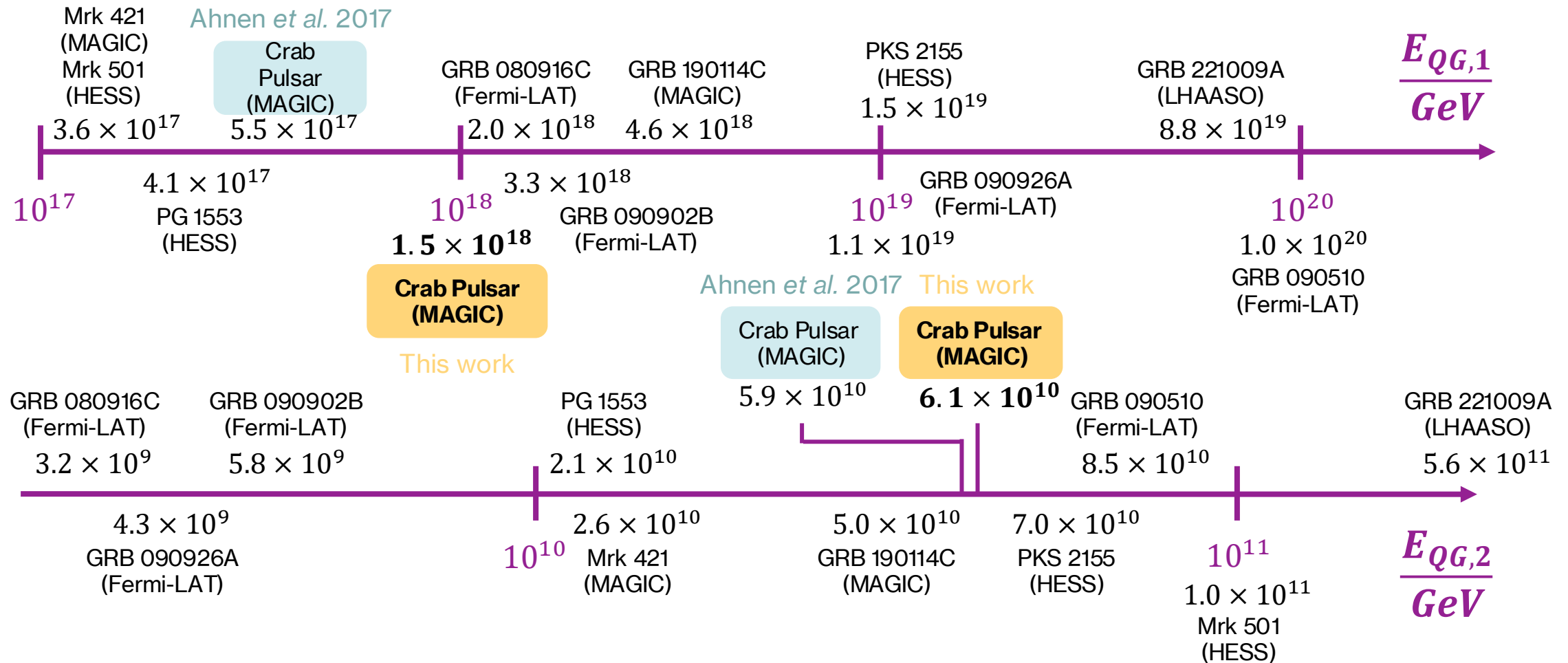
This work

Case	$E_{QG,1}$ (GeV)
$S_1 = +1$	3.5×10^{17}
$S_1 = -1$	1.5×10^{18}
	$E_{QG,2}$ (GeV)
$S_2 = +1$	1.8×10^{10}
$S_2 = -1$	6.1×10^{10}

95% CL bounds from the Profile Likelihood Method (*without systematics)

*Asymmetric Lorentzian pulse shape with the asymmetry parameter fixed.

II. Crab Pulsar LIV Analysis



*Most stringent lower bounds based on time-of-flight studies, expressed at 95% CL.

III. Conclusions

- Spectral analysis:
 - Results marginally compatible with Ansoldi *et al.* 2016
- LIV analysis:
 - Asymmetry of the pulse shape
 - Results compatible with Ahnen *et al.* 2017
- General outlook:
 - Plan observations to improve the sensitivity
 - Upcoming CTAO
 - Joint LIV analysis: LIVelihood



Thank you for your attention!

- i. Amelino-Camelia, G., & Smolin, L. (2009). **Prospects for constraining quantum gravity dispersion with near term observations.** *Phys. Rev. D*, 80(8), 084017.
- ii. Otte, A. (2012). **Prospects of performing Lorentz invariance tests with VHE emission from pulsars.** *Proceedings of the 32nd International Cosmic Ray Conference (ICRC 2011)*, 7.
- iii. Guerrero, M., Campoy-Ordaz, A., Potting, R., Gaug, M. (2025). **Bounding anisotropic Lorentz invariance violation from measurements of the effective energy scale of quantum gravity.** *Phys. Rev. D*, 112(10), 104002.
- iv. Ansoldi *et al.* 2016. **Teraelectronvolt pulsed emission from the Crab Pulsar detected by MAGIC.** *Astronomy & Astrophysics*, 585, A133.
- v. Ahnen *et al.* 2017. **Constraining Lorentz Invariance Violation Using the Crab Pulsar Emission Observed up to TeV Energies by MAGIC.** *ApJS*, 232, 9.

BACK UP

II. Crab Pulsar LIV Analysis

Ahnen *et al.* 2017

Fitting both ON and OFF regions

$$\mathcal{L}(\lambda_n; \boldsymbol{\nu} | \mathbf{X}) = \mathcal{L}(\lambda_n; f, \alpha, \phi_{P2}, \sigma_{P2} | \{ \{E'_i, \phi'_i\}_{i=0}^{N_k} \}_{k=0}^{N_s})$$

$$= P(\boldsymbol{\nu}) \cdot \prod_{k=0}^{N_s} \exp \left(-g_k(\lambda_n; \boldsymbol{\nu}) - b_k \cdot \frac{1 + \tau}{\tau} \right) \cdot \prod_{m=0}^{N_k^{OFF}} b_k \cdot \prod_{i=0}^{N_k^{ON}} (g_k(\lambda_n; \boldsymbol{\nu}) + b_k/\tau) \cdot \mathcal{P}_k(E'_i, \phi'_i | \lambda_n; \boldsymbol{\nu}) ,$$

Individual event PDF depends on:

(fitted) background spectral energy distribution $h(E')$ and PDF of the signal $S(E', \Phi' | \lambda_n; \boldsymbol{\nu})$

$$\mathcal{P}_k(E'_i, \phi'_i | \lambda_n; \boldsymbol{\nu}) = \frac{b_k/\tau \cdot h_k(E') + g_k(\lambda_n; \boldsymbol{\nu}) \cdot S_k(E'_i, \phi'_i | \lambda_n; \boldsymbol{\nu})}{g_k(\lambda_n; \boldsymbol{\nu}) + b_k/\tau}$$

Uses signal normalization $\mathbf{g}_k$ (which depends on all nuisance parameters!) and background norm $\mathbf{b}_k/\tau$

II. Crab Pulsar LIV Analysis

Ahnen *et al.* 2017

$$\begin{aligned}\mathcal{L}(\lambda_n; \boldsymbol{\nu} | \mathbf{X}) &= \mathcal{L}(\lambda_n; f, \alpha, \phi_{P2}, \sigma_{P2} | \{ \{ E'_i, \phi'_i \}_{i=0}^{N_k} \}_{k=0}^{N_s}) \\ &= P(\boldsymbol{\nu}) \cdot \prod_{k=0}^{N_s} \exp \left(-g_k(\lambda_n; \boldsymbol{\nu}) - b_k \cdot \frac{1 + \tau}{\tau} \right) \cdot \prod_{m=0}^{N_k^{OFF}} b_k \cdot \prod_{i=0}^{N_k^{ON}} (g_k(\lambda_n; \boldsymbol{\nu}) + b_k/\tau) \cdot \mathcal{P}_k(E'_i, \phi'_i | \lambda_n; \boldsymbol{\nu}) ,\end{aligned}$$

PDF of the signal:

$$\begin{aligned}S_k(E'_i, \phi'_i | \lambda_n; \boldsymbol{\nu}) \\ = \frac{\Delta t_k \int_0^\infty R_k(E | E'_i) \cdot \Gamma_{P2}(E, f, \alpha) \cdot F_{P2}(\phi'_i, E | \lambda_n; \phi_{P2}, \sigma_{P2}) dE}{g_k(\lambda_n; \boldsymbol{\nu})},\end{aligned}$$

With Γ_{P2} : pulsar spectrum & F_{P2} : **pulsar phaseogram model**.

II. Crab Pulsar LIV Analysis

➤ Asymmetric Lorentzian

$$f(x) = \begin{cases} \frac{A}{\pi} \frac{\sigma_1}{(x - x_0)^2 + \sigma_1^2} + C, & x < x_0 \\ \frac{A}{\pi} \frac{\sigma_2}{(x - x_0)^2 + \sigma_2^2} \frac{\sigma_2}{\sigma_1} \left(1 + \frac{\pi \sigma_1 C}{A} \right), & x > x_0 \end{cases}$$

$$\sigma_2 = \sigma_1 \cdot (1 + a)$$

Asymmetry parameter