

Cosmology-informed constraints on the fuzzy dark matter mass from dwarf-spheroidal stellar kinematics

Shunichi Horigome

Astronomical Institute, Tohoku University

with Elisa Ferreira (Kavli IPMU) and Shin'ichiro Ando (GRAPPA, Kavli IPMU) [in prep.]

Cosmology-informed FDM constraints

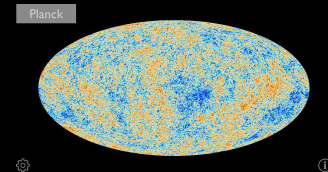
- Goal: Constrain the FDM mass from dSph stellar kinematics
 - parameter degeneracy
- Method: Combine spherical Jeans analysis with cosmology-informed constraints using semi-analytical models (SASHIMI)
 - Motivation: The same kinematics + cosmology strategy yielded stringent SIDM constraints
- Main results — Segue 1
 - 95% CL: FDM mass $> 9.1 \times 10^{-22}$ eV
 - 68% CL: $> 3.3 / 4.0 \times 10^{-21}$ eV ($V_{50} = 10.5 / 18$ km/s)

Introduction

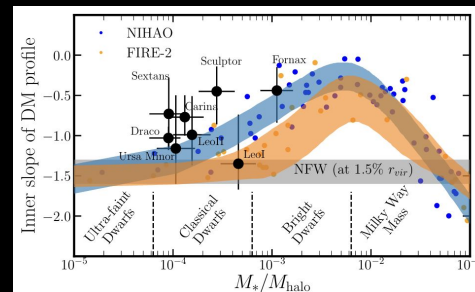
Dark matter in our universe

Λ CDM explains the large-scale structure of the universe very well

What happens on small scales?



- Satellite-count mismatch
 - Missing (or too many) satellites? Homma et al. (2023) [arXiv:2311.05439]
 - Too-big-to-fail?
- Structure of low-mass (sub)halos
 - Core vs. cusp?
 - Systematics in profile estimation?
 - Baryonic physics?
 - Dark matter (SIDM, FDM)?



Hayashi et al. (2020)

Dwarf spheroidal galaxies (dSphs)

Important targets for probing small-scale dark-matter structure

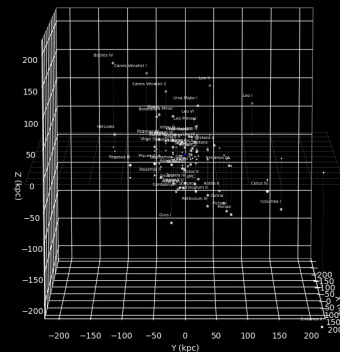
- Large mass-to-light ratio: $Y=10\text{-}100\text{ M/L}$
→ dark-matter-dominated systems
- Close to us

How to observe the dark matter density profile in dSphs?

Fornax dSph



Dwarf spheroidal galaxies in the Milky Way



Dynamical analysis

Jeans equation:

$$\frac{1}{\nu} \frac{d(\nu(r) \sigma_r^2(r))}{dr} + \frac{2\beta(r) \sigma_r^2(r)}{r} = -\frac{G M(r)}{r^2}$$

$\nu(r)$: Number density (e.g. Plummer model)

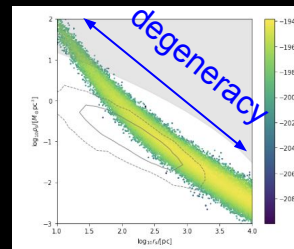
$\beta(r)$: Anisotropy (e.g. flat model, Baes & van Hese (2007) model)

$\sigma_r(r)$: Velocity dispersion

$\rho_{\text{DM}}(r)$: DM density profile

$$M(r) = \int dr 4\pi r^2 \rho_{\text{DM}}(r)$$

plausible region from kinematical analysis



Horigome et al. (2022)

We have only limited data. Additional constraints to break degeneracy?

- ← Proper motions
- ← Higher-order analysis
- ← **Cosmology**

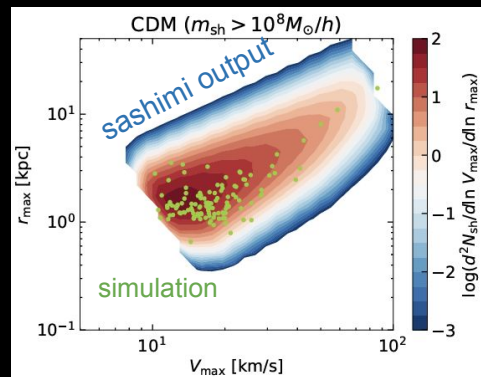
Cosmology

SASHIMI (Semi-Analytical SubHalo Inference Modeling)

Ando et al.

[[arXiv:1803.07691](https://arxiv.org/abs/1803.07691)], [[arXiv:1903.11427](https://arxiv.org/abs/1903.11427)]

- Efficiently calculates various subhalo properties using semi-analytical models
 - Before accretion: Extended Press-Schechter formalism
 - After accretion: empirical tidal stripping formula
- Output: catalog of subhalos
 - accretion redshift
 - mass
 - maximum circular velocity
 - $V_{\max}$, $r_{\max}$
 - ...



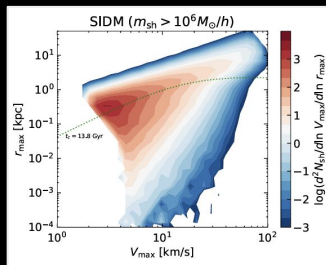
Stringent Constraints on Self-Interacting Dark Matter Using Milky Way Satellite-Galaxy Kinematics

[Shin'ichiro Ando](#), [Kohei Hayashi](#), [Shunichi Horigome](#),
[Masahiro Ibe](#), [Satoshi Shirai](#) [[arXiv:2503.13650](#)]

Kinematics of dSphs

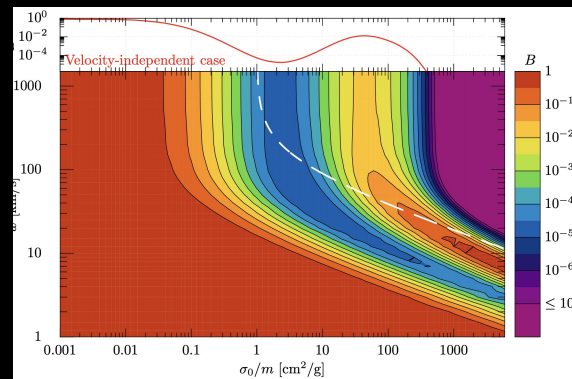
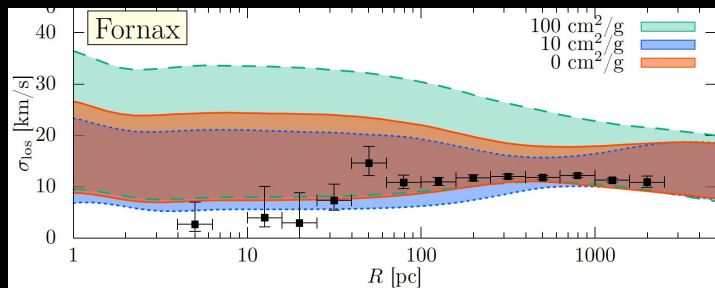


SIDM cosmology



- We test SIDM models by comparing
 - theory (SIDM cosmology)
 - observations (DM halos in dSphs)
- We obtain constraints on SIDM parameters
 - e.g. in the velocity-independent case:

$$\sigma/m \lesssim 0.2 \text{ cm}^2/\text{g}$$



FDM analysis

FDM profile

Shunichi Horigome, Elisa Ferreira, Shin'ichiro Ando [in prep.]

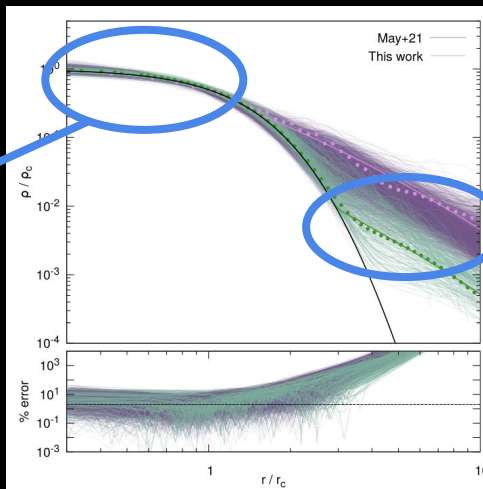
Schrödinger–Poisson equations

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2ma^2}\nabla^2\psi + \frac{m\Phi}{a}\psi$$

Soliton core

$$\rho(r) = \rho_c \left[1 + 0.091 \left(\frac{r}{r_c} \right)^2 \right]^{-8}$$

NOTE: We assume **smooth** density profile



NFW-like

$$\rho(r) = \rho_s \left(\frac{r}{r_s} \right)^{-1} \left[1 + \left(\frac{r}{r_s} \right) \right]^{-2}$$

Chan et al. (2021) [arXiv:2110.11882]

FDM Cosmology?

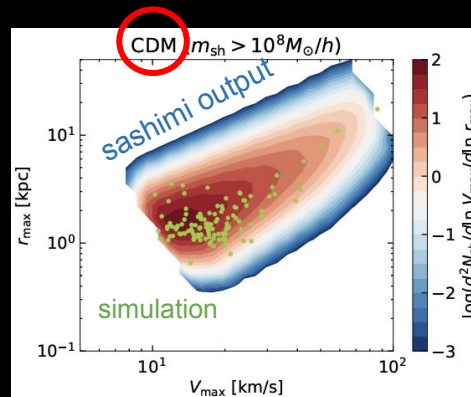


SASHIMI (Semi-Analytical SubHalo Inference Modeling)

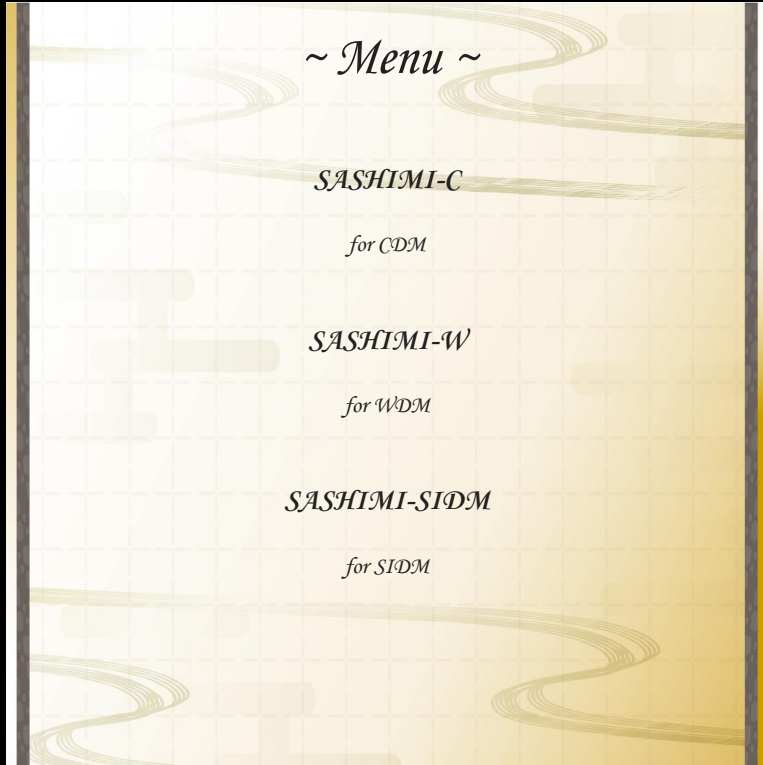
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 - mass
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 - ...



Various SASHIMI



cold



warm



self-interacting



- <https://github.com/shinichiroando/sashimi-c>
- <https://github.com/shinichiroando/sashimi-w>
- <https://github.com/shinichiroando/sashimi-si>

Various SASHIMI

~ Menu ~

SASHIMI-C
for CDM

SASHIMI-W
for WDM

SASHIMI-SIDM
for SIDM

Today's Special
SASHIMI-F
(or Aburi: axionlike bosons, ultralight, rarely interacting)
for FDM

cold



warm



self-interacting



fuzzy or aburi (炙り, burned)?



How to prepare SASHIMI-F/ABURI

Choose/define cosmology



EPS



Subhalo mass function



Tidal stripping



$$\dot{m}(z) = -A \frac{m(z)}{\tau_{\text{dyn}}(z)} \left[\frac{m(z)}{M(z)} \right]^\zeta,$$

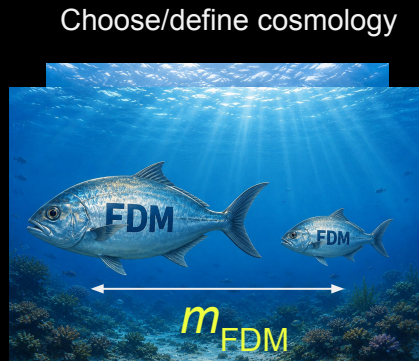
Subhalo Catalog



$r_s, \rho_s, r_{\text{trunc}}$

$$\Omega_m, \Omega_b, h$$

How to prepare SASHIMI-F/ABURI



EPS

Subhalo mass function



Tidal stripping

Subhalo Catalog'



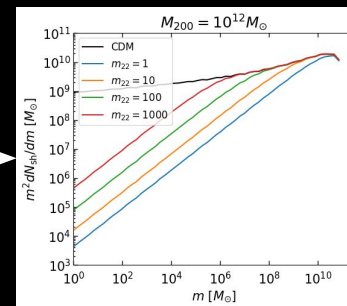
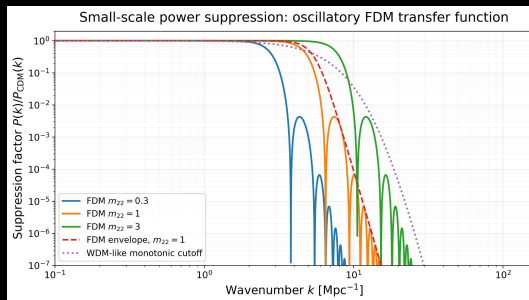
r_s , ρ_s , r_{trunc}

&

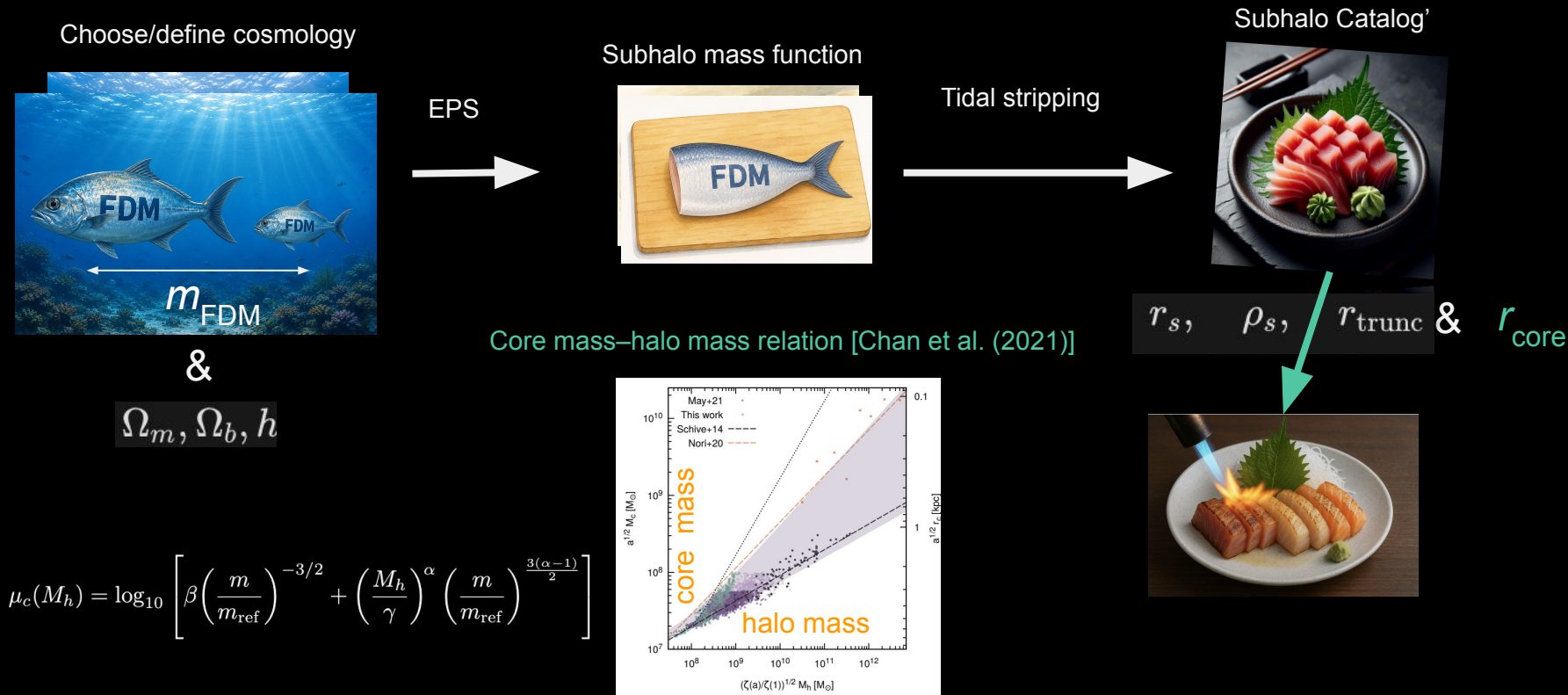
Ω_m, Ω_b, h

Power spectrum suppression [Hu et al. (2000)]

$$P_{\text{FDM}}(k) = |T(k)|^2 P_{\text{CDM}}(k)$$



How to prepare SASHIMI-F/ABURI



FDM cosmology

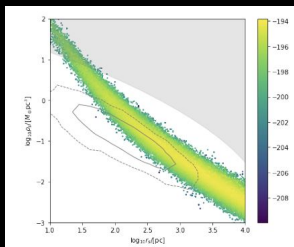
Shunichi Horigome, Elisa Ferreira, Shin'ichiro Ando [in prep.]

- SASHIMI-F(Aburi)

Contours: SASHIMI output (r_s , ρ_s)
for specific FDM masses

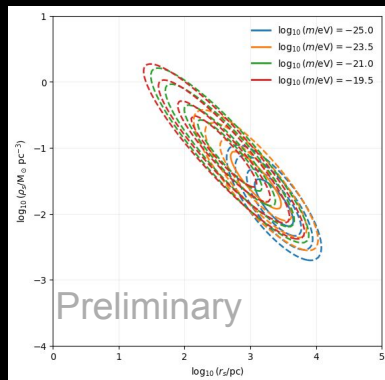


Jeans



Horigome et al. (2022)

×



$$\pi_{\text{sat}}(\Theta_{\text{sat}} | m) = \mathcal{N}(\theta_{\text{sat}}; \boldsymbol{\mu}(m), \boldsymbol{\Sigma}(m))$$

$$\chi^2_{\text{sat}}(\Theta_{\text{sat}}) \equiv -2 \ln \pi_{\text{sat}}(\Theta_{\text{sat}})$$

FDM analysis

Shunichi Horigome, Elisa Ferreira, Shin'ichiro Ando [in prep.]

- Profile likelihood: Chi-square

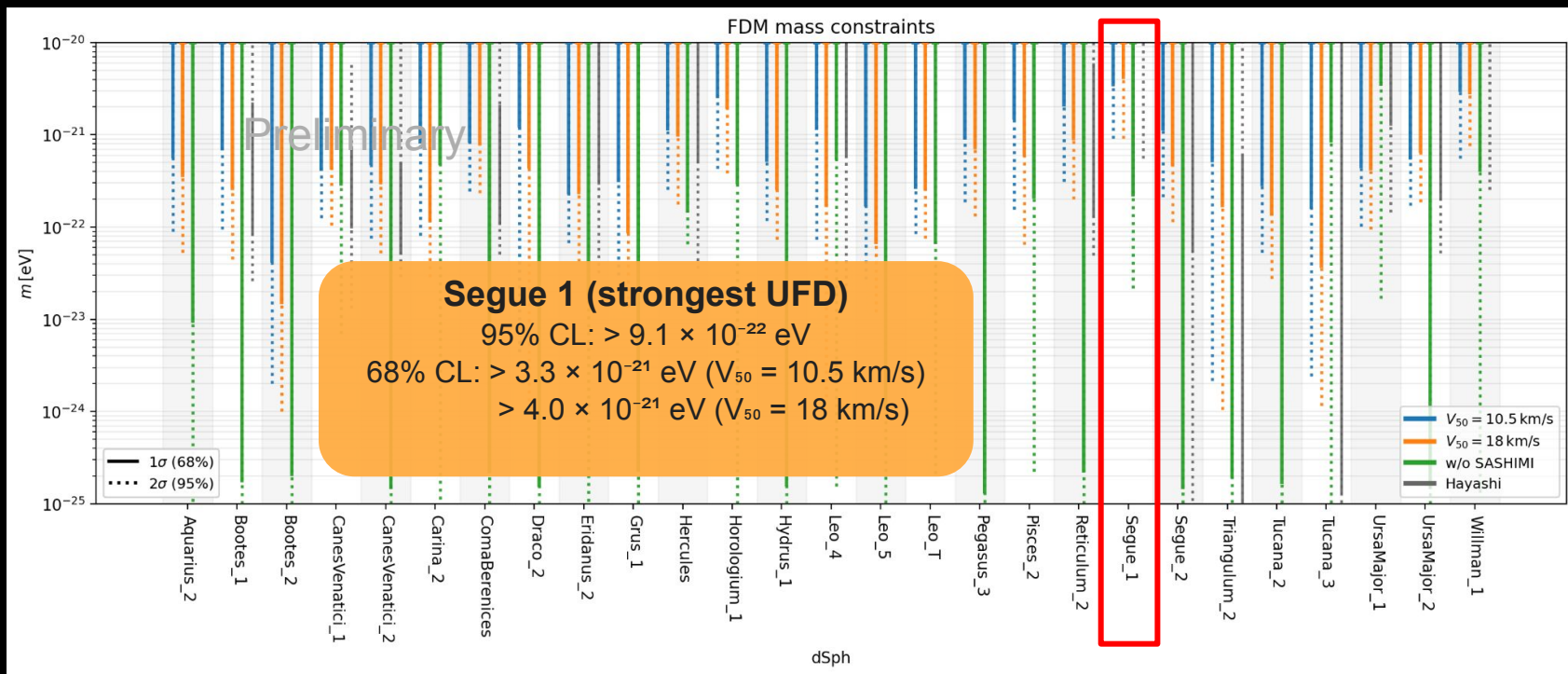
$$\chi_{\text{tot}}^2(\Theta) = \sum_{k \in \{\text{kin}, M, \text{photo}, \text{sat}, \text{flat}\}} \chi_k^2(\Theta_k)$$

- χ_{flat}^2 : hard bounds on the parameters
- χ_{kin}^2 : kinematical likelihood based on the Jeans analysis
- χ_M^2 : deviation from the core-halo mass relation
- χ_{photo}^2 : prior on the photometric parameters
- χ_{sat}^2 : prior on the satellite parameters



FDM results — Segue 1

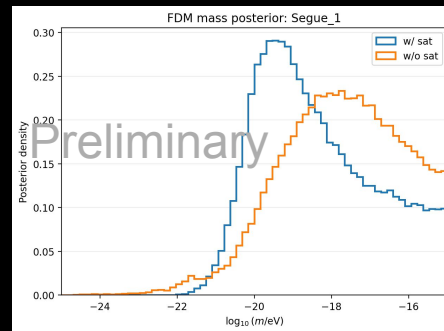
Shunichi Horigome, Elisa Ferreira, Shin'ichiro Ando [in prep.]



Discussion

- **What drives the constraint**
 - Joint use of dSph kinematics and SASHIMI cosmology
- **Main caveat**
 - The lower bound changes with the statistical treatment and the cosmological prior
- **Current interpretation**
 - The bound is weaker than aggressive previous Jeans limits, but cosmologically more explicit

Segue 1
**FDM mass > 9.1
 $\times 10^{-22}$ eV**
95% CL (both V50 choices)



Summary

- **Cosmology-informed FDM constraints from dSphs**
- Method
 - **Spherical Jeans kinematics**
 - **Cosmology-informed SASHIMI constraints**
 - for FDM subhalo structure
- Strongest UFD limit - Segue 1
 - 95% CL: FDM mass $> 9.1 \times 10^{-22}$ eV
 - 68% CL: $> 3.3 / 4.0 \times 10^{-21}$ eV ($V_{50} = 10.5 / 18$ km/s)
- Interpretation and next steps
 - The bound depends on cosmological and statistical assumptions (Bayes / Freq.)
 - FDM-specific subhalo modeling and higher-order dynamics

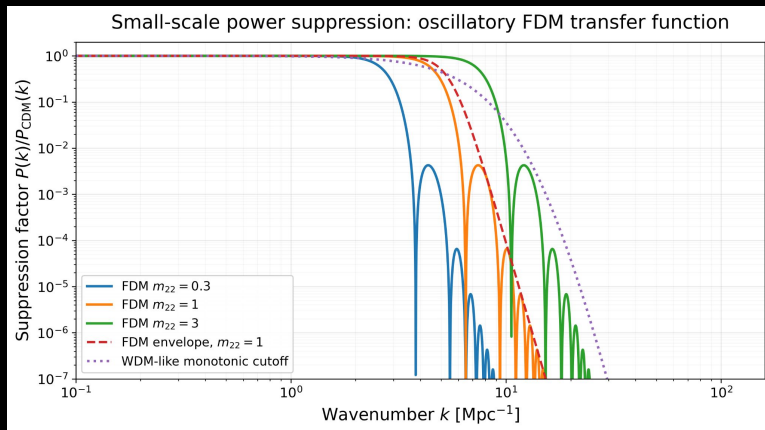
Backup

FDM cosmology

Shunichi Horigome, Elisa Ferreira, Shin'ichiro Ando [in prep.]

- Power-spectrum suppression [Hu et al. (2000)]

$$P_{\text{FDM}}(k) = |T(k)|^2 P_{\text{CDM}}(k)$$



$$T_{\text{FDM}}(k) = \frac{\cos(x^3)}{1+x^8},$$

$$\frac{P_{\text{FDM}}(k)}{P_{\text{CDM}}(k)} = T_{\text{FDM}}^2(k),$$

$$x = 1.61 m_{22}^{1/18} \frac{k}{k_J},$$

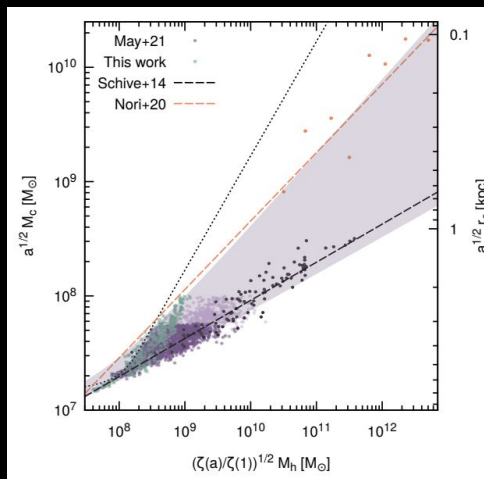
$$k_J = 9 m_{22}^{1/2} \text{Mpc}^{-1},$$

$$m_{22} = \frac{m}{10^{-22} \text{eV}}.$$

FDM cosmology

Shunichi Horigome, Elisa Ferreira, Shin'ichiro Ando [in prep.]

- Core mass–halo mass relation



Chan et al. (2021)



$$\mu_c(M_h) = \log_{10} \left[\beta \left(\frac{m}{m_{\text{ref}}} \right)^{-3/2} + \left(\frac{M_h}{\gamma} \right)^\alpha \left(\frac{m}{m_{\text{ref}}} \right)^{\frac{3(\alpha-1)}{2}} \right]$$

$$\chi_M^2(\Theta_M) = \frac{(\mu_c - \mu_{c,\text{best}}(M_h))^2}{\sigma_{\mu_c}^2(M_h)}$$

$$\sigma_{\mu_c}(M_h) = \max_{a \in \{\text{lower, upper}\}} |\mu_{c,a}(M_h) - \mu_{c,\text{best}}(M_h)|,$$

FDM cosmology

Shunichi Horigome, Elisa Ferreira, Shin'ichiro Ando [in prep.]

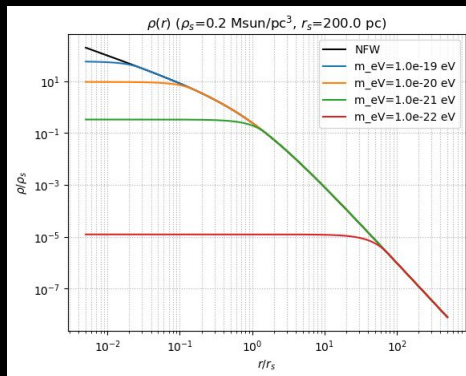
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- Efficiently calculates various subhalo properties using semi-analytical models

- Power spectrum suppression

$$P_{\text{FDM}}(k) = |T(k)|^2 P_{\text{CDM}}(k)$$

- $M_c - M_h$ relation



DM density profile with different FDM mass

What is V_{50} ?

- **Definition**

- V_{50} is the V_{peak} where 50% of subhalos are assumed to host a visible galaxy.

- **Occupation model**

- $f_{\text{galaxy}}(>V_{\text{peak}}) = \frac{1}{2}[1 + \text{erf}((V_{\text{peak}} - V_{50})/(\sqrt{2} \sigma))]$

- **Choices used here**

- 18 km/s: conventional reionization-quenching scale.
10.5 km/s: lower-threshold model motivated by Milky Way satellite counts.

Galaxy occupation
 $f_{\text{galaxy}}(V_{50}) =$
0.5

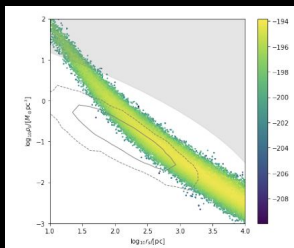
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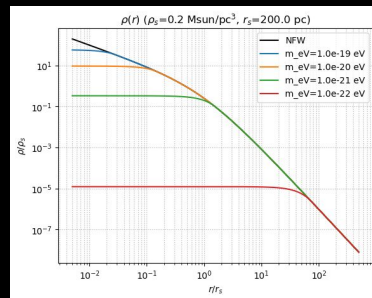
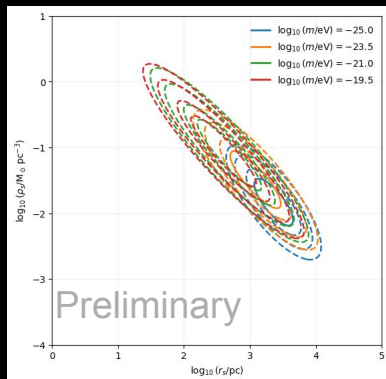
- SASHIMI-F(Aburi)

Contours: 3D satellite prior projected onto the (r_s, ρ_s) plane



Horigome et al. (2022)

×



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Dynamical heating: stronger constraints, more assumptions

- **Existing heating limit**

- FDM wave interference heats stellar orbits; Segue 1/2 give $m_{\text{FDM}} > 3 \times 10^{-19}$ eV (99% CL) — a much higher lower bound than ours, though with different assumptions and CL.

- **Why is it so strong?**

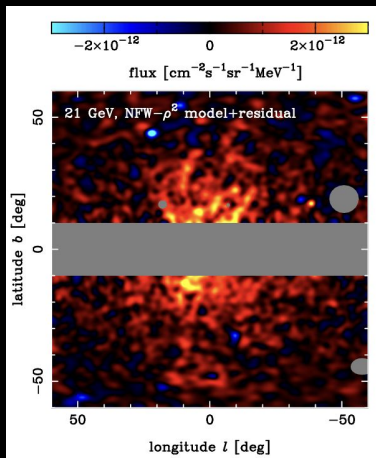
- It is a cumulative Gyr-scale effect: very compact, dynamically cold UFDs leave little room for repeated potential fluctuations.

- **Important ingredients / caveats**

- Inference depends on stellar age/initial phase-space structure; halo/soliton structure and fluctuations; heating regime (diffusive vs coherent/tidal); tides/environment; and UFD kinematic systematics. Full Schrödinger–Poisson simulations support strong heating, but mapping it to m_{FDM} is more history-dependent than an equilibrium Jeans fit.

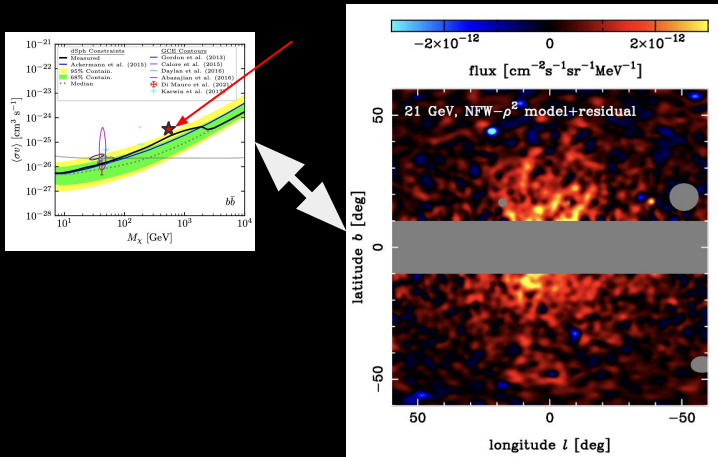
Has dark matter finally been detected?

- Halo gamma-ray excess [[Totani \(2025\)](#)]
 - Tension with dSph searches....



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Resonant Annihilation of WIMP Dark Matter for Halo Gamma Ray Signal

Hitoshi Murayama^{1,2,3}

Modified annihilation scenario

Electroweak Doublet Dark Matter for a Galactic Halo Gamma-Ray Excess

Yasunori Nomura^{1,2,3,4} and Tomonori Totani^{5,6}

Particle-physics model for cross-section enhancement

Legacy analysis of Milky Way dwarf spheroidal satellite galaxies: an update

ANTONIO CIRIELLO,¹ MATTIA DI MAURO,² MARCO AJELLO,¹ CHRISTOPHER KIRWIN,¹ ALEX DRICA-WAGNER,^{3,4,5} AND MIGUEL A. SÁNCHEZ-CONDE^{6,7}

Weak signal from a reanalysis of dSph indirect searches

Constraining the dark matter origin of the halo-like 20 GeV γ -ray excess with the AMS-02 antiproton data

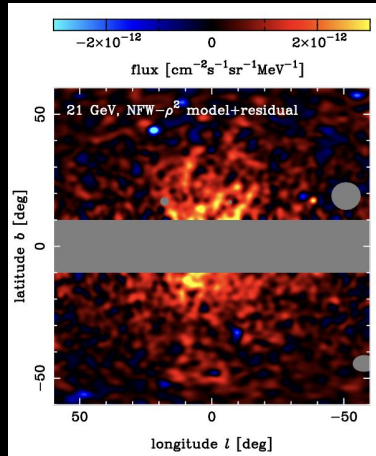
Xiao Wang, Kai-Kai Duan

Tension with AMS-02 antiproton observations

Intriguing, but far from a confirmed discovery.

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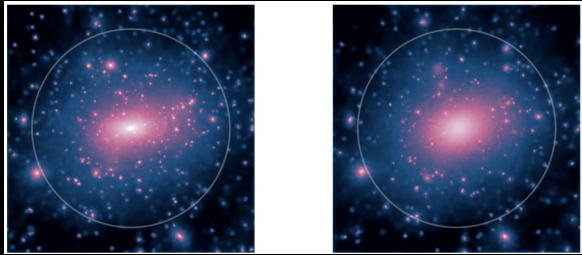
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Alternative DM models for small-scale challenges

SIDM (Self-Interacting Dark Matter)

- Dark-matter self-interactions can prevent the formation of cuspy profiles

$$\rho(r_{\text{core}}) \frac{\langle \sigma v \rangle t_{\text{age}}}{m_{\text{SIDM}}} \simeq 1$$

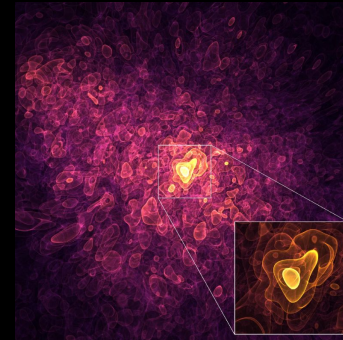


[Brinckmann et al. \(2017\) \[arXiv:1705.00623\]](#)

FDM (Fuzzy Dark Matter)

- Cored profile due to the wave nature of an ultralight particle
 - Soliton core
 - Interference

$$r_c \simeq 1.6 \text{ kpc } m_{22}^{-1} \left(\frac{M_h}{10^9 M_\odot} \right)^{-1/3} a^{1/2}$$



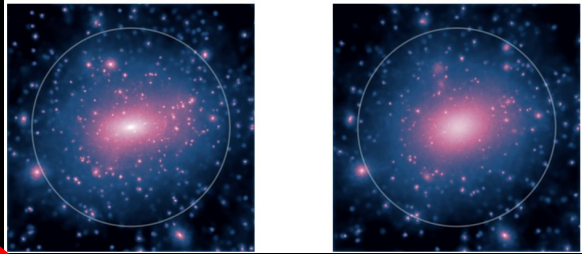
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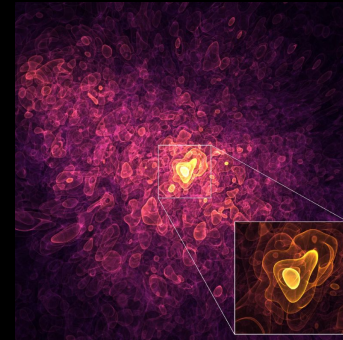


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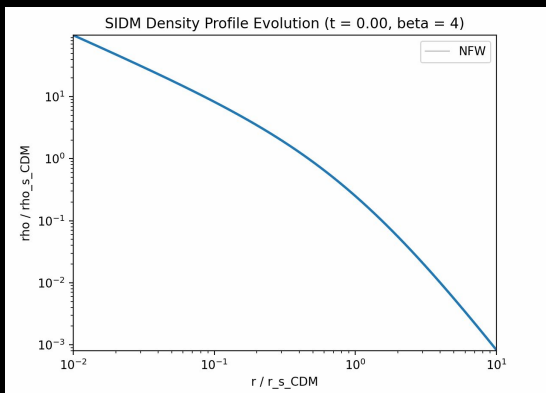
[Schive \(2025\) \[arXiv:2509.23231\]](#)

Self-Interacting Dark Matter (SIDM)

Large self-scattering cross section

$$\sigma/m \simeq 1 \text{ cm}^2/\text{g}$$

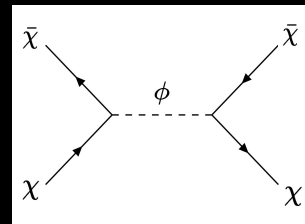
→ Gravothermal evolution of halos



cf. CDM (wino DM)

$$\sigma/m \approx G_F^2 m \approx 10^{-12} \left(\frac{m_\chi}{100 \text{ GeV}} \right) \text{ cm}^2/\text{g}$$

$$\frac{d\sigma}{d\cos\theta} = \frac{\sigma_0}{2[1 + (v/w)^2 \sin^2(\theta/2)]^2}$$

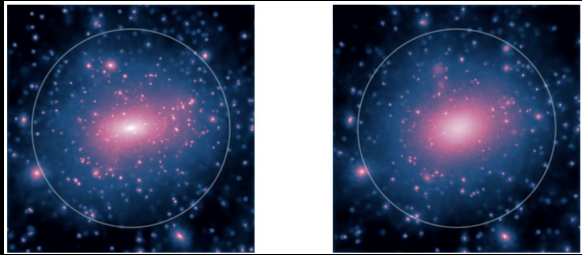


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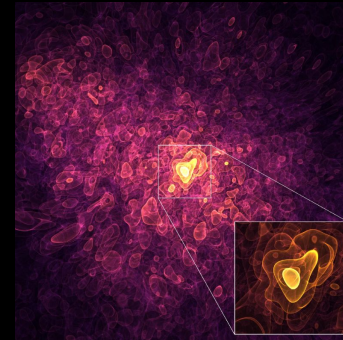


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 - Soliton core
 - Interference

$$r_c \simeq 1.6 \text{ kpc } m_{22}^{-1} \left(\frac{M_h}{10^9 M_\odot} \right)^{-1/3} a^{1/2}$$



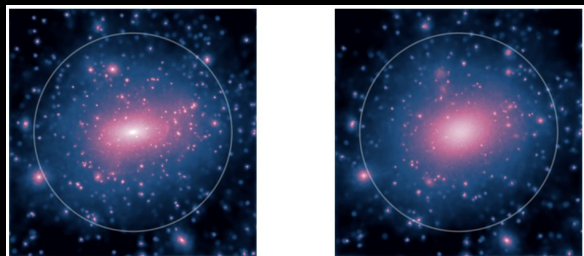
[Schive \(2025\) \[arXiv:2509.23231\]](#)

Alternative DM models for small-scale challenges (in prep....)

SIDM (Self-Interacting Dark Matter)

- Dark-matter self-interactions can prevent the formation of cuspy profiles

$$\rho(r_{\text{core}}) \frac{\langle \sigma v \rangle t_{\text{age}}}{m_{\text{SIDM}}} \simeq 1$$

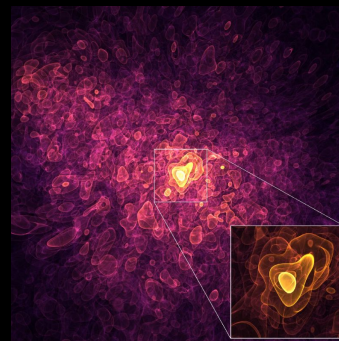


[Brinckmann et al. \(2017\) \[arXiv:1705.00623\]](#)

FDM (Fuzzy Dark Matter)

- Cored profile due to the wave nature of an ultralight particle
 - Soliton core
 - Interference

$$r_c \simeq 1.6 \text{ kpc } m_{22}^{-1} \left(\frac{M_h}{10^9 M_\odot} \right)^{-1/3} a^{1/2}$$



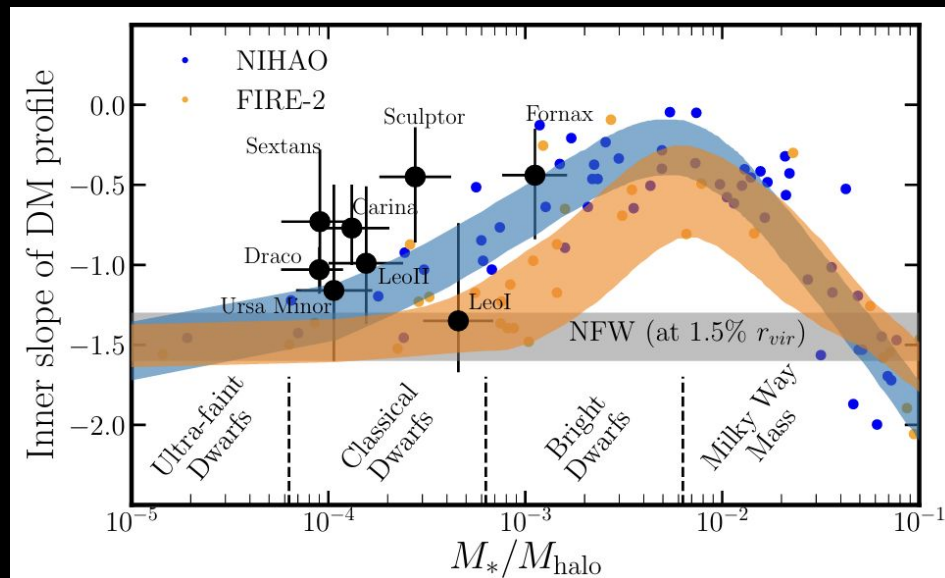
[Schive \(2025\) \[arXiv:2509.23231\]](#)

Core v.s. cusp?

- Neither fully cored, nor fully cuspy
→ Diversity

Origin of the diversity?

- Baryonic effect?
- Dark matter?



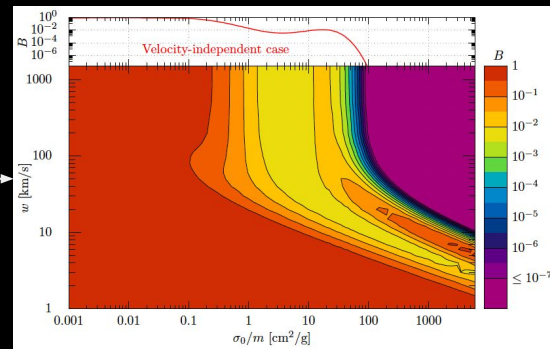
[Hayashi+\(2020\)\[arxiv:2007.13780\]](#)

Discussion & Future direction

[Shin'ichiro Ando, Kohei Hayashi, Shunichi Horigome, Masahiro Ibe, Satoshi Shirai \[arXiv:2503.13650\]](#)

- Baryonic feedback
 - Classical dSphs might be affected
- Galaxy formation model
 - Uncertainty: DM halo $\rightarrow$ Galaxy
- Spatial SASHIMI
 - Simulate precise mass-loss history of dSphs by tracing their orbital information
- Axisymmetric SASHIMI
 - Considering axisymmetry & tidal disruption

(e.g. $V_{50} = 18$ km/s)



Method: Jeans analysis

Jeans equation:

$$\frac{1}{\nu} \frac{d(\nu(r) \sigma_r^2(r))}{dr} + \frac{2\beta(r) \sigma_r^2(r)}{r} = -\frac{G M(r)}{r^2}$$

$$M(r) = \int dr 4\pi r^2 \rho_{\text{DM}}(r)$$

$\nu(r)$: Number density (e.g. Plummer model)

$\beta(r)$: Anisotropy (e.g. flat model, [Baes & van Hese \(2007\)](#) model)

$\sigma_r(r)$: Velocity dispersion

$\rho_{\text{DM}}(r)$: DM density profile

SIDM: $\rho(r) = \frac{\rho_s r_s^3}{(r^4 + r_c^4)^{1/4} (r + r_s)^2}$

$r \gg r_c$
 $r \ll r_c$

→

$\frac{\rho_s r_s^3}{r(r + r_s)^2}$
 $\rho_s (r_s/r_c)^3$

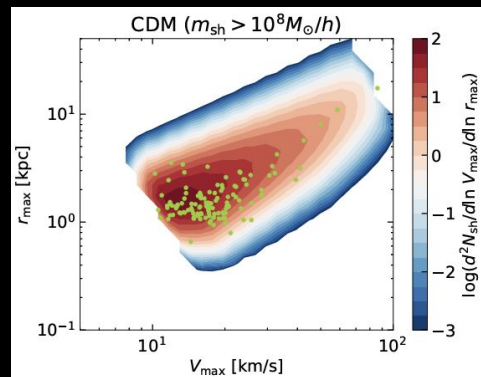
NFW

const. (cored)

Prior: Cosmological constraint of SASHIMI-SIDM

SASHIMI (Semi-Analytical SubHalo Inference Modeling) [\[arXiv:1803.07691\]](#), [\[arXiv:1903.11427\]](#)

- calculate various subhalo properties efficiently using semi-analytical models
 - Extended Press-Schechter formalism
 - Tidal stripping effect
- Outputs: catalogue of subhalos
 - accretion redshift
 - mass
 - $V_{\max}$, $r_{\max}$
 - ...

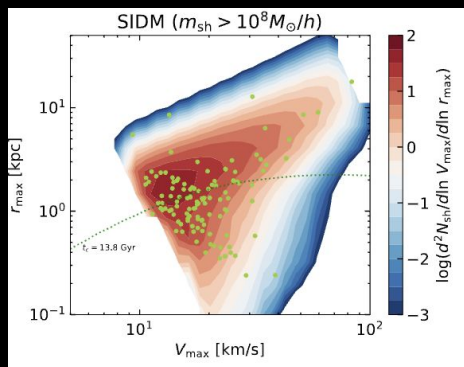


For CDM: [\[arXiv:2403.16633\]](#)

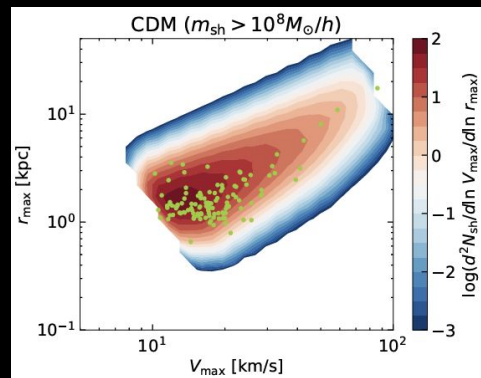
Prior: Cosmological constraint of SASHIMI-SIDM

SASHIMI (Semi-Analytical SubHalo Inference Modeling) for SIDM [\[arXiv:2403.16633\]](#)

- Parametric model of SIDM halos [\[Yang+\(2023\)\]](#) to reinterpret CDM $\rightarrow$ SIDM



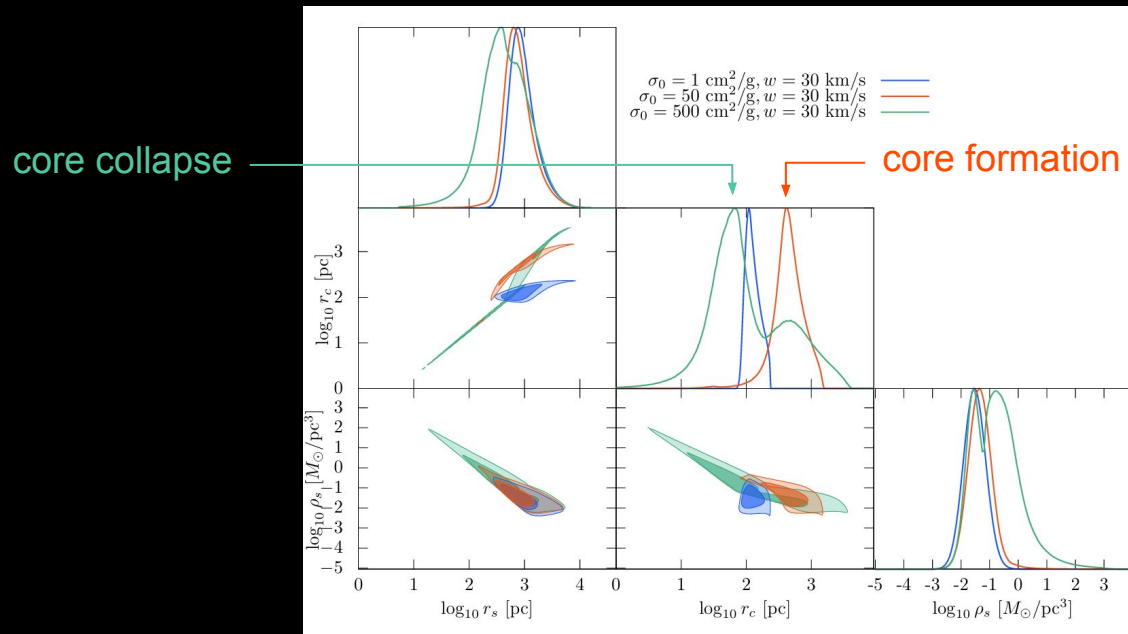
For SIDM: [\[arXiv:2403.16633\]](#)



For CDM: [\[arXiv:2403.16633\]](#)

Prior: Cosmological constraint of SASHIMI-SIDM

Example: Prior distributions for different SIDM parameters

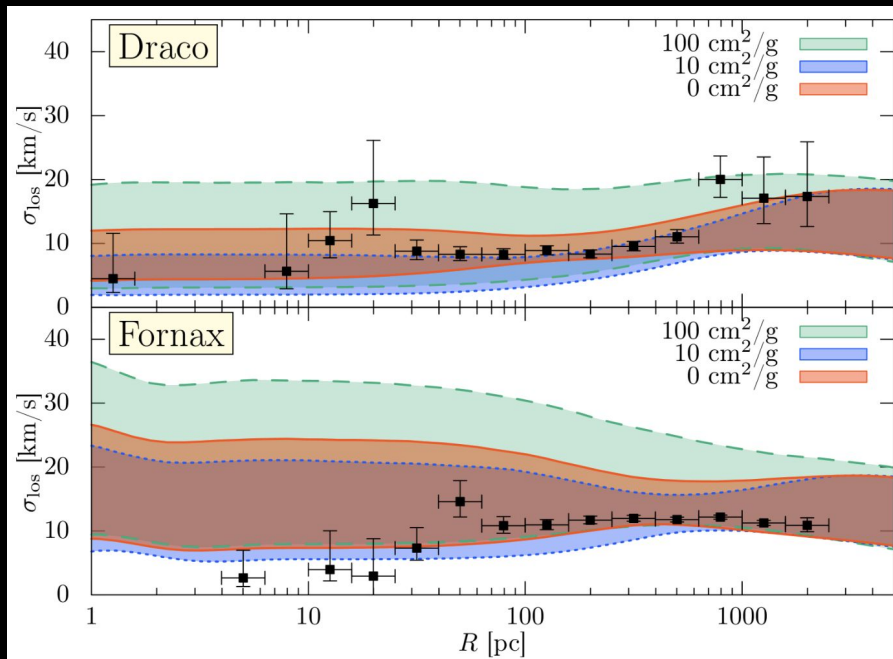


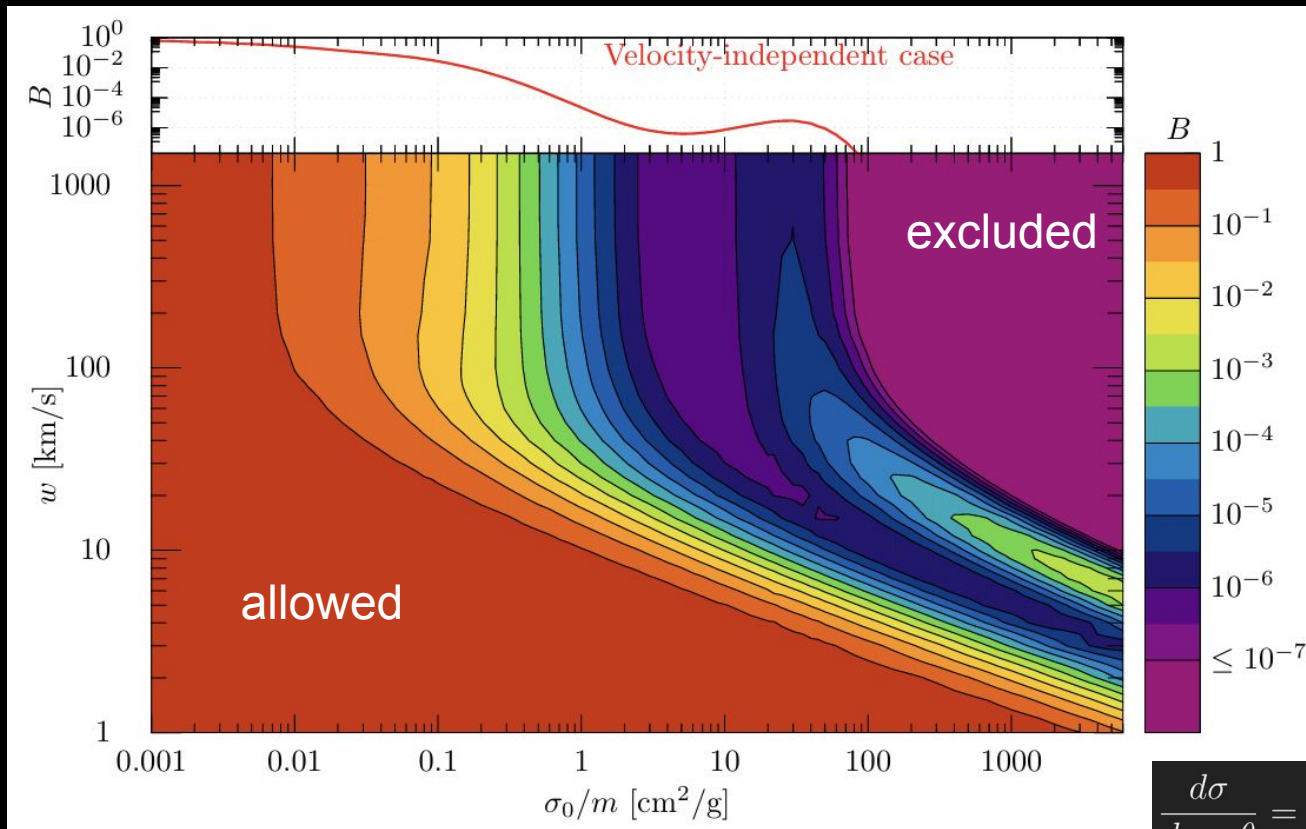
Results (SIDM)

Results

[Shin'ichiro Ando, Kohei Hayashi, Shunichi Horigome, Masahiro Ibe, Satoshi Shirai \[arXiv:2503.13650\]](#)

- Too large σ/m cannot fit observed velocity dispersion of stars
- To marginalize the diversity of dSphs, we compute total Bayes factor $B = \prod_i B_i$

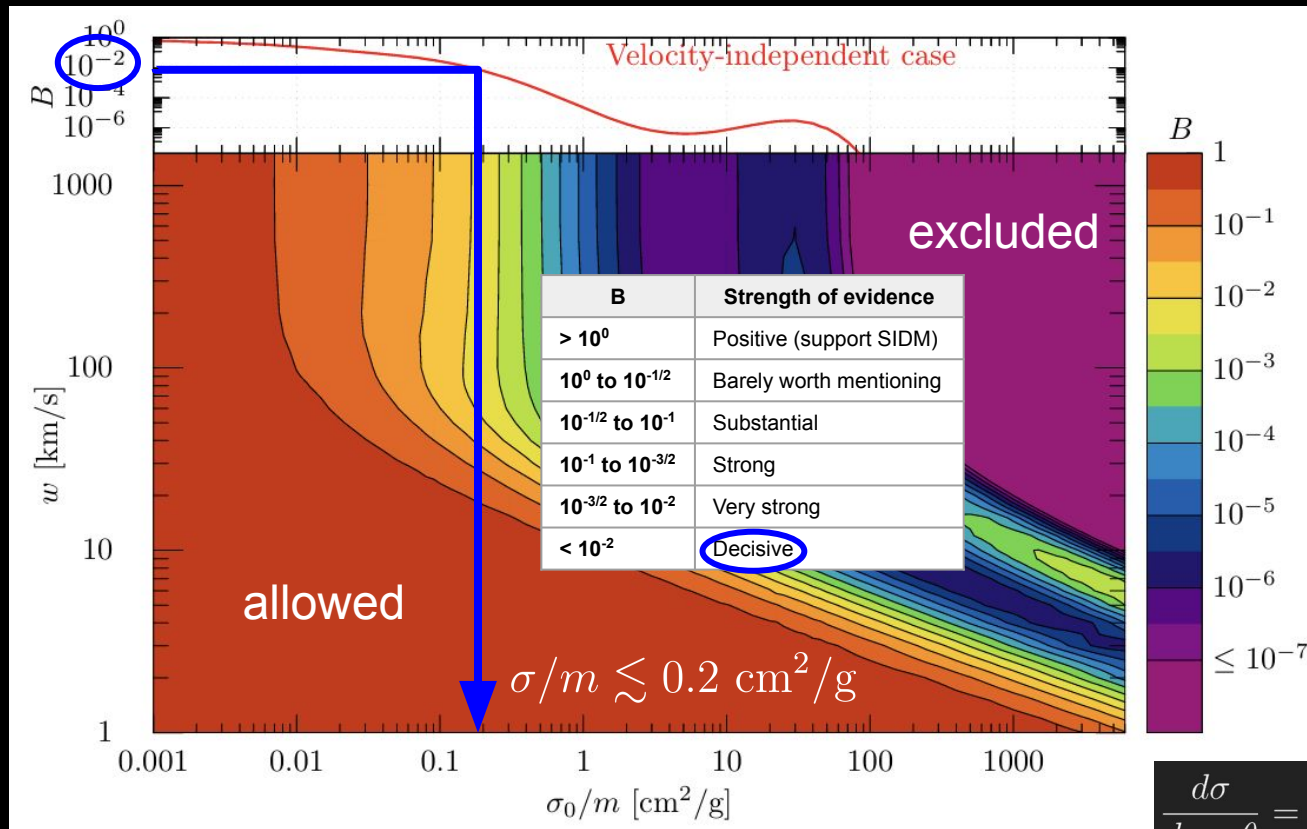




$$B = \prod_i B_i$$

$$B_i = \frac{Z_{\text{SIDM},i}}{Z_{\text{CDM},i}}$$

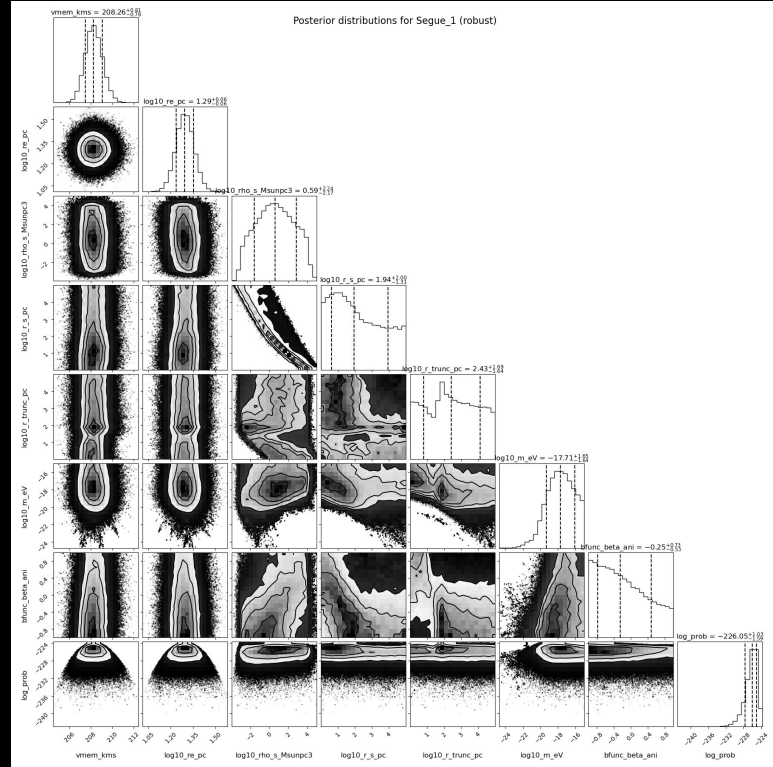
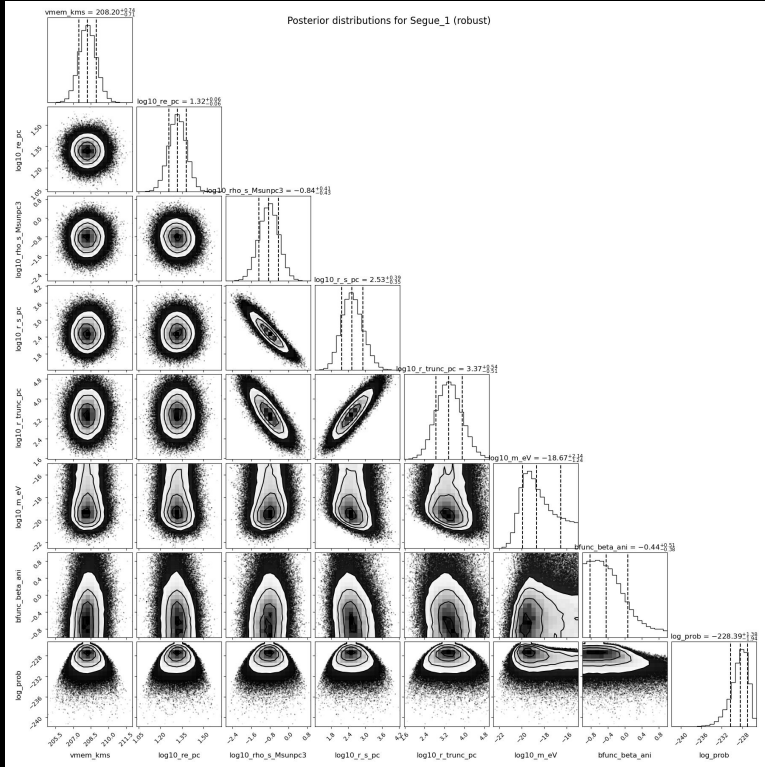
$$\frac{d\sigma}{d\cos\theta} = \frac{\sigma_0}{2[1 + (v/w)^2 \sin^2(\theta/2)]^2}$$



$$B = \prod_i B_i$$

$$B_i = \frac{Z_{\text{SIDM},i}}{Z_{\text{CDM},i}}$$

$$\frac{d\sigma}{d\cos\theta} = \frac{\sigma_0}{2[1 + (v/w)^2 \sin^2(\theta/2)]^2}$$



$$\begin{aligned}
-1000 &\leq v_{\text{mem}}/(\text{km s}^{-1}) \leq 1000, \\
0 &\leq \log_{10}(R_e/\text{pc}) \leq 5, \\
-5 &\leq \log_{10}(\rho_s/(M_{\odot} \text{ pc}^{-3})) \leq 5, \\
0 &\leq \log_{10}(r_s/\text{pc}) \leq 5, \\
0 &\leq \log_{10}(r_{\text{trunc}}/\text{pc}) \leq 5, \\
-25 &\leq \log_{10}(m/\text{eV}) \leq -15, \\
-1 &\leq b \equiv \log_{10}(1 - \beta_{\text{ani}}) \leq 1.
\end{aligned}$$

Prior: Cosmological constraint of SASHIMI-SIDM

SASHIMI (Semi-Analytical SubHalo Inference Modeling) for SIDM [\[arXiv:2403.16633\]](https://arxiv.org/abs/2403.16633)

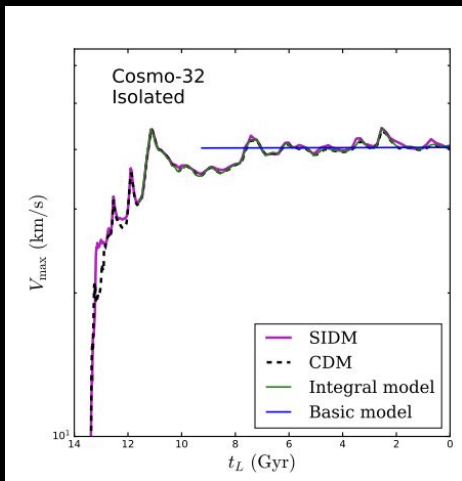
- Parametric model of SIDM halos [\[Yang+\(2023\)\]](#) to reinterpret CDM $\rightarrow$ SIDM
 - Empirical function of $\tau = t/t_c$
 - t_c : collapse time scale:

$$t_c = \frac{150}{C} \frac{1}{(\sigma_{\text{eff}}/m)\rho_{\text{eff}}r_{\text{eff}}} \frac{1}{\sqrt{4\pi G\rho_{\text{eff}}}},$$

SIDM

$\frac{V_{\text{max}}}{V_{\text{max},0}} = 1 + 0.1777\tau - 4.399\tau^3 + 16.66\tau^4 - 18.87\tau^5 + 9.077\tau^7 - 2.436\tau^9$

CDM



[Yang+\(2023\)](#)

Bayesian analysis

$$\mathcal{L}_{\text{eff}}(\overbrace{r_s, \rho_s, r_c}^{\text{DM halo parameters}}) = \int d[\overbrace{r_\beta, \eta, \beta_{0,\infty}, r_{1/2}}^{\text{nuisance parameters}}] \mathcal{L}$$

$$\underbrace{Z(\sigma_0/m, w)}_{\text{SIDM paramters}} = \int d\rho_s dr_s dr_c \mathcal{L}_{\text{eff}}(r_s, \rho_s, r_c) P(\rho_s, r_s, r_c | \sigma_0/m, w)$$

- L : Likelihood of dSph kinematics (Jeans analysis)
- P : Prior from SIDM Cosmology
- $Z(\sigma/m, w)$: marginal likelihood (model *evidence*)
- $B(\sigma/m, w) = Z(\sigma/m, w)/Z(0,0)$: Bayes factor

Likelihood: Jeans analysis

$$-2 \log(\mathcal{L}) = \sum_i \left[\frac{(v_i - V)^2}{\sigma_i^2} + \log(2\pi\sigma_i^2) \right]$$

$\nu(r)$: number density
 $\sigma(r)$: velocity dispersion
 $\beta(r)$: anisotropy

Jeans equation:

$$\frac{\partial \nu(r) \sigma_r^2(r)}{\partial r} + \frac{2\beta_{\text{ani}}(r) \sigma_r^2(r)}{r} = -\nu(r) \frac{\partial \Phi}{\partial r}$$

Projection:

$$\sigma_{\text{los}}^2(R) = \frac{2}{\Sigma(R)} \int_R^\infty dr \left(1 - \beta_{\text{ani}}(r) \frac{R^2}{r^2} \right) \frac{\nu(r) \sigma_r^2(r)}{\sqrt{1 - R^2/r^2}}$$

Likelihood: Jeans analysis

Models

- $v(r)$: Plummer profile
- $\beta(r)$: [Baes & van Hese \(2007\)](#)
- SIDM profile:

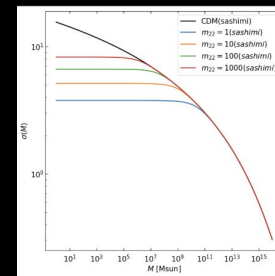
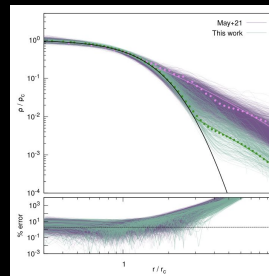
$$\rho(r) = \frac{\rho_s r_s^3}{(r^4 + r_c^4)^{1/4} (r + r_s)^2}$$

$r \gg r_c$ $\frac{\rho_s r_s^3}{r(r + r_s)^2}$ NFW

$r \ll r_c$ $\rho_s (r_s / r_c)^3$ const. (cored)

Difference from Dafa

- Smooth transition
- Cosmological prior
- Higher order moment
- Core mass–halo mass relation

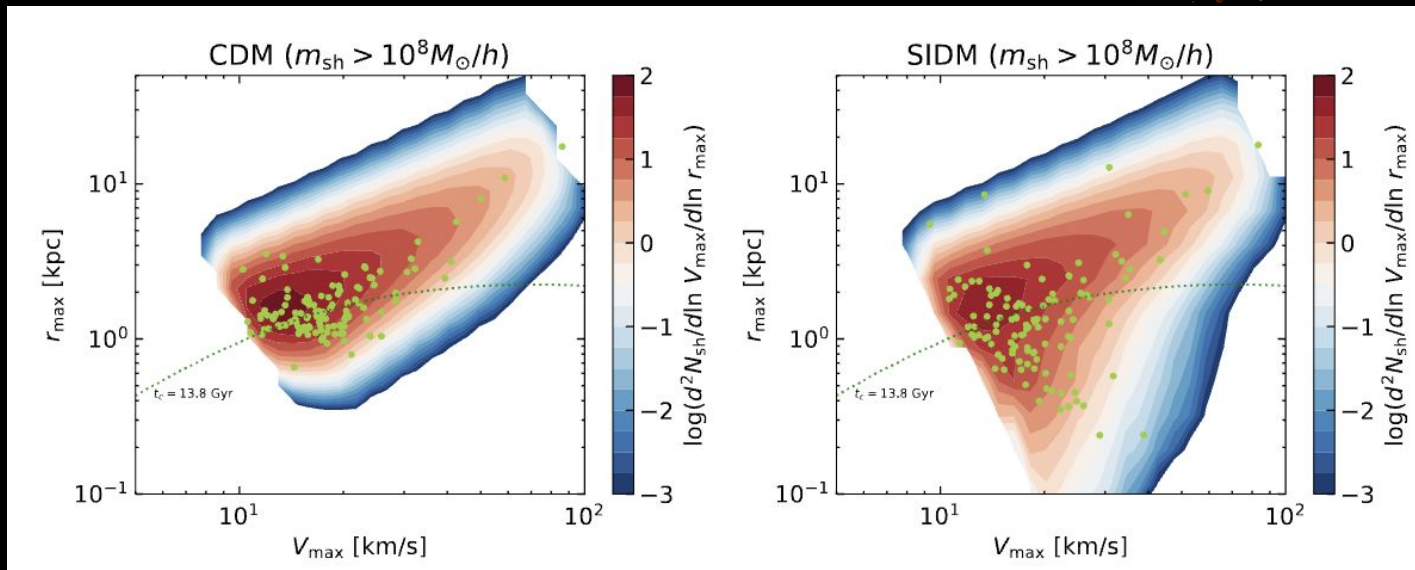


Various SASHIMI

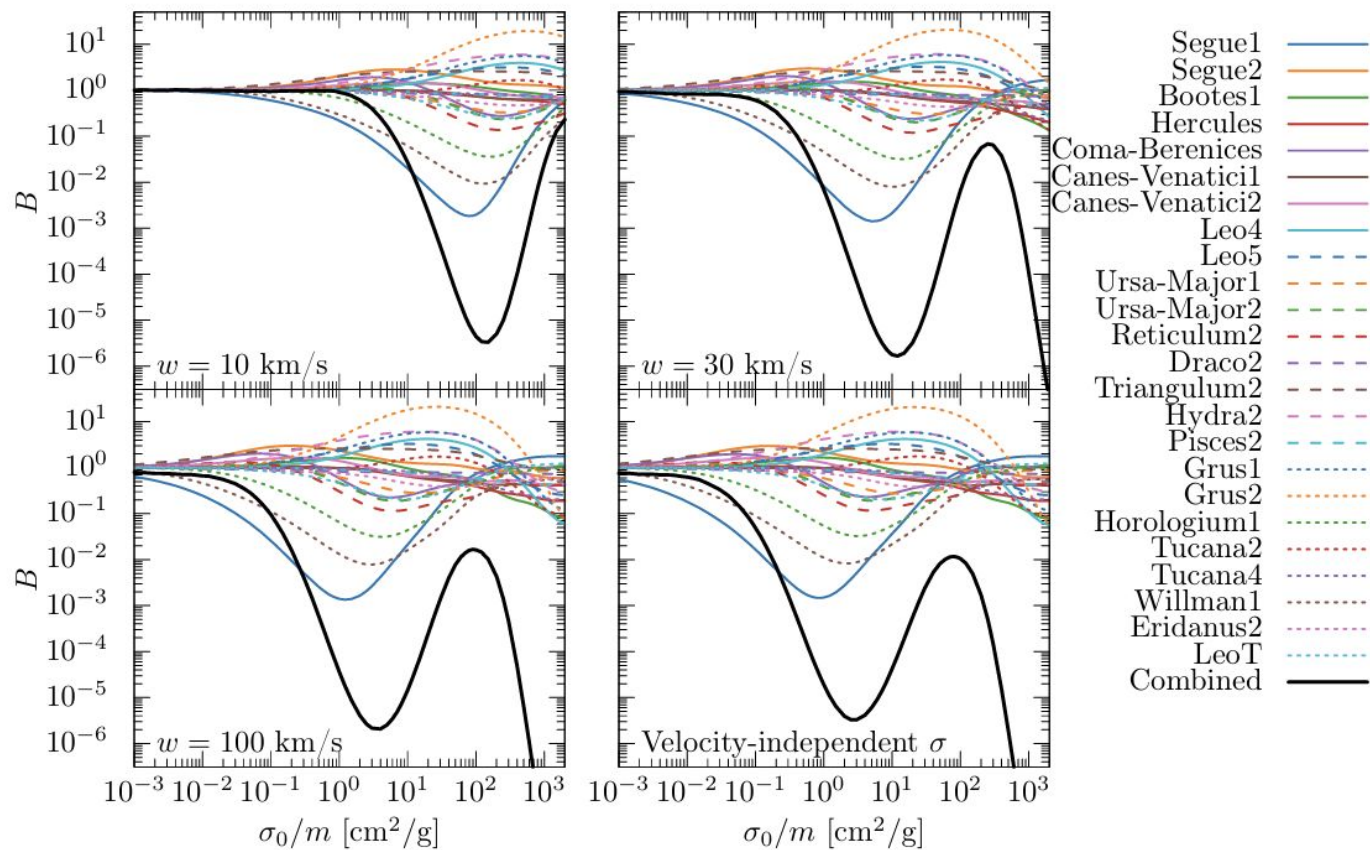


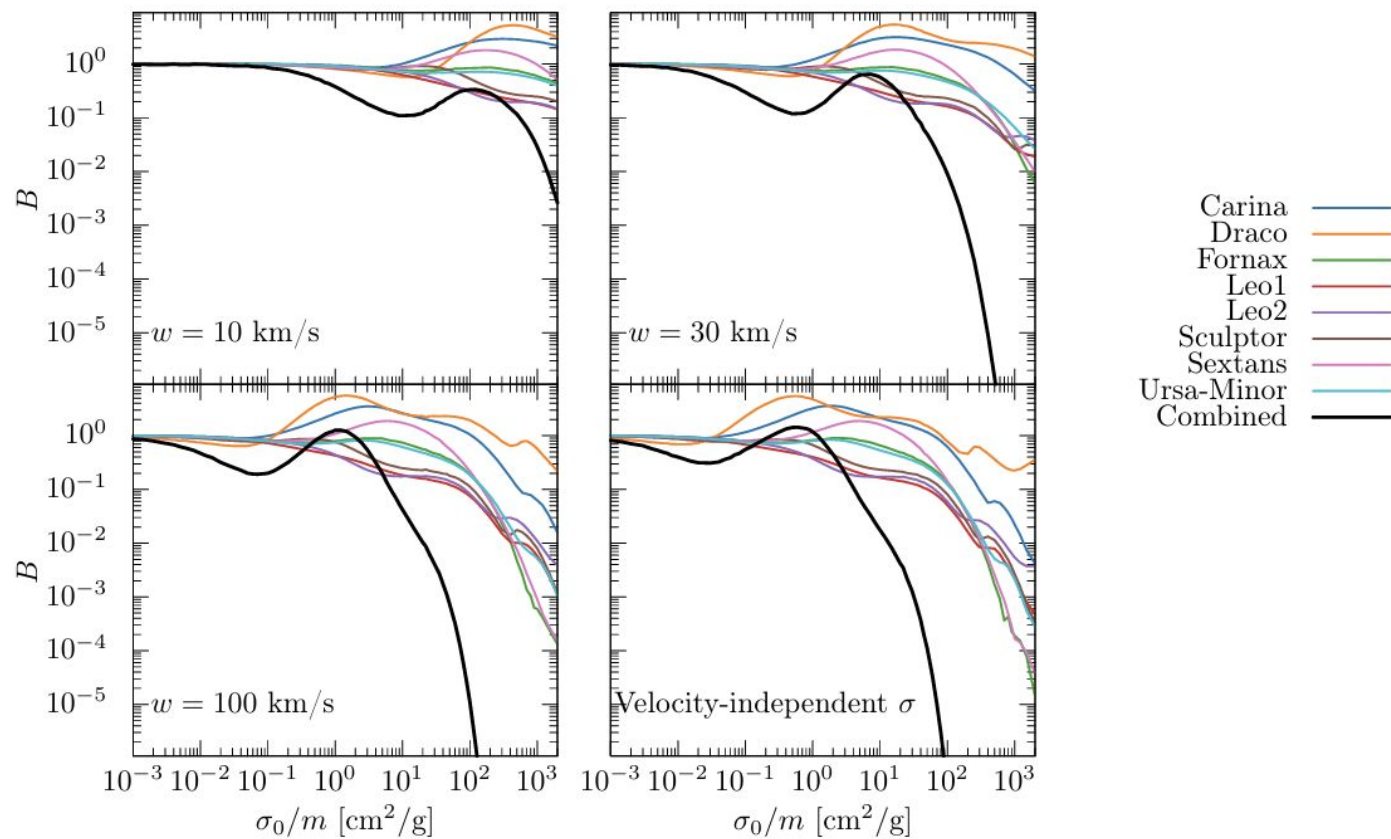
Testing and Tasting SASHIMI-SIDM

SASHIMI-SIDM can reproduce N-body results



SASHIMI-SIDM allows us to play with SIDM cosmology easily in a few minutes





Interpretation and next steps

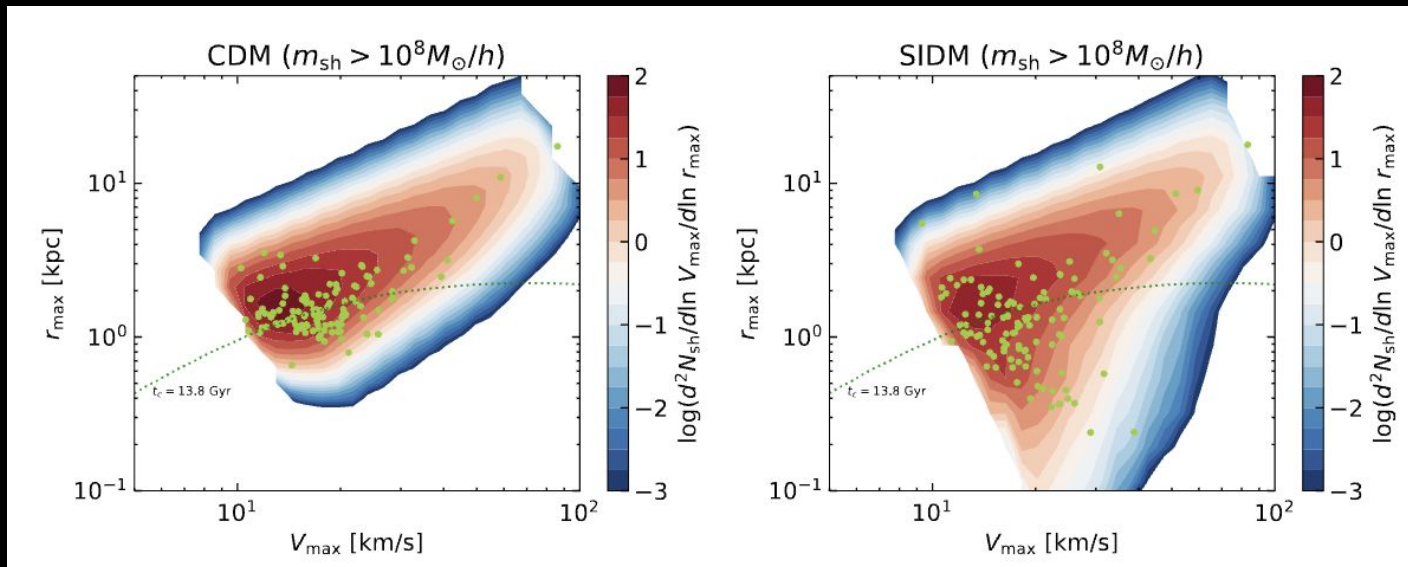
- Baryonic feedback
 - Classical dSphs might be affected
- Formation history
 -
- Spatial SASHIMI
 - Simulate precise mass-loss history of dSphs by tracing their orbital information
- Axisymmetric SASHIMI
 -

SIDM halo structure and evolution: Semi-analytical and effective models

S. Ando, S. Horigome, E. O. Nadler,
D. Yang, H.-B. Yu [in prep.]

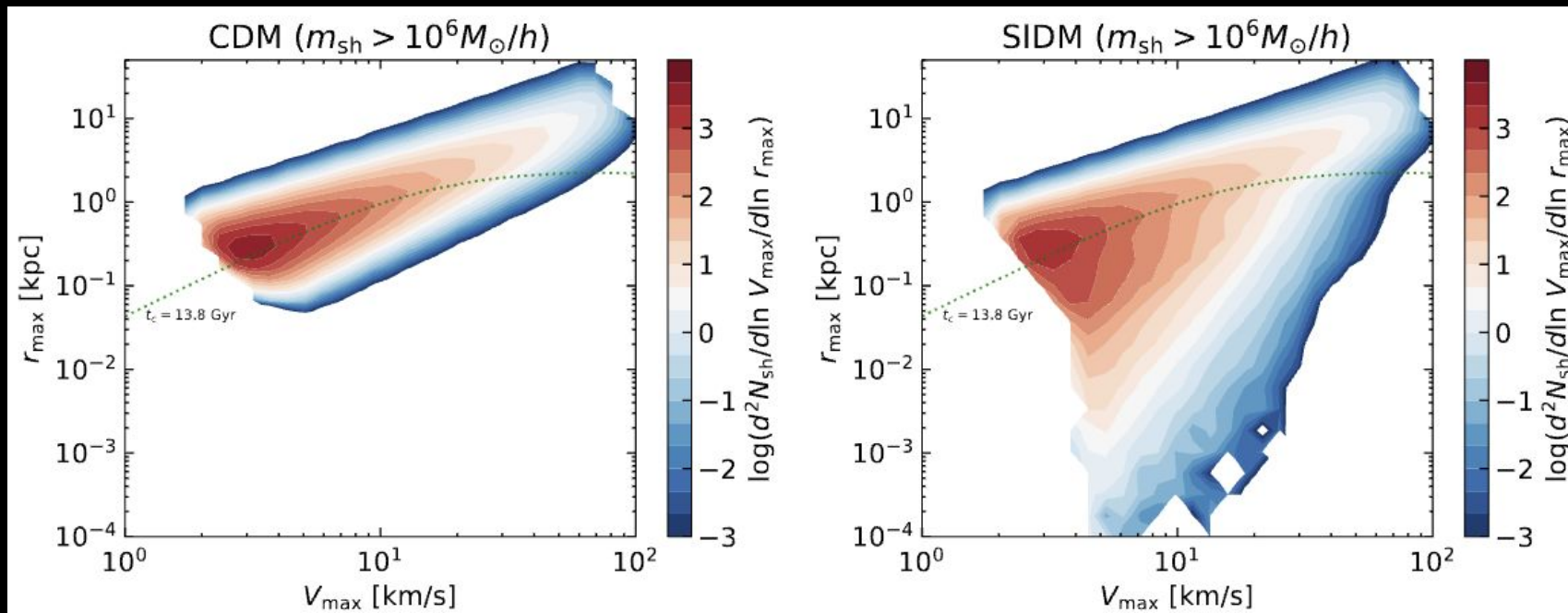
SASHIMI-SIDM: Semi-analytical approach to simulate SIDM models

- Quick calculation of SIDM halo properties and subhalo mass functions



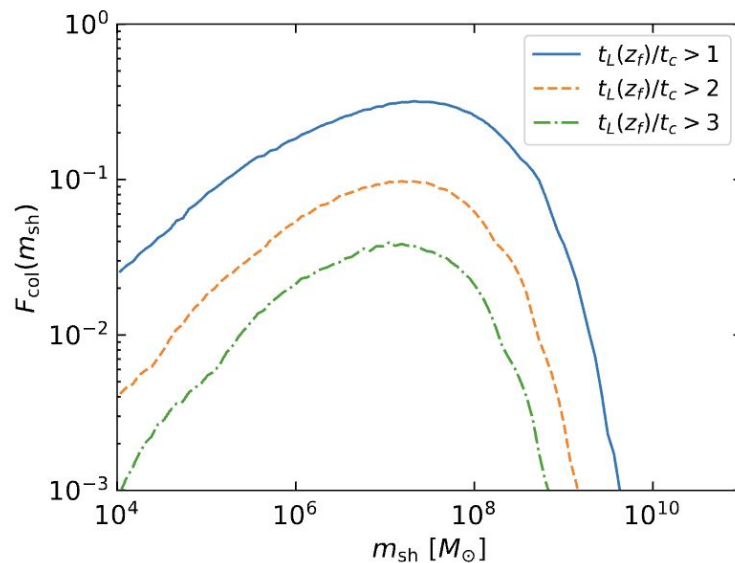
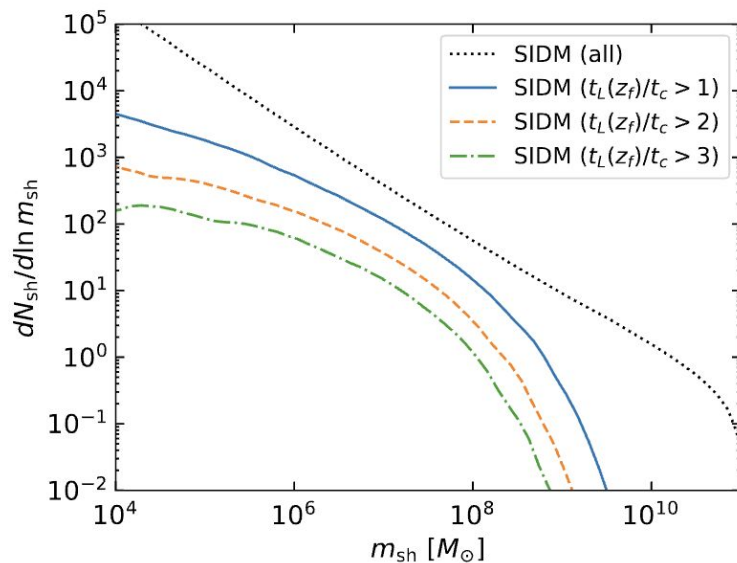
SASHIMI-SIDM

Use Case: Property for lighter subhalos



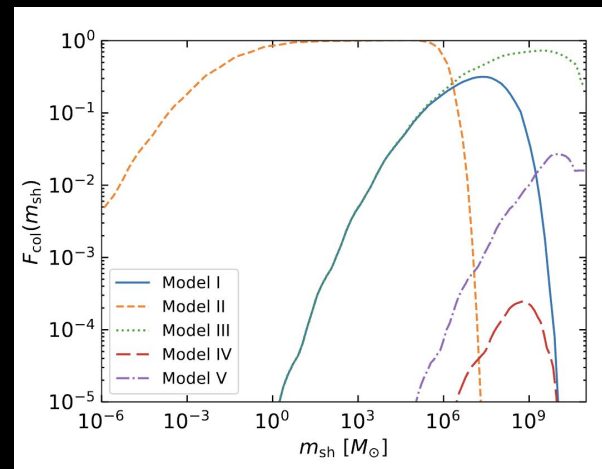
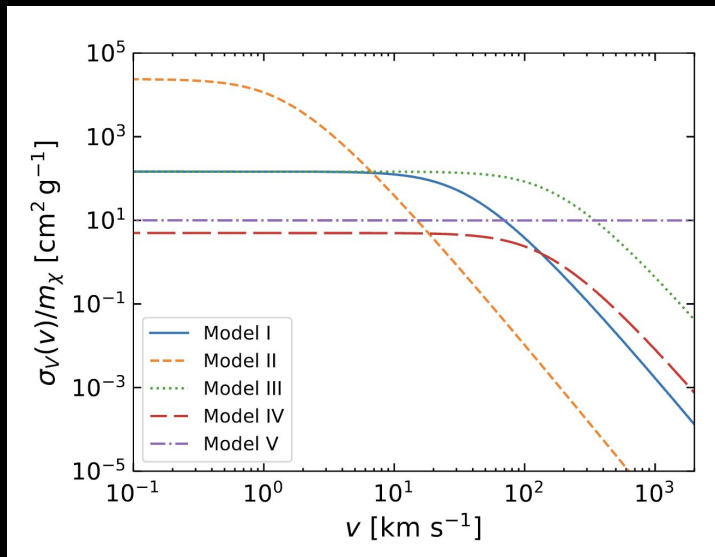
SASHIMI-SIDM

Use case: Subhalo mass function



SASHIMI-SIDM

Subhalo mass function for various SIDM models



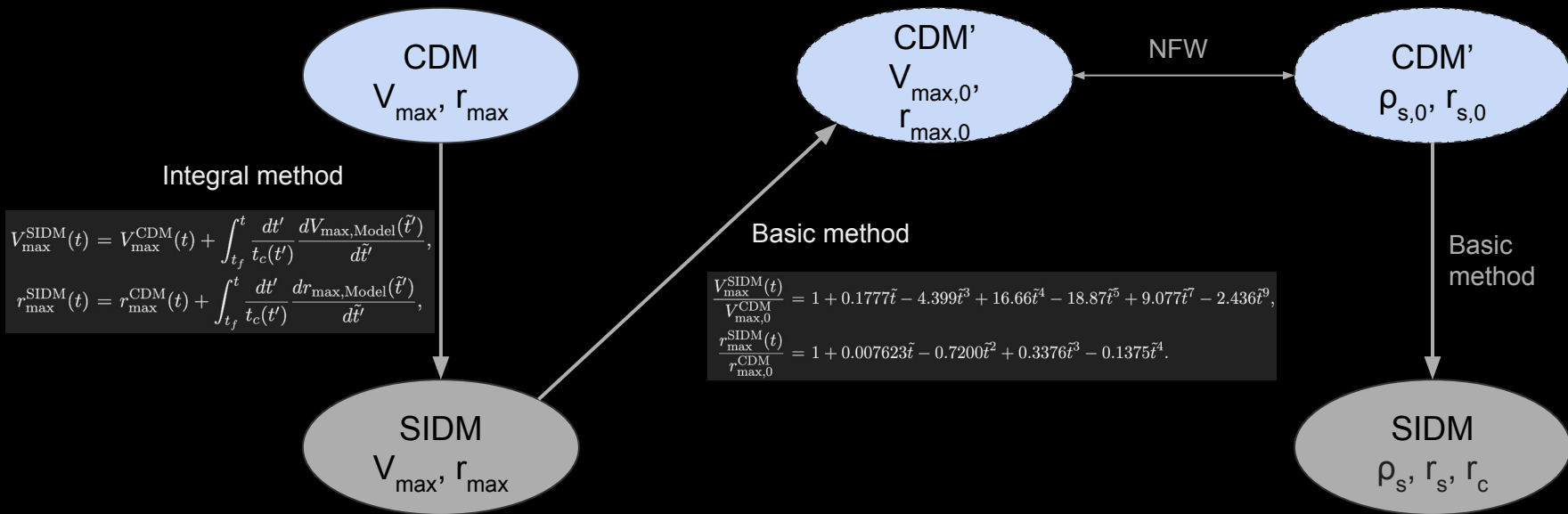
It works for lighter subhalo masses

Simulation setup behind the SIDM model

- **What simulations?**
 - DM-only N-body simulations (not hydrodynamical; no baryonic disk/feedback).
 - GADGET-2 + SIDM scattering module.
- **Calibration: isolated BM2 halo**
 - Initial NFW: $\rho_s = 2.74 \times 10^8 \text{ M}_\odot \text{ kpc}^{-3}$; $r_s = 0.141 \text{ kpc}$.
 - 4 million DM particles; full gravothermal evolution from core formation to collapse.
 - Rutherford-like velocity/angular-dependent scattering + constant-cross-section tests.
 - $\sigma_{\text{eff}}/m = 7.1 \text{ cm}^2 \text{ g}^{-1}$..
- **Validation: cosmological MW zoom-in**
 - Matched CDM/SIDM zoom-in of an MW-like host + LMC analog.
 - $m_p = 4 \times 10^4 \text{ M}_\odot \text{ h}^{-1}$; $\varepsilon = 80 \text{ pc h}^{-1}$.
 - High-res region $\approx 3 \text{ Mpc}$ ($\approx 10 R_{\text{vir}}$).
 - SIDM benchmark: $\sigma_0/m = 147.1 \text{ cm}^2 \text{ g}^{-1}$; $w = 24.33 \text{ km s}^{-1}$.
 - Test cut: $M_{\text{vir}} > 10^8 \text{ M}_\odot \text{ h}^{-1}$; > 3500 particles.

Refs: [Yang+2024] [arXiv:2305.16176] — parametric SIDM / isolated N-body calibration
[Yang+2023] [arXiv:2211.13768] — DMO cosmological MW zoom-in
[Ando+2024] [arXiv:2403.16633] — SASHIMI-SIDM implementation

Parametric Model

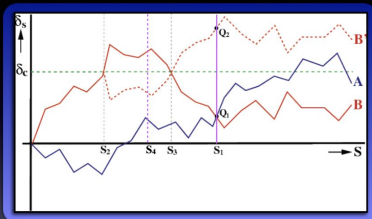


EPS formalism

[Press & Schechter(1974)]

Ref: http://www.astro.yale.edu/vdbosch/astro610_lecture9.pdf

The Excursion Set Formalism



Three trajectories corresponding to three different mass elements in a Gaussian random field. Note that B' is obtained mirroring trajectory B in the line $\delta_S = \delta_c$ for $S > S_1$. Since the trajectories are Markovian B and B' are equally likely!

The problem with the PS ansatz is that it fails to account for trajectories such as B when counting mass elements in haloes with mass $M > M_1$.

Correcting for this is easy though, by realizing that each trajectory B has a mirror version, B' , that is equally likely (as a result of the Markovian nature of the trajectories).

Double-counting trajectories with $\delta_S > \delta_c$ at S_1 corrects for 'missed trajectories'....



→ A natural explanation for the fudge-factor two in PS formalism!

ASTR 610: Theory of Galaxy Formation

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The Excursion Set Formalism

In the excursion set formalism, also called the Extended Press-Schechter (EPS) formalism, one uses the (statistics of) Markovian random walks (the trajectories of mass elements in (S, δ_S) -space) to infer the halo mass function (and more).

PS ansatz:

fraction of mass elements with $\delta_S > \delta_c(t)$ is equal to the mass fraction that at time t resides in haloes with masses $> M$, where S and M are related according to $S = \sigma^2(M)$



EPS ansatz:

fraction of trajectories with a first upcrossing (FU) of the barrier $\delta_S = \delta_c(t)$ at $S > S_1 = \sigma^2(M_1)$ is equal to the mass fraction that at time t resides in haloes with masses $< M_1$

Since, each trajectory is guaranteed to upcross the barrier $\delta_S = \delta_c(t)$ at some (arbitrarily large) S , the EPS ansatz predicts that every mass element is in a halo of some (arbitrarily low) mass

$$\Rightarrow F(< M_1) = 1 - F(> M_1)$$

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The EPS Mass Function

Based on the EPS ansatz, we can write the EPS mass function as:

$$n(M, t) dM = \frac{\bar{\rho}}{M} \frac{\partial F(> M)}{\partial M} dM = - \frac{\bar{\rho}}{M} \frac{\partial F(< M)}{\partial M} dM \\ = - \frac{\bar{\rho}}{M} \frac{\partial f_{FU}(> S)}{\partial S} \frac{dS}{dM} dM = \frac{\bar{\rho}}{M} f_{FU}(S, \delta_c) \left| \frac{dS}{dM} \right| dM$$

Here $f_{FU}(S, \delta_c) dS$ is the fraction of trajectories that have their first upcrossing of barrier $\delta_c(t)$ between S and $S + dS$.

Without proof:

$$f_{FU}(\nu) = \frac{1}{\sqrt{2\pi}} \frac{\delta_c}{S^{3/2}} \exp\left[-\frac{\delta_c^2}{2S}\right] = \frac{1}{2S} f_{PS}(\nu) \quad (\text{see MBW §7.2.2 for derivation})$$

where, as before, we defined $\nu = \delta_c(t)/\sigma(M) = \delta_c/\sqrt{S}$ and we expressed the result in terms of the PS multiplicity function $f_{PS}(\nu) = \sqrt{2/\pi} \nu \exp(-\nu^2/2)$

It is straightforward to show that this yields exactly the same halo mass function as before, but this time there has been no need for a fudge factor....

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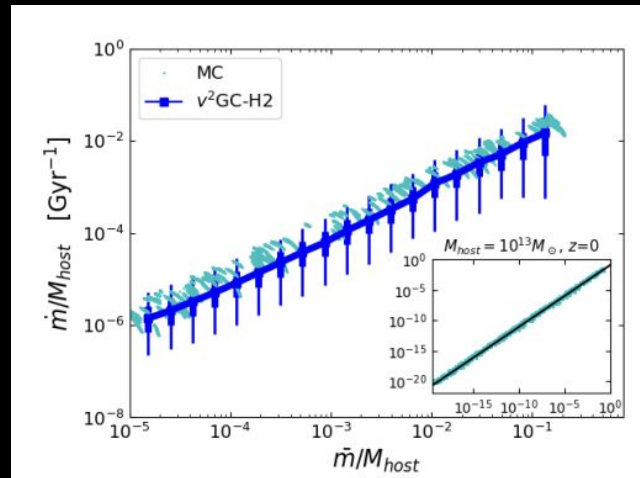
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SASHIMI

Tidal stripping: [[Hiroshima+\(2018\)](#)]

$$\dot{m}(z) = -A \frac{m(z)}{\tau_{\text{dyn}}(z)} \left[\frac{m(z)}{M(z)} \right]^\zeta,$$

$$\begin{aligned} \log A &= \left[-0.0003 \log \left(\frac{M_{\text{host}}}{M_\odot} \right) + 0.02 \right] z \\ &\quad + 0.011 \log \left(\frac{M_{\text{host}}}{M_\odot} \right) - 0.354, \\ \zeta &= \left[0.00012 \log \left(\frac{M_{\text{host}}}{M_\odot} \right) - 0.0033 \right] z \\ &\quad - 0.0011 \log \left(\frac{M_{\text{host}}}{M_\odot} \right) + 0.026. \end{aligned}$$



SIDM review

Sean Tulin, Hai-Bo Yu, "Dark Matter Self-interactions and Small Scale Structure"