



2026 9/3 TeVPA

Gravitational-Wave signals in Pulsar Timing Array from Confined Flux Tubes as Cosmic Strings

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Particle and Cosmology group D1
(arXiv:2512.16821 published in PRD)

contents

- Background
- Color flux tube as cosmic string
- Theory for string networks
- Our result: Delayed scaling and GWs spectrum
- Summary

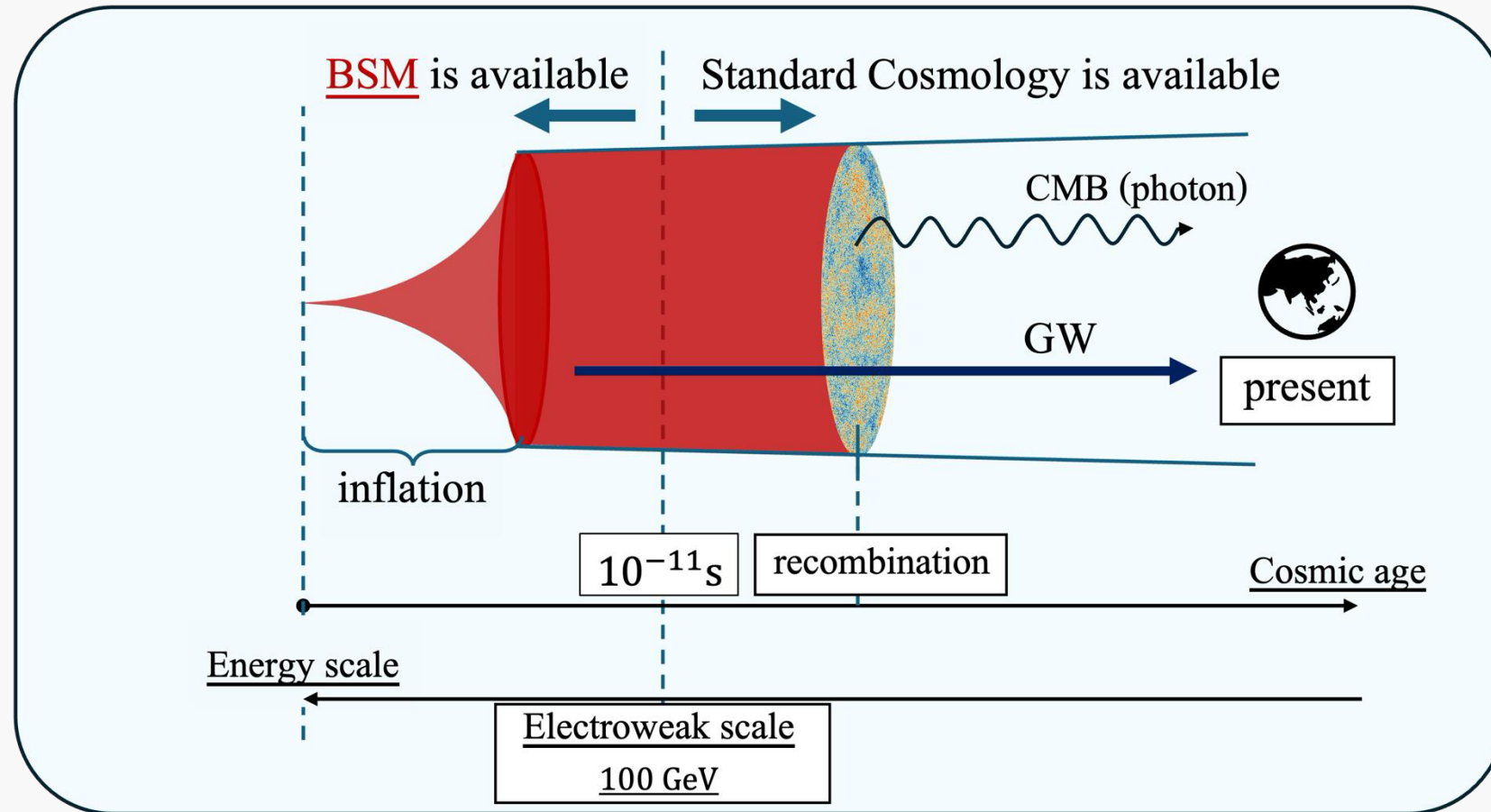
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Gravitational waves from BSM physics in the early universe can reach today

- GWs undergo almost no scattering and carry early-universe information
- They are observed as a stochastic GW background



Cosmic Superstring(CSS) can be a source of GWs

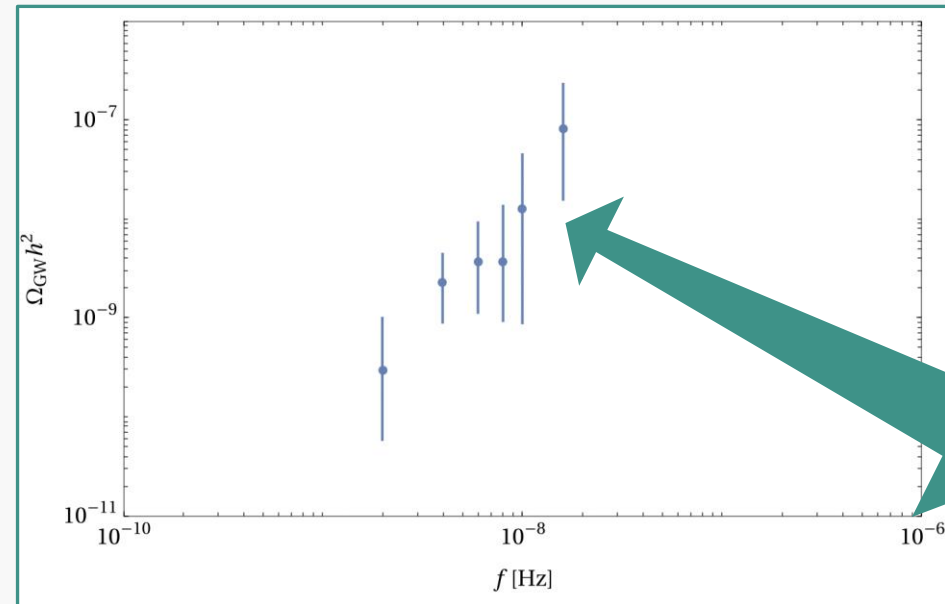
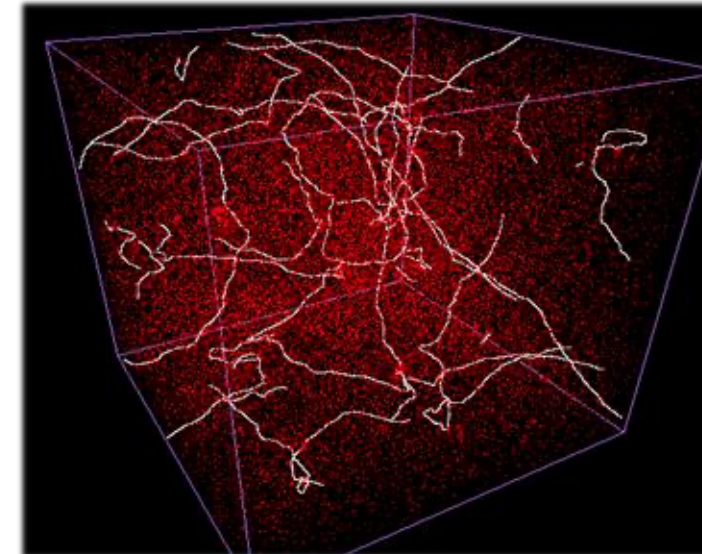
Multi-type CSS also arise
in Gauge theory

Yamada, Yonekura 2022

(Bodiut, Victor Andrei. (2016). The Multiverse. 10.13140/RG.2.1.4131.0323)

Cosmic Superstring

- Macroscopic 1D object in string theory
- GWs are distributed over a wide range
- Quantum property



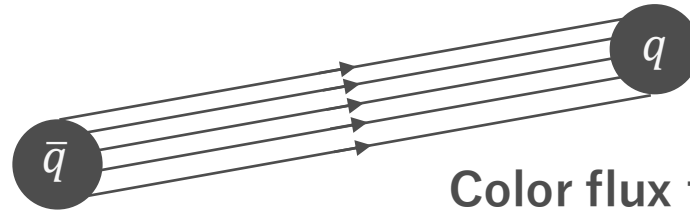
NANOGrav collaboration, G. Agazie et al.,
Astrophys. J. Lett. 951 (2023) L8 [2306.16213].

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- **Color flux tube as cosmic string**
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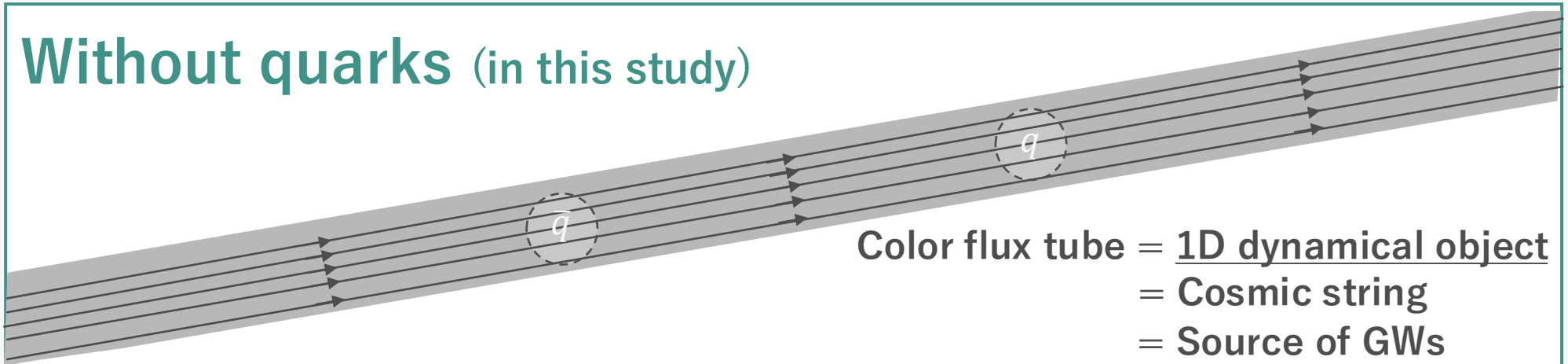
Stabel color flux tube behave as **dynamical object**

With quarks



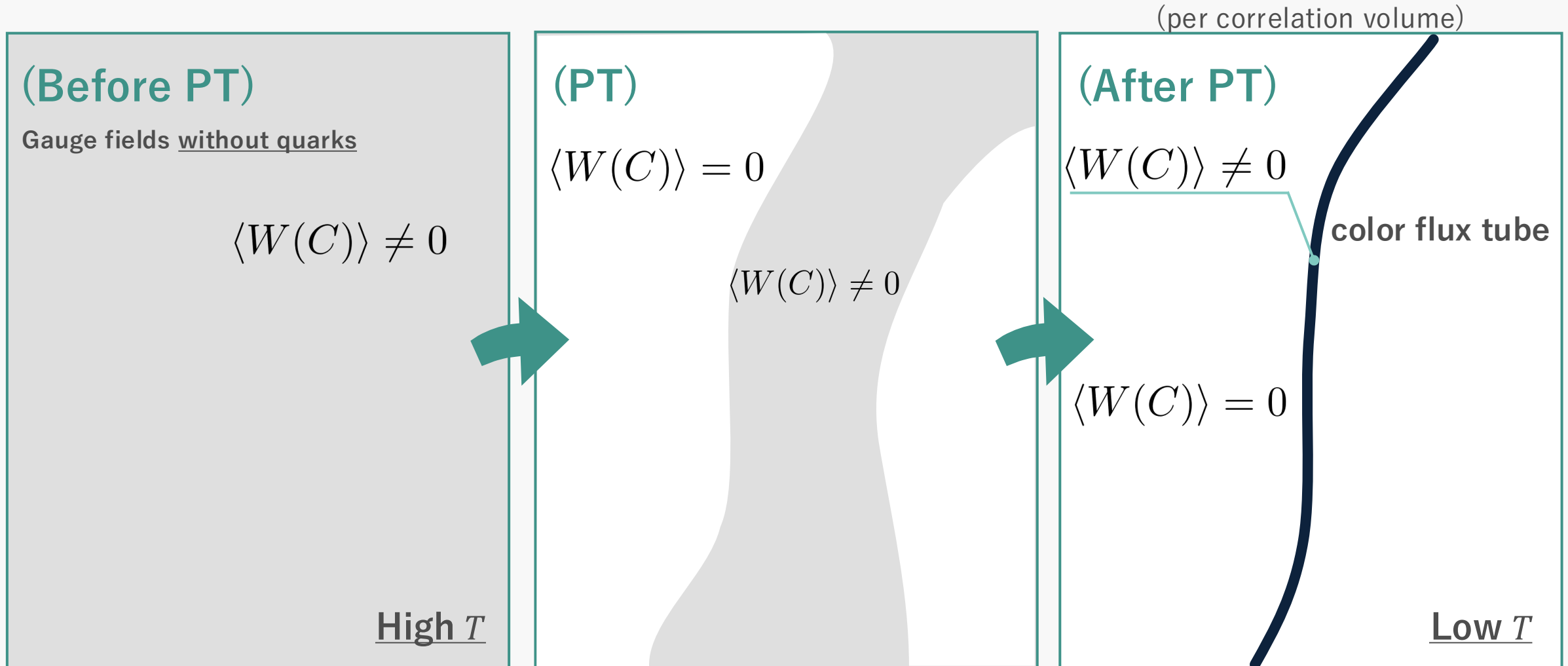
Color flux tube is cut by quarks

Without quarks (in this study)



Color flux tube = 1D dynamical object
= Cosmic string
= Source of GWs

1st order Phase Transitions make color flux tube

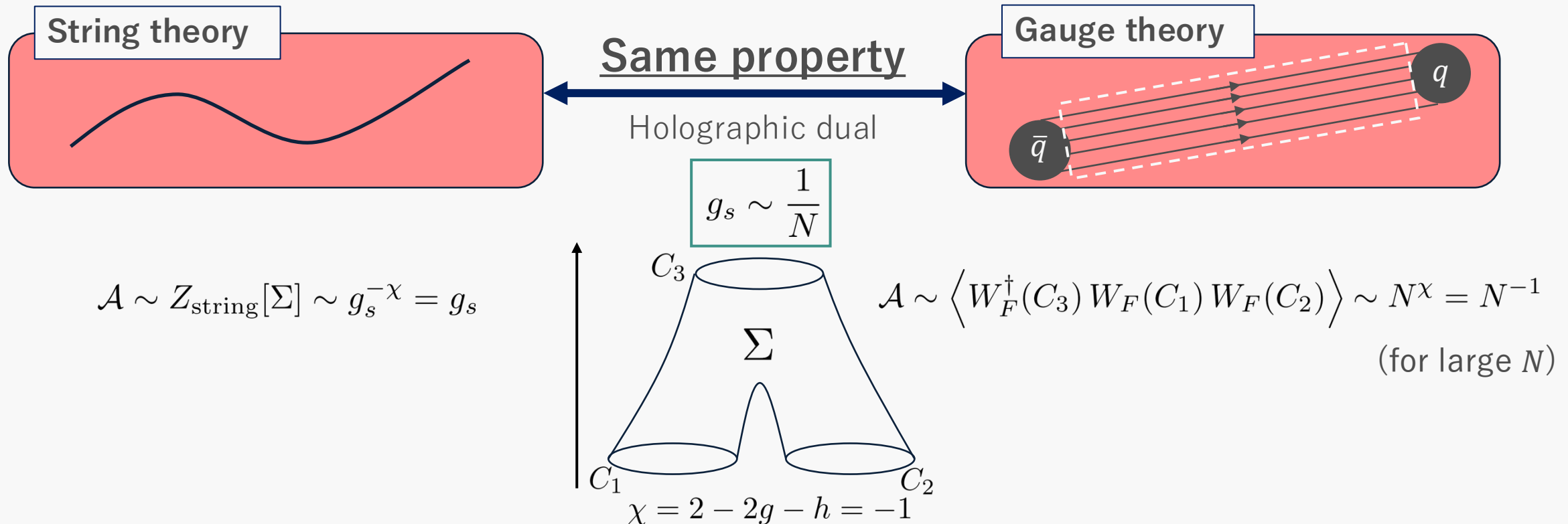


Color flux tube has Holographic dual description

$$S_{\text{ws}} = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{h} h^{ab} G_{\mu\nu}(X) \partial_a X^\mu \partial_b X^\nu + \frac{1}{4\pi} \int d^2\sigma \sqrt{h} \Phi(X) R^{(2)} + \dots$$

$$S_{\text{YM}} = -\frac{1}{4g^2} \int d^4x F_{\mu\nu}^a F^{a\mu\nu},$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc} A_\mu^b A_\nu^c$$



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The Network approaches the scaling regime

Reconnection of strings

- When strings collide, loops are produced via interactions called reconnection.

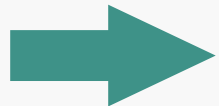


When long strings are dense

- **Frequent** reconnection
⇒ String number density decreases **faster** than cosmic expansion.

When long strings are dilute

- **Suppressed** reconnection
⇒ String number density decreases **more slowly** than cosmic expansion.



Quantitative modeling of this network
= **VOS model**

We can use the VOS model to study network

Statistical quantities

$$v, \rho = \frac{\mu L}{L^3} = \frac{\mu}{L^2}, L \equiv \gamma(t) \cdot t$$

Nambu-Goto action

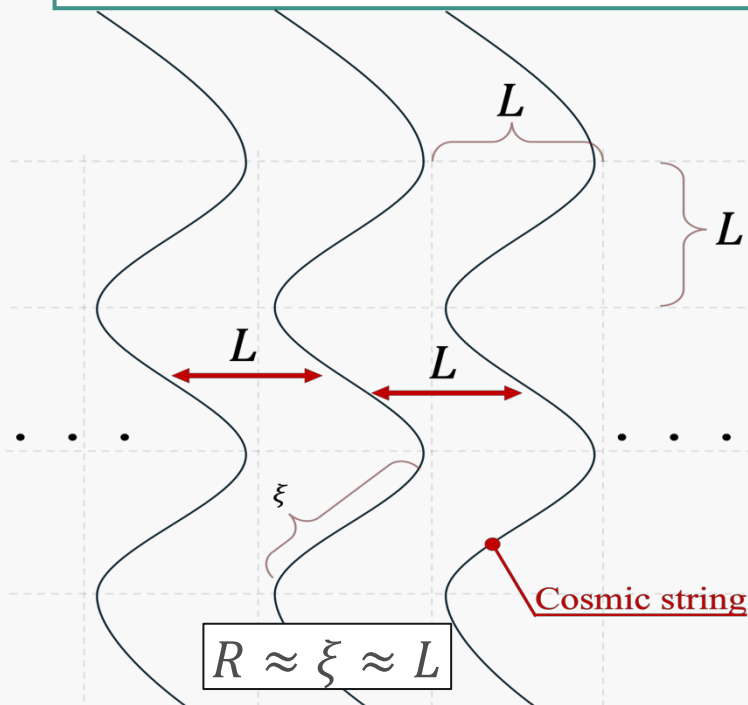
$$S_{\text{NG}} = -\mu \int d^2\zeta \sqrt{-\gamma}$$

(Background:
expanding universe)

$$\left. \begin{array}{l} \mu : \text{Tension of string} \\ k(v) = \frac{2\sqrt{2}}{\pi} \frac{1 - 8v^6}{1 + 8v^6} \\ \tilde{c} : \text{loop chopping efficiency} \end{array} \right\}$$

$$\left. \begin{array}{l} \frac{\dot{\gamma}}{\gamma} = \frac{1}{2t} \left(2Ht(1 + v^2) - 2 + \frac{\tilde{c}v}{\gamma} \right) \\ \dot{v} = \frac{1 - v^2}{t} \left(\frac{k(v)}{\gamma} - 2Htv \right) \end{array} \right\}$$

VOS model
(Velocity-dependent
One Scale model)



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We consider color flux tube in pure Yang-Mills theory

Feature①

Color flux tubes in $\text{Spin}(4N)$ ($N \geq 2$)

- We consider the gauge group $\text{Spin}(4N)$ ($N \geq 2$).
- Fermion doesn't exist.
⇒ color flux tube is stable
even when it is macroscopic
⇒ color flux tube
behave as cosmic string

Feature②

Three types of strings coexist

- The center of $\text{Spin}(4N)$ ($N \geq 2$) is given by $\mathbb{Z}_2 \times \mathbb{Z}_2$.
- Three string types arise:
 $(1,0), (0,1), (1,1)$.
- Two of them are symmetric.

Feature③

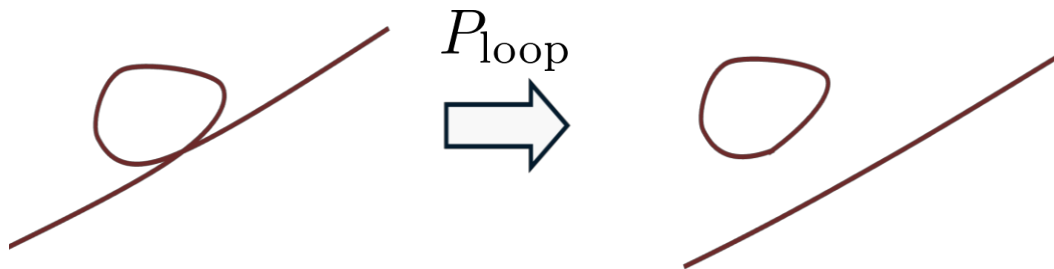
Interactions with reconnection probabilities $P_i < 1$

- All three strings possess quantum properties.
- Each string type has two types of interaction.
 - Loop formation
 - Zipper interaction

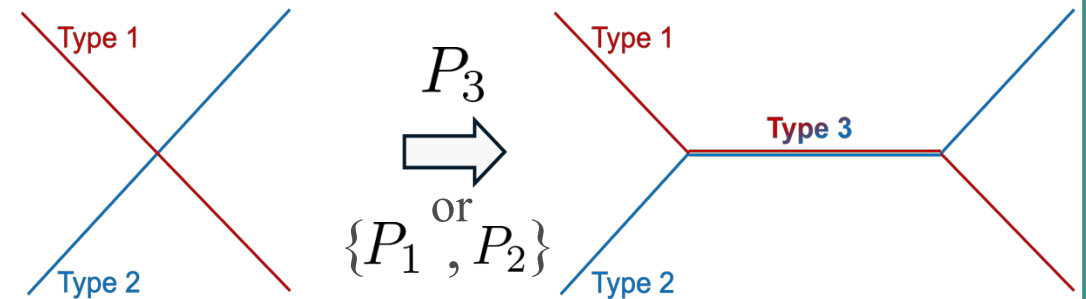
String networks have two quantum interactions

- Two interactions are included phenomenologically

Loop production



Zipper type interaction



Due to quantum effects, interactions occur **probabilistically**

⇒ Introduced the reconnection probability P_i

VOS model with interaction and reconnection probability

VOS equations derived in this work $\{i, j, k\} = \{1, 2, 3\}$

$$\frac{\dot{\gamma}_i}{\gamma_i} = \frac{1}{2t} \left(2Ht(1 + v^2) - 2 + P_{\text{loop}} \frac{c_\xi \tilde{c} v_i}{\gamma_i^2} + P_k d \bar{v}_{ij} \frac{c_\xi}{\gamma_j^2} + P_j d \bar{v}_{ki} \frac{c_\xi}{\gamma_k^2} - P_i d \bar{v}_{jk} \frac{c_\xi \gamma_i^2}{\gamma_j^2 \gamma_k^2} \right)$$

$$\dot{v}_i = \frac{1 - v_i^2}{t} \left(\frac{k(v_i)}{c_\xi} - 2Hv_i t \right) \left[\begin{array}{l} P_{\text{loop}} : \text{reconnection probability for loop production} \\ P_i : \text{reconnection probability for Zipper type interaction} \\ \bar{v}_{ij} : \text{average relative velocity between type } i \text{ and type } j \end{array} \right]$$

const parameters

$$d, \tilde{c}, c_\xi$$

Relation

$$P_1 = P_2$$

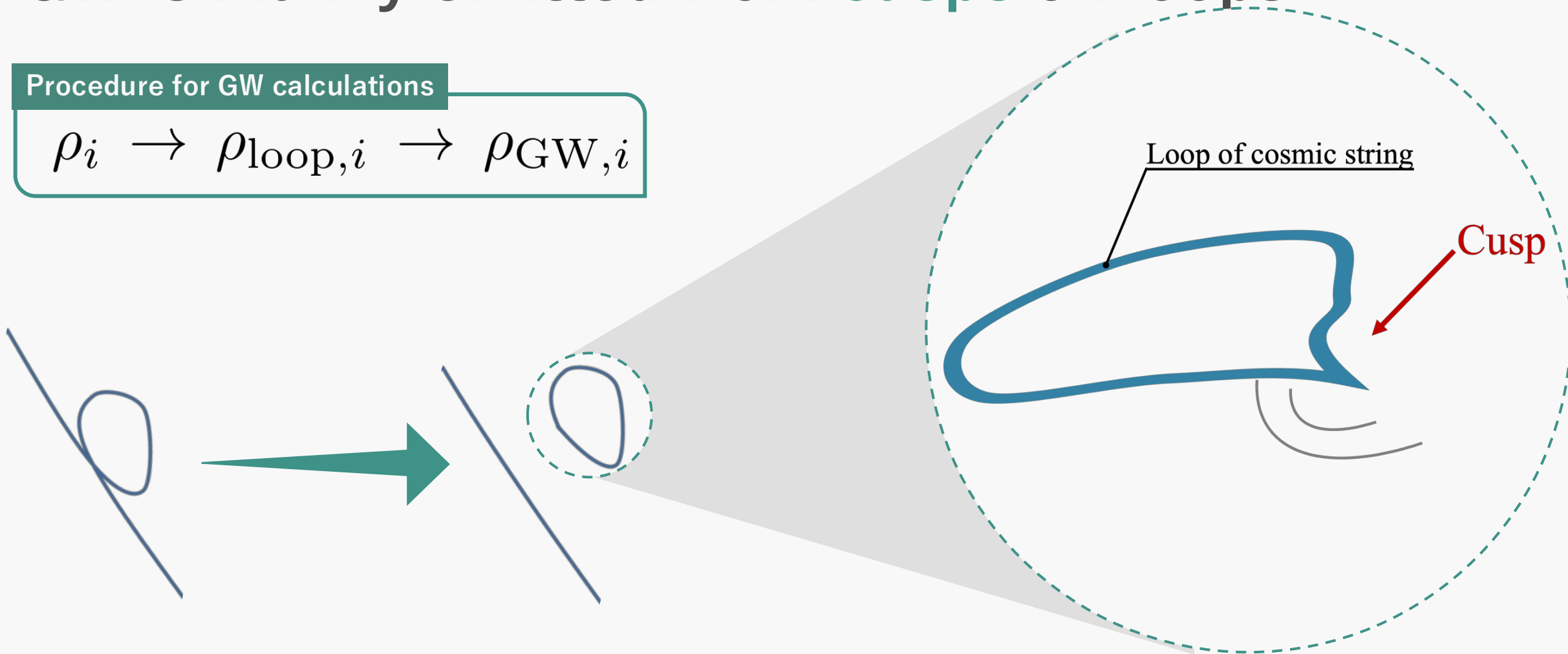
Two of three string types are symmetric.

⇒ Only $\gamma_1 = \gamma_2, \gamma_3$ are required.

GW is mainly emitted from **cusps** on loops

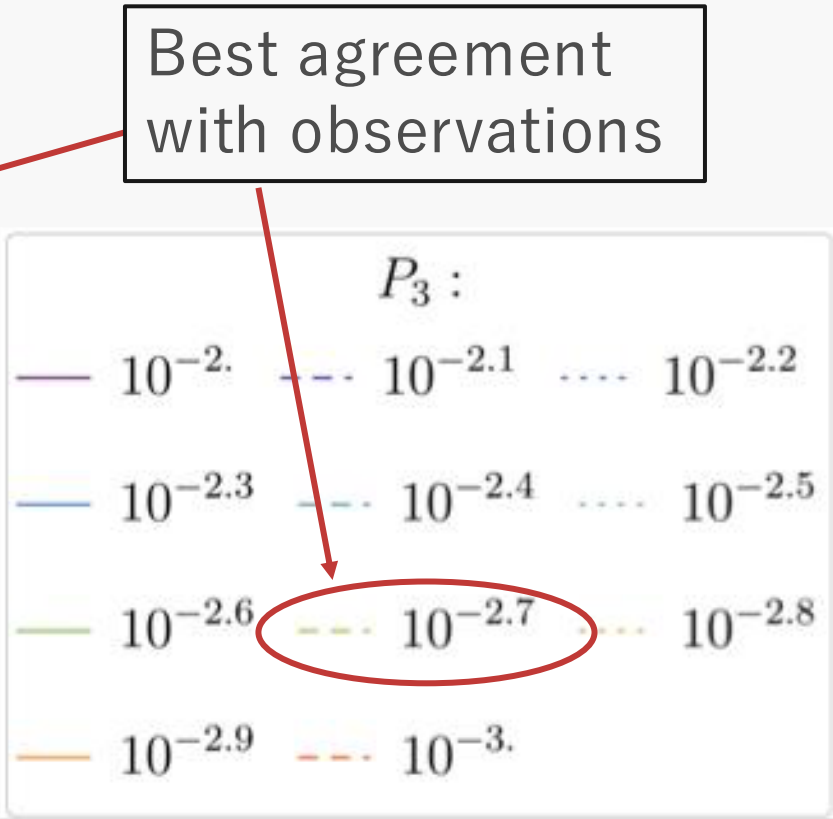
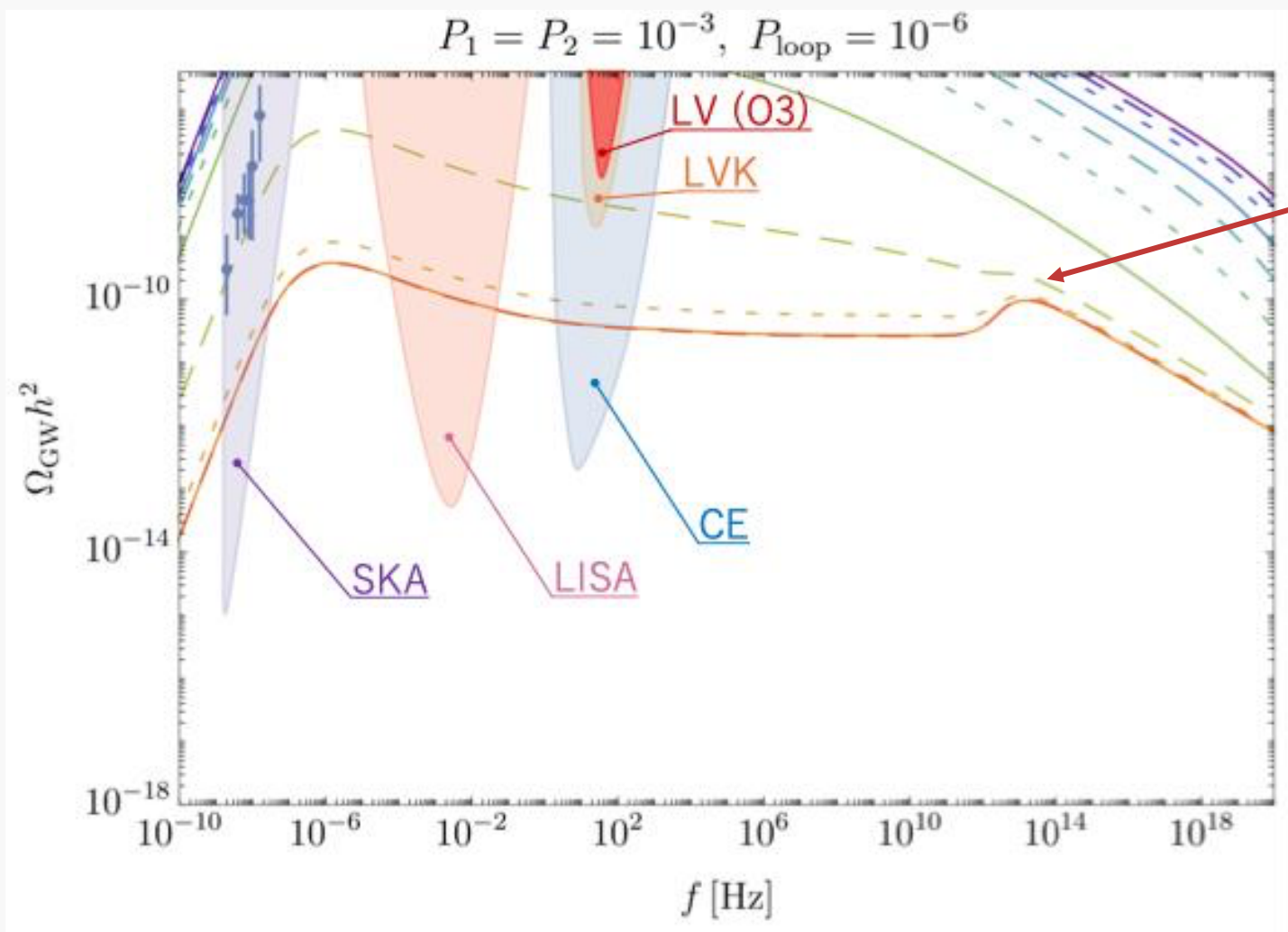
Procedure for GW calculations

$$\rho_i \rightarrow \rho_{\text{loop},i} \rightarrow \rho_{\text{GW},i}$$



The GW signal can be explained for certain $P_{\{i\}}$

K. N, Yamada 2025



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Distinct dynamics and GW emission are obtained

Dynamics of multi-species CSS networks

- Revealed the $P_{\{i\}}$ dependence of the scaling solution. (not shown here)
- Revealed the $P_{\{i\}}$ dependence of the scaling time. (not shown here)
- Demonstrated the existence of enhanced scaling time.

GW emission from multi-species CSS networks

- Revealed the $P_{\{i\}}$ dependence of the GW spectrum.
- Revealed the existence of parameter regions where high-frequency GWs are suppressed.

Comparison with GW observations

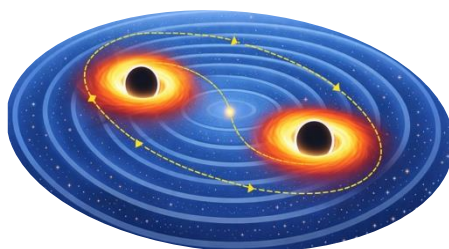
- May provide parameter regions that evade existing observational constraints while explaining PTA observations.
- Testable with future GW observation.

Other sources of GW

Candidate①

SMBHB

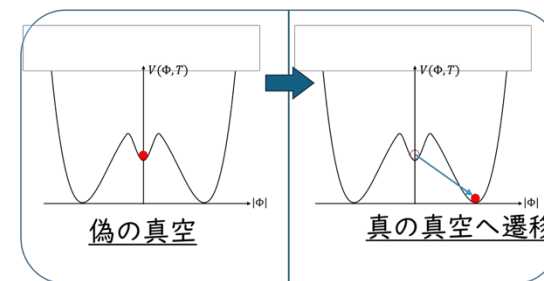
- Without BSM
- Final pc problem



Candidate②

1st-order PT in Dark Sector

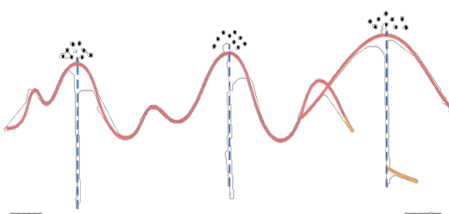
- Structure of DS
- Bubble creation



Candidate③

Scalar induced GW

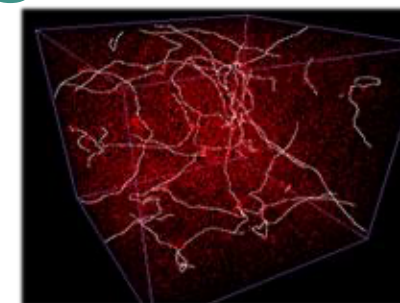
- Non-linear int
- PBH formation



Candidate④

Cosmic Superstring

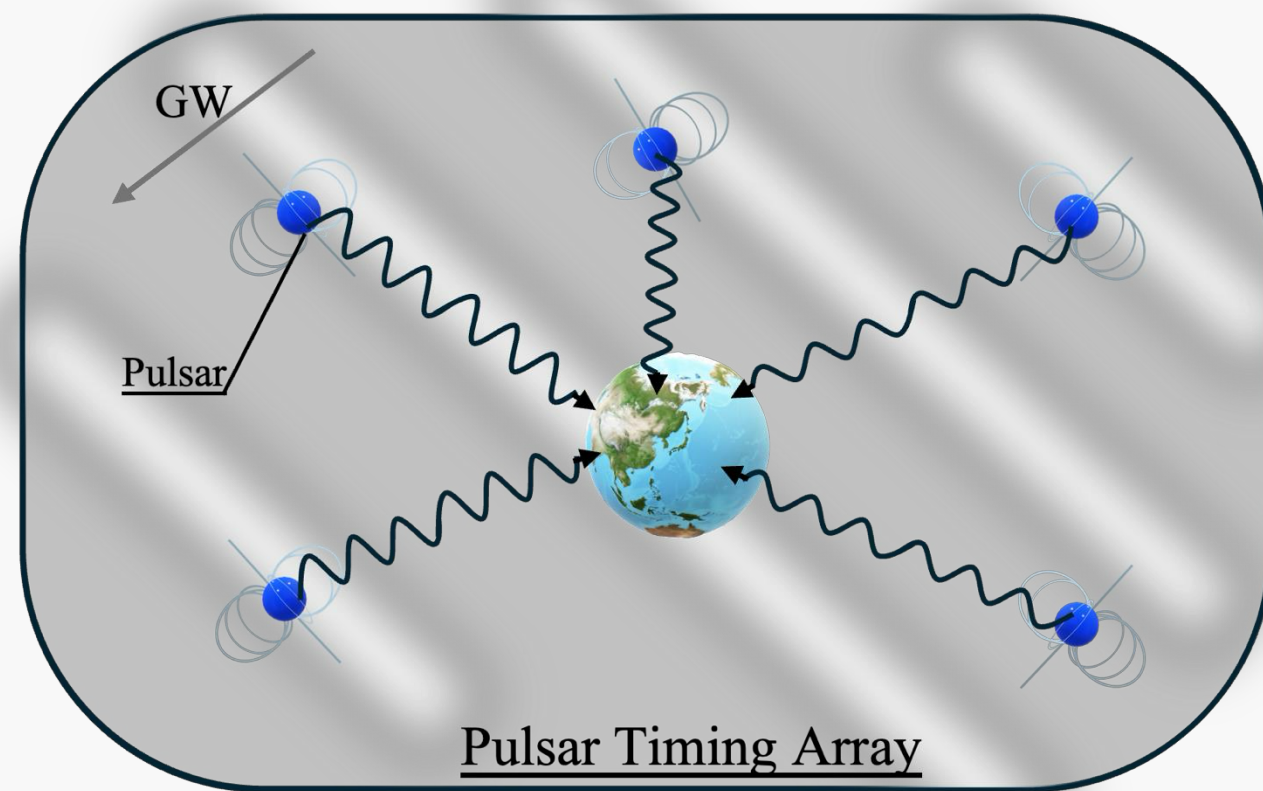
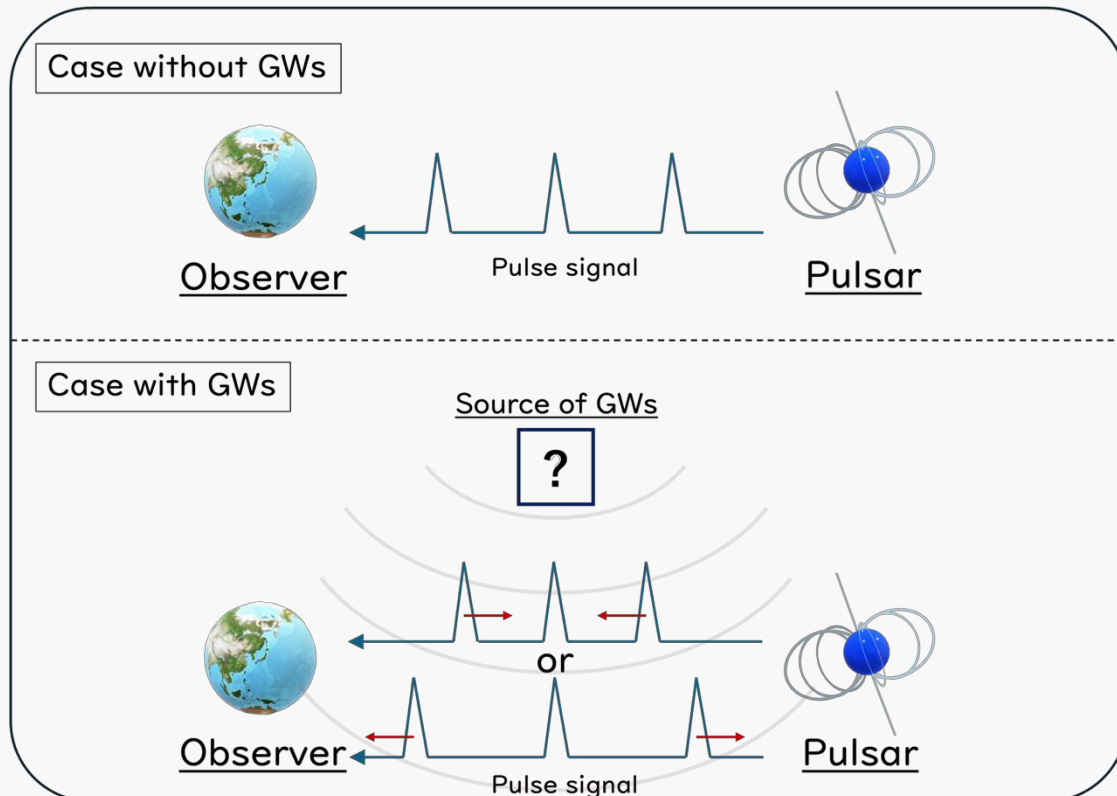
- Appear in string theory
- Over wide range



PTAs have detected a GW signal of unknown origin

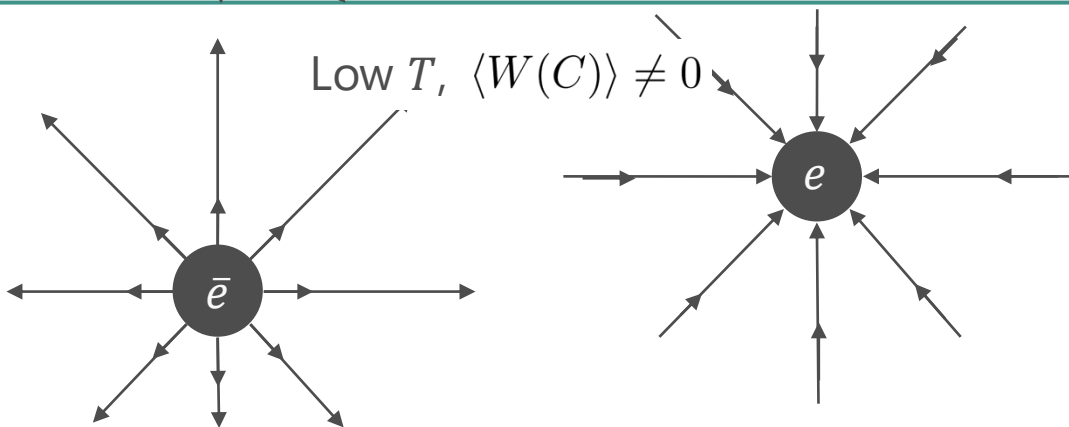
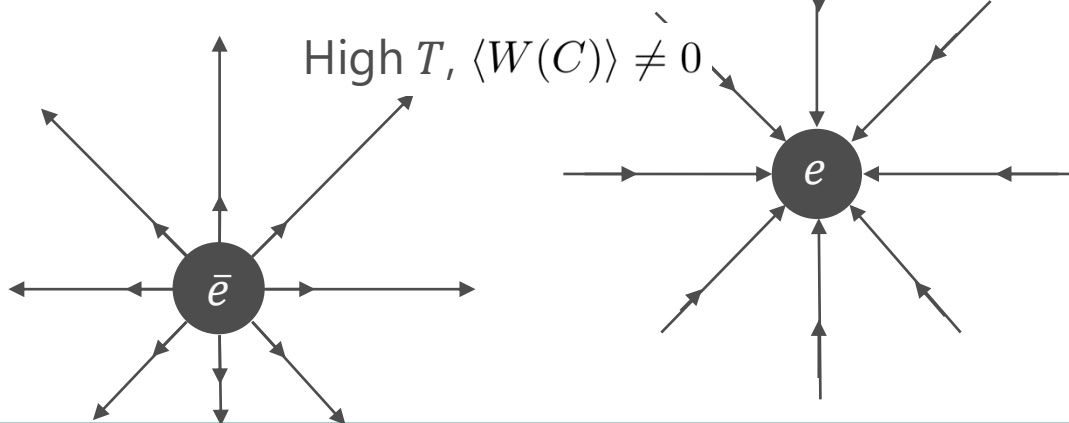
- Galaxy-scale GW detection with pulsars

Sensitive to low-frequency (long-wavelength) GWs.

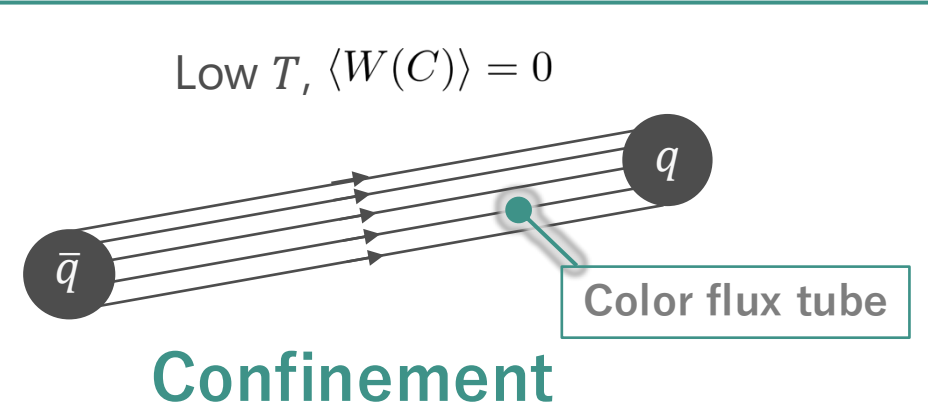
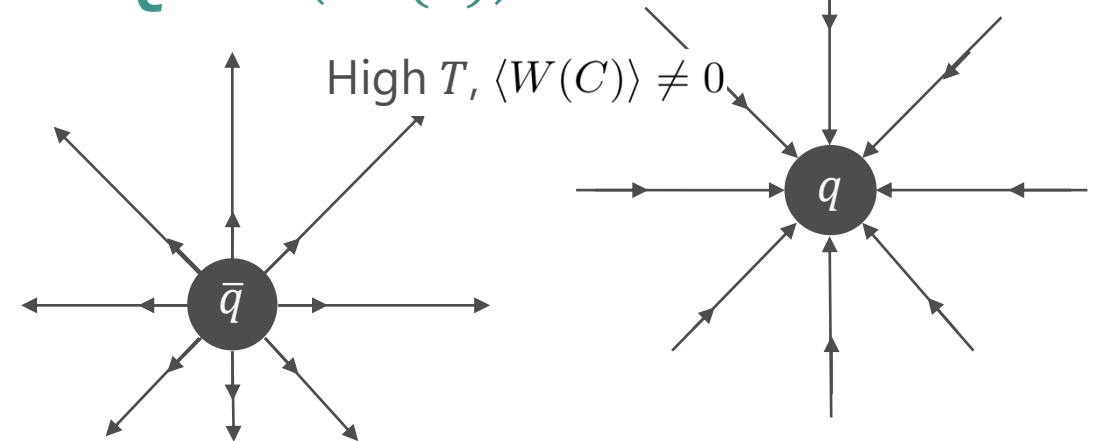


Existence of the confined/deconfined phase

In Maxwell theory



In QCD ($SU(3)$)



Color flux tube is formed in Yang-Mills theory

$$S_{\text{YM}} = -\frac{1}{4g^2} \int d^4x F_{\mu\nu}^a F^{a\mu\nu},$$

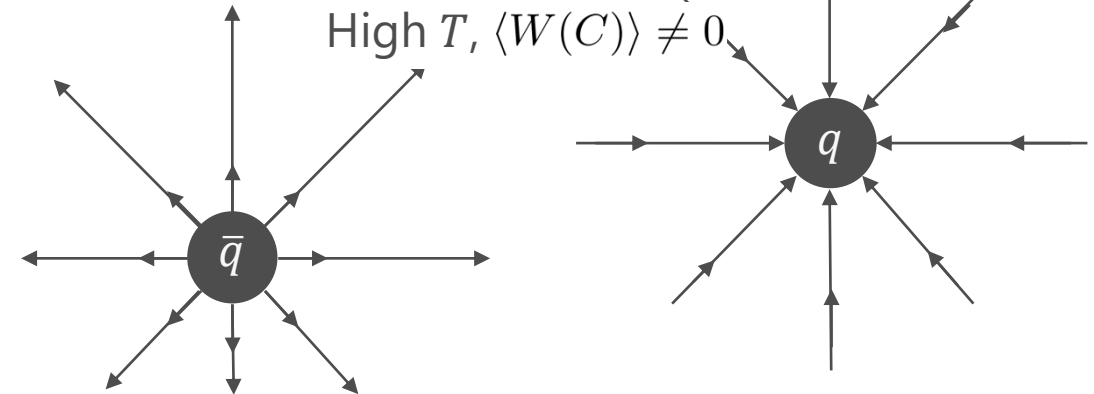
$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \underline{\underline{f^{abc} A_\mu^b A_\nu^c}}$$

Confinement happen in the other Yang-Mills (non-abelian gauge) theory.

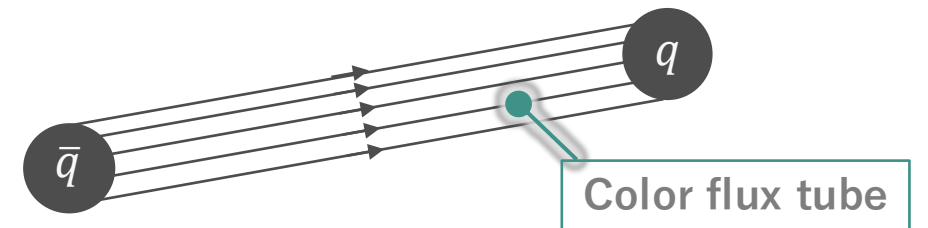


Color flux tube is also created in the other Yang-Mills theory.

In QCD ($SU(3)$)



Low T , $\langle W(C) \rangle = 0$



Confinement

Color flux tube is stable due to 1-form symmetry

Stability of particle (electron)

0-form symmetry
(conventional symmetry)

$$\hat{U}(t)\hat{\psi}(x,t)\hat{U}^\dagger(t) = e^{-iq\alpha}\hat{\psi}(x,t)$$

Symmetry exist

⇒ Electron is stable due to
the conserved charge q



No possible decay channel

Stability of color flux tube in pure YM

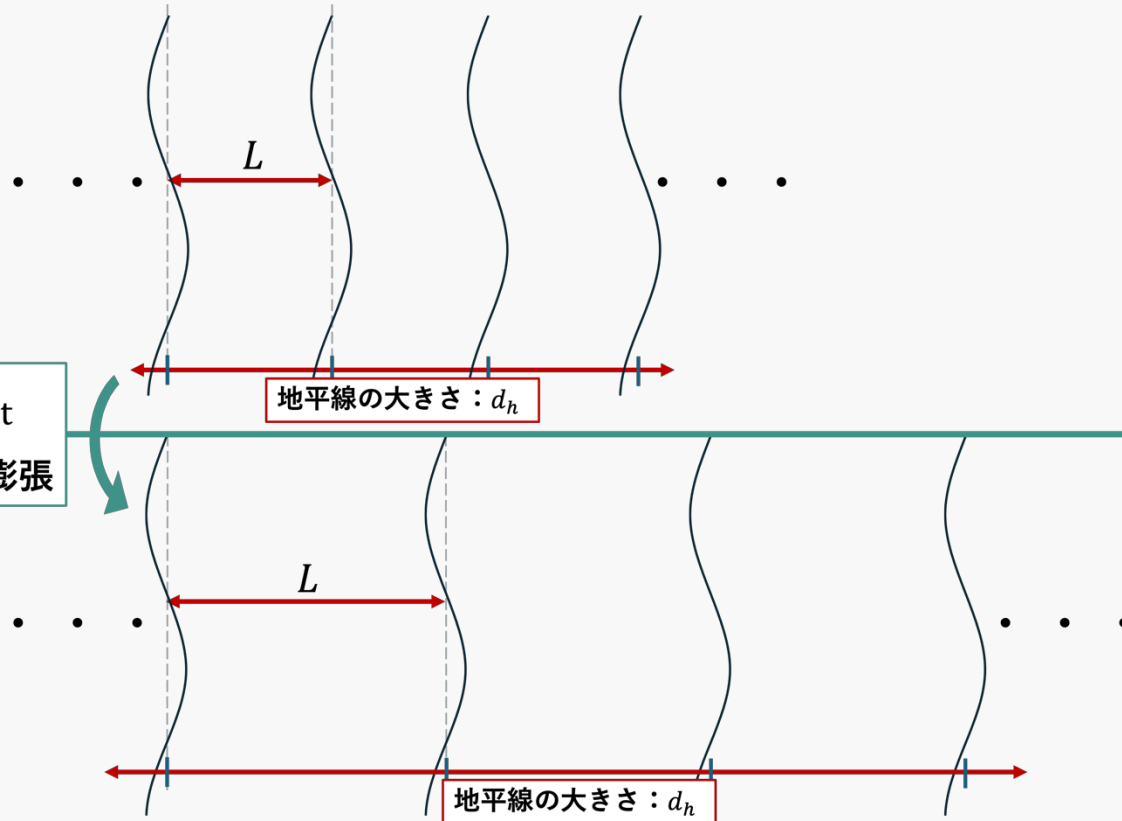
1-form center symmetry
(charge of Wilson loop)

$$\begin{aligned}\hat{U}(\Sigma)\hat{W}(q,C)\hat{U}^\dagger(\Sigma) \\ = e^{-iq\alpha \text{Link}(\Sigma,C)}\hat{W}(q,C)\end{aligned}$$

1-form symmetry exist

⇒ Color flux tube is stable due to
the conserved charge q

Scaling law is defined by $\gamma(t) = \text{const}$



Horizon

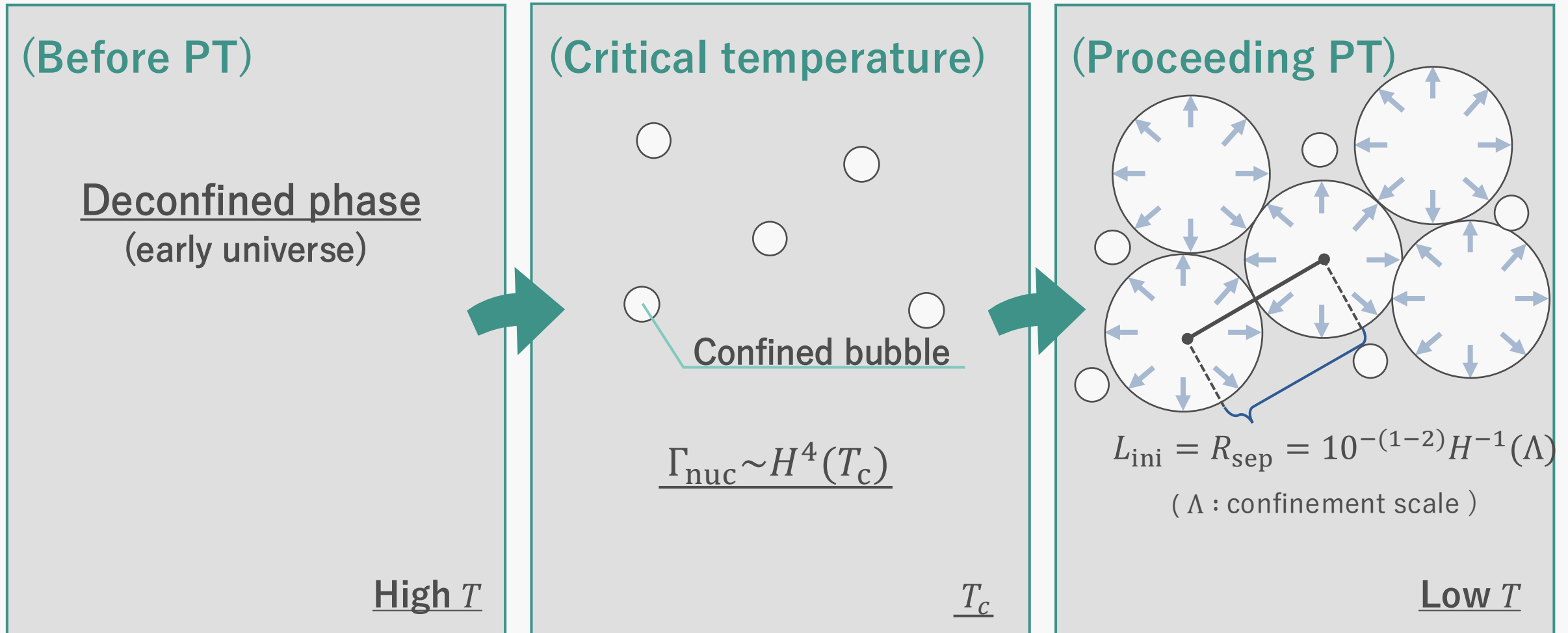
$$d_h(t) = a(t) \int_0^t \frac{c dt'}{a(t')} \sim t$$



Scaling law $\Leftrightarrow \gamma(t) = \text{const}$

$$L \equiv \gamma(t) \cdot t$$

1st order Phase Transitions make color flux tube



Baryonic glue ball and Domain wall

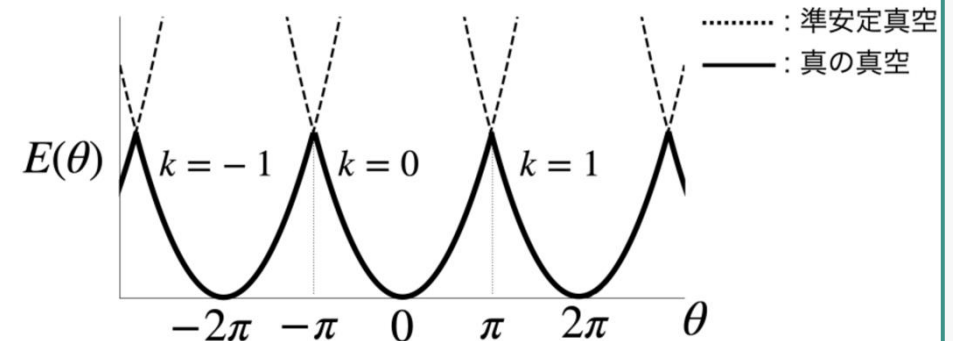
Baryonic glue ball come from accidental $\mathbb{Z}_2$ symmetry

$$O(2N)/SO(2N) = \mathbb{Z}_2$$

$$B_{\mu_1 \cdots \mu_{2N}} = \epsilon_{i_1 \cdots i_{2N}} (F_{\mu_1 \mu_2})_{i_1 i_2} \cdots (F_{\mu_{2N-1} \mu_{2N}})_{i_{2N-1} i_{2N}}$$

Θ term produce domain wall around $\theta = \pi$

$$S_\theta = \frac{\theta}{32\pi^2} \int d^4x \epsilon^{\mu\nu\rho\sigma} \text{Tr}(F_{\mu\nu} F_{\rho\sigma})$$



Hellings-Down curve

Frequency shift : $\frac{\delta\nu_i}{\nu} = \alpha_i h(t) + n_i(t)$

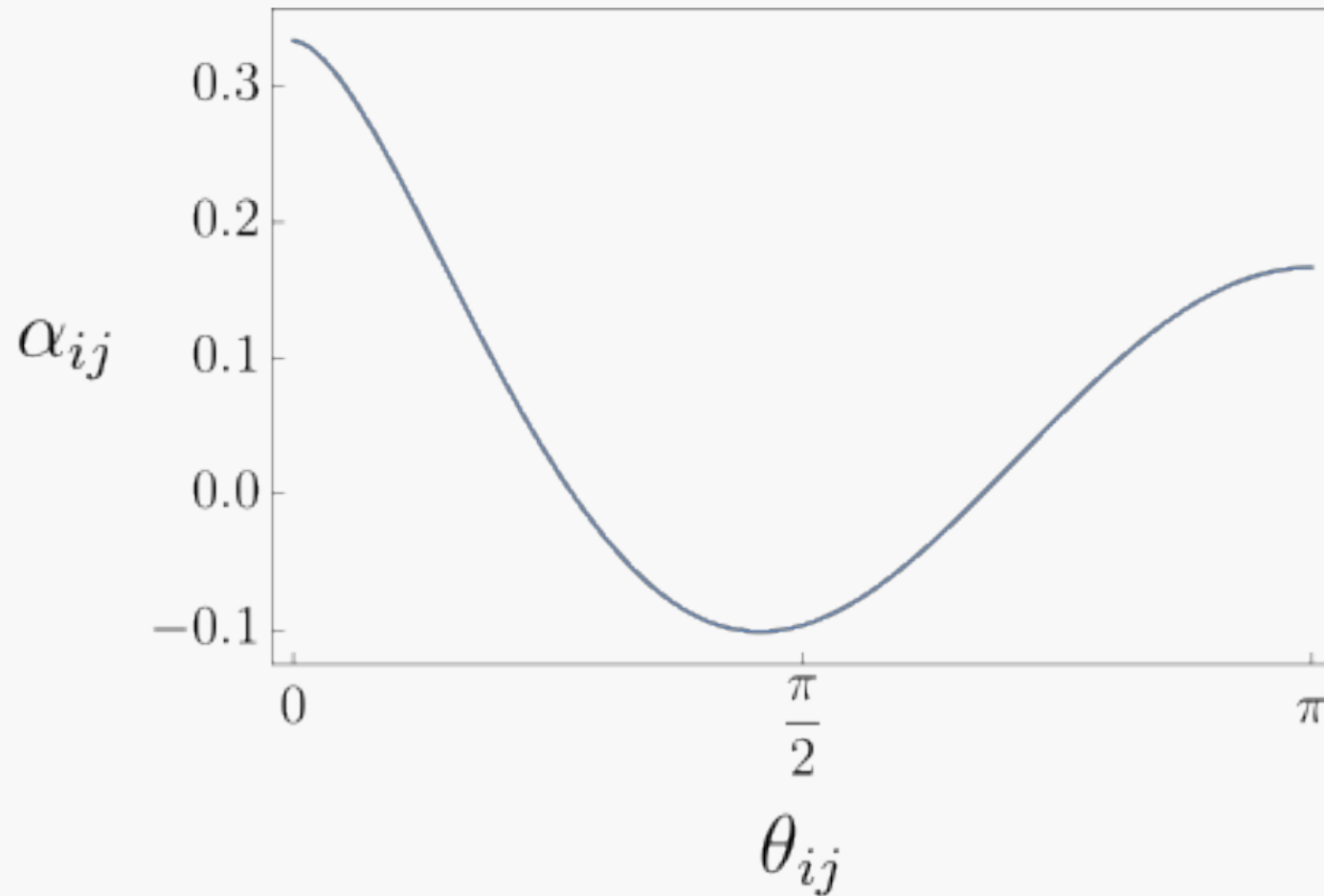
Correlation of the shifts : $c_{ij}(\tau) = \alpha_{ij} \langle h^2 \rangle + \delta c_{ij}$

$$\alpha_{ij} = \frac{1}{4\pi} \int d\hat{\Omega} \sum_A F_a^A(\hat{\Omega}) F_b^A(\hat{\Omega}) = \frac{1}{4\pi} \int d\hat{\Omega} F_a(\hat{\Omega}) F_b^*(\hat{\Omega})$$

$$= \frac{1 - \cos \theta_{ij}}{2} \ln \left(\frac{1 - \cos \theta_{ij}}{2} \right) - \frac{1}{6} \frac{1 - \cos \theta_{ij}}{2} + \frac{1}{3}$$

$$\left(\begin{array}{ll} F_a^A(\hat{\Omega}) \equiv \frac{1}{2} \frac{\hat{p}^i \hat{p}^j}{1 + \hat{\Omega} \cdot \hat{p}} e_{ij}^A(\hat{\Omega}) & e_{ij}^A(\hat{\Omega}) : \text{Polarization tensor} \\ & \hat{\Omega} : \text{Unit vector along the GW propagation direction} \\ F \equiv F_+ + iF_{\times} & \hat{p} : \text{Unit vector pointing from Earth to the pulsar} \end{array} \right)$$

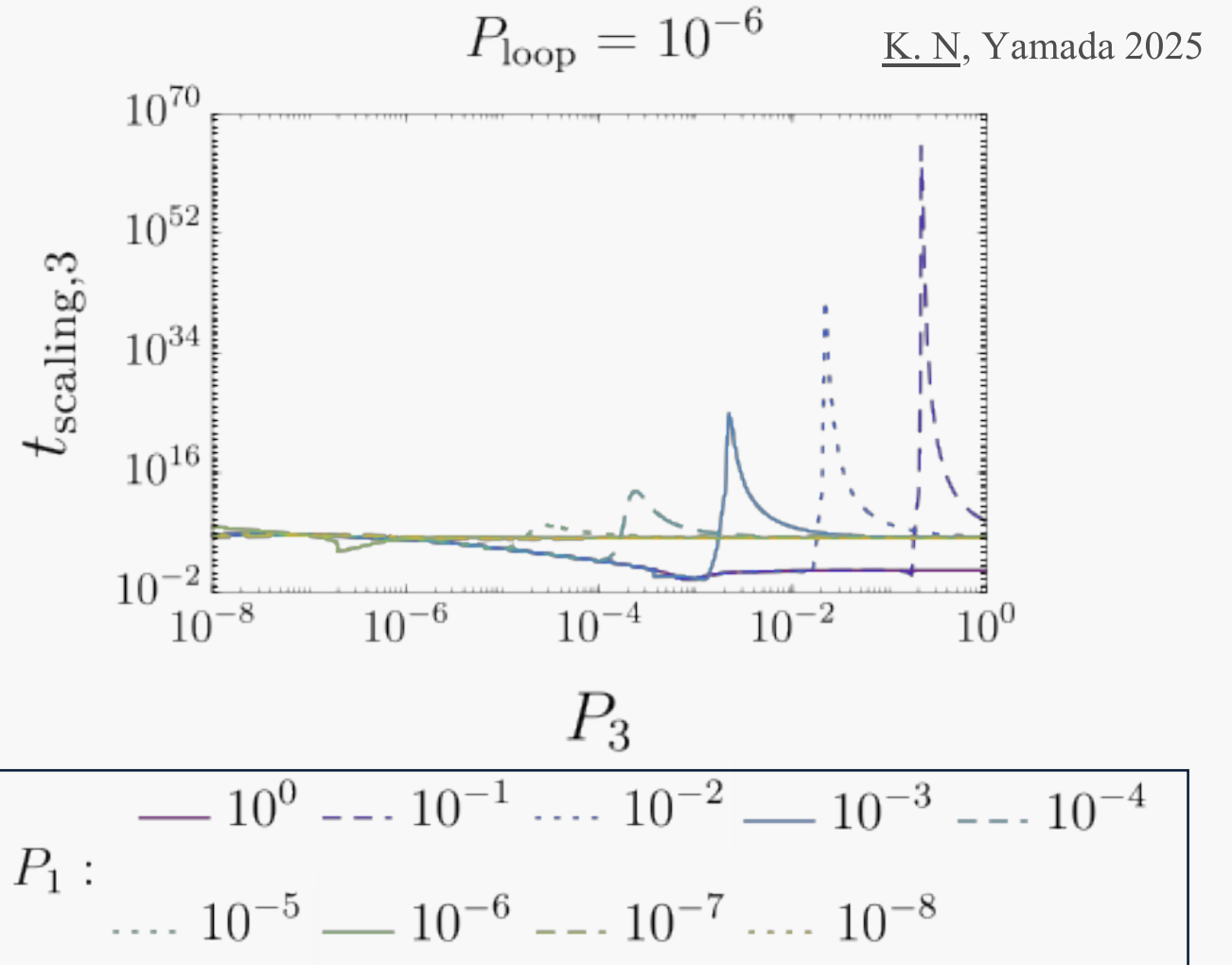
Hellings-Down curve



Longer scaling time for certain P_i

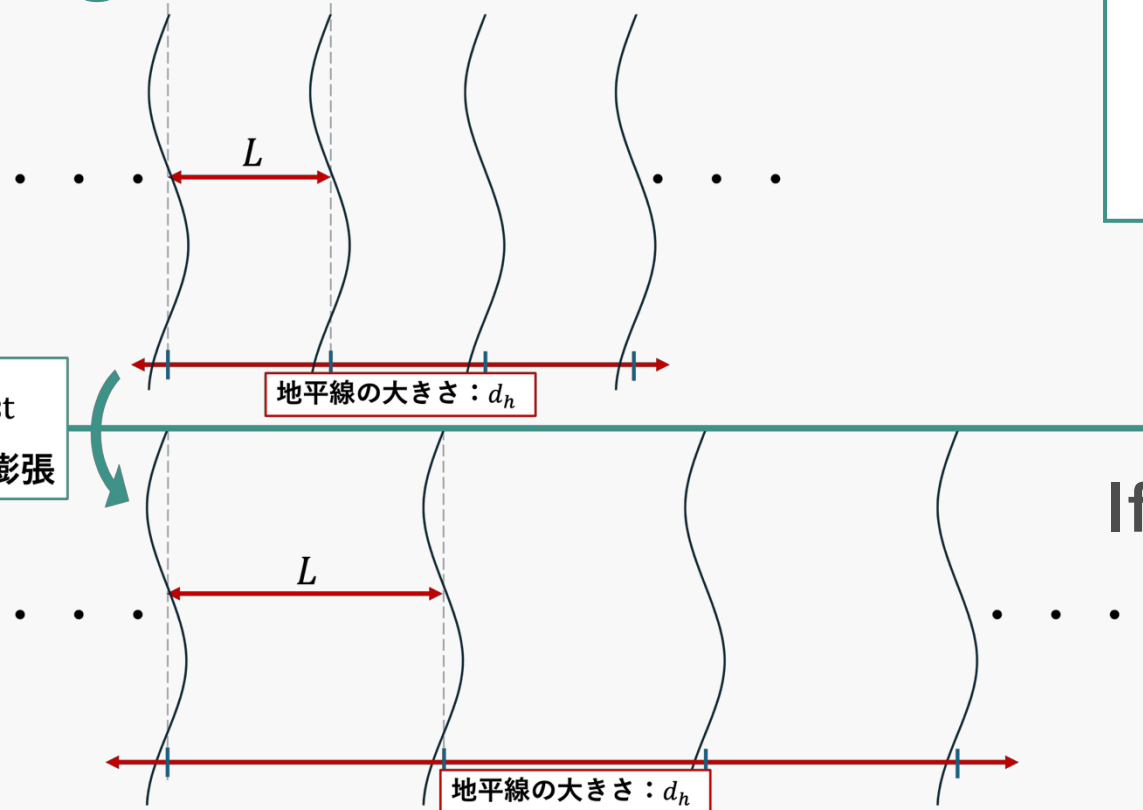
On the scaling time

- Cosmic strings typically reach the scaling regime relatively quickly.
- The significant increase in the scaling time is a new effect found in this work
- However, it requires fine-tuning.
- The increased scaling time plays an important role in the calculation of GWs.



The scaling law corresponds to $\gamma(t) = \text{const}$

Scaling law :



Horizon (causally connected distance)

$$d_h(t) = a(t) \int_0^t \frac{c dt'}{a(t')} \sim t$$



If the network obeys the scaling law,
 $\gamma(t) = \text{const}$

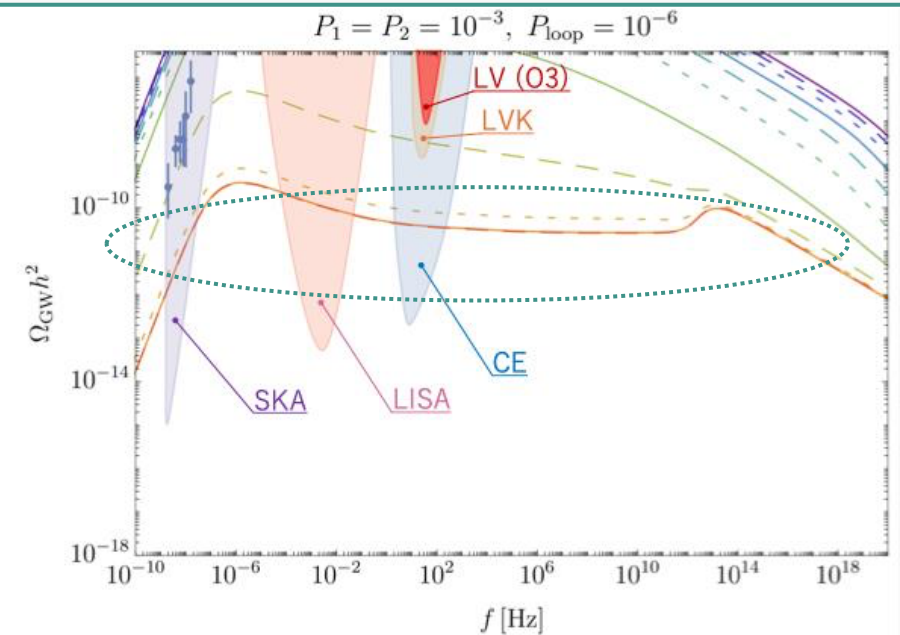
$$L \equiv \gamma(t) \cdot t$$

$P_{\{i\}}$ dependence appear in the GW spectrum

- Including the P_i dependence of the scaling solution of γ_i

$$\Omega_{\text{GW}} h^{2(\text{scaling})} \propto \sum_i \Omega_{\text{rad}} \sqrt{G\mu_i} \frac{P_{\text{loop}}}{\gamma_i^4}$$

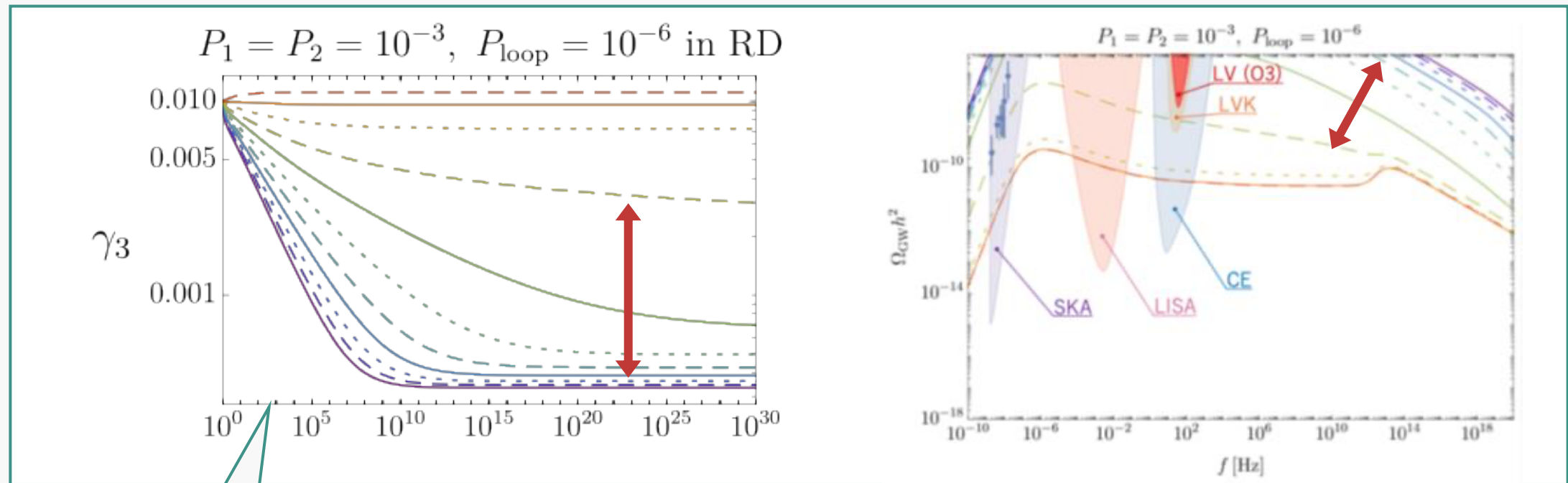
$$\propto \begin{cases} P_{\text{loop}}/P_3^2 & \text{for } P_{\text{loop}} \ll P_3 \ll P_1 \\ P_{\text{loop}}^{-1} & \text{for otherwise} \end{cases}$$



GWs emitted from a network in the scaling regime
produce a flat spectrum

GW is suppressed in the high-frequency region

- Non-flat spectrum = Longer scaling time



If the scaling time is strongly delayed,
high-freq suppression and characteristic slope appear

The path to the scaling solution varies

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