International Center for
Quantum-field Measurement Systems for
Studies of the Universe and Particles
WPI research center at KEK

Enhanced Dark Matter Quantum Sensing via Squeezing-amplified Geometric Phase

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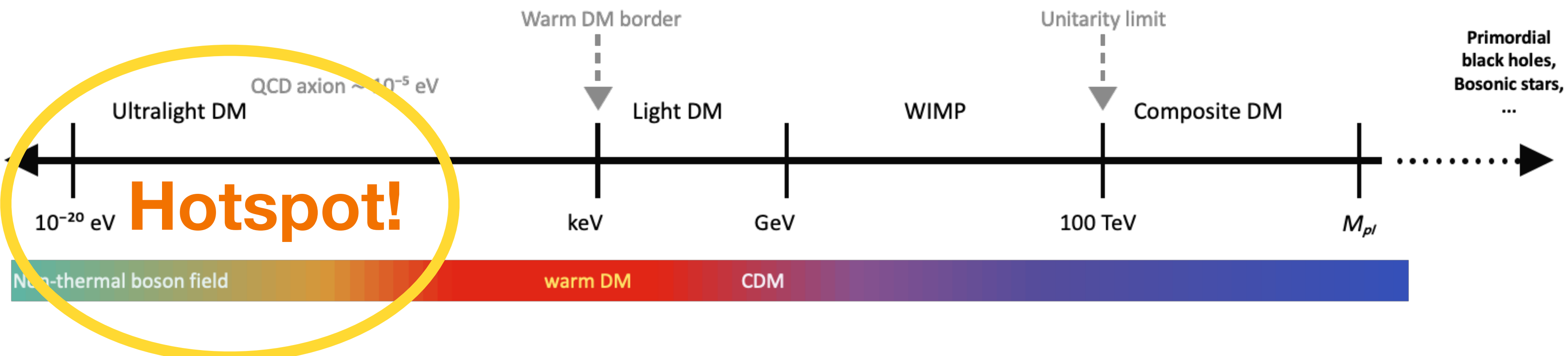
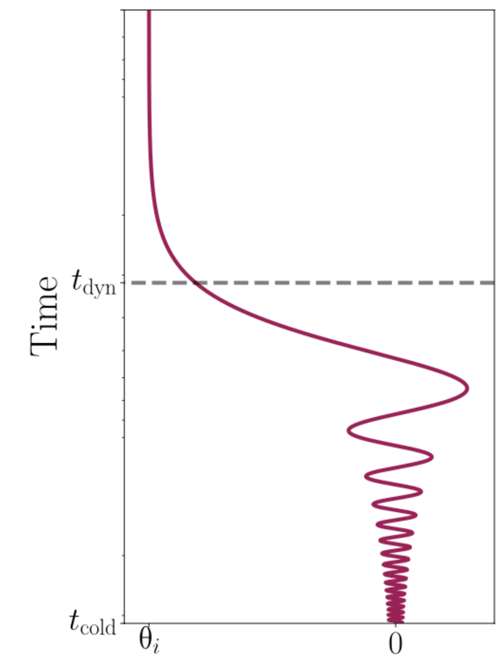
arXiv: 2603.23599

(KEK/QUP)

Dark Matter at μeV Matters





- Ultralight bosonic dark matter is well motivated: axions and dark photons are prime WISP candidates.

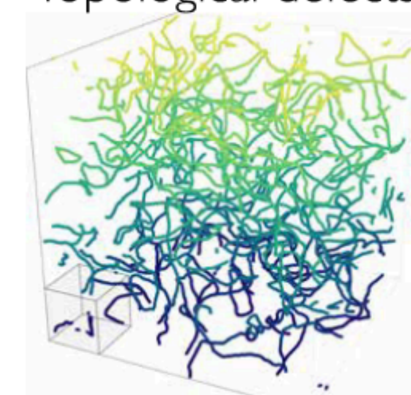
- The μeV axion(dark photon) dark matter with various production mechanisms.

$$\theta_a \sim \mathcal{O}(1) \qquad m_a \simeq 5.7\ \mu\text{eV} \times \left(\frac{10^{12}\text{GeV}}{f_a}\right)$$ **Axion**

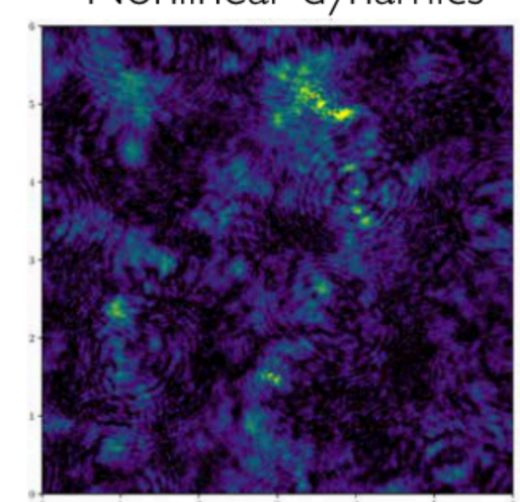
$$m_{A'} \simeq 6\mu\text{eV} \left(\frac{10^{14}\ \text{GeV}}{H_I}\right)^4 .$$ **Dark photon**

Graham, 1504.02102

Topological defects



Nonlinear dynamics



Chadha-Day, Sci.Adv. 8 (2022) 8

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Cavity Searches For $\mu\mathrm{eV}$ DM

Axion and Dark photon naturally interact with Standard model photon:

$$\epsilon F_{\mu\nu}F'^{\mu\nu}$$

(Kinetic mixing with photon)

$$g_{a\gamma\gamma} a F_{\mu\nu}\tilde{F}^{\mu\nu}$$

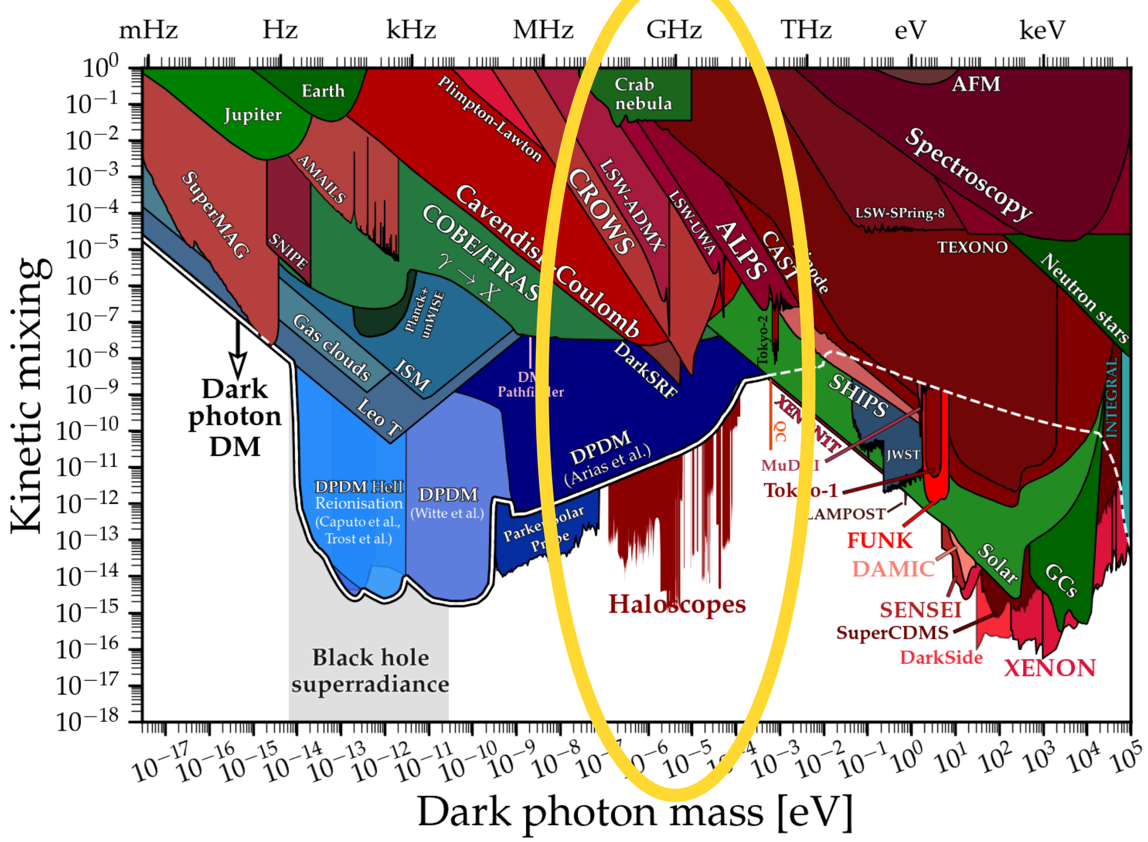
(Peccei–Quinn current EM anomaly)

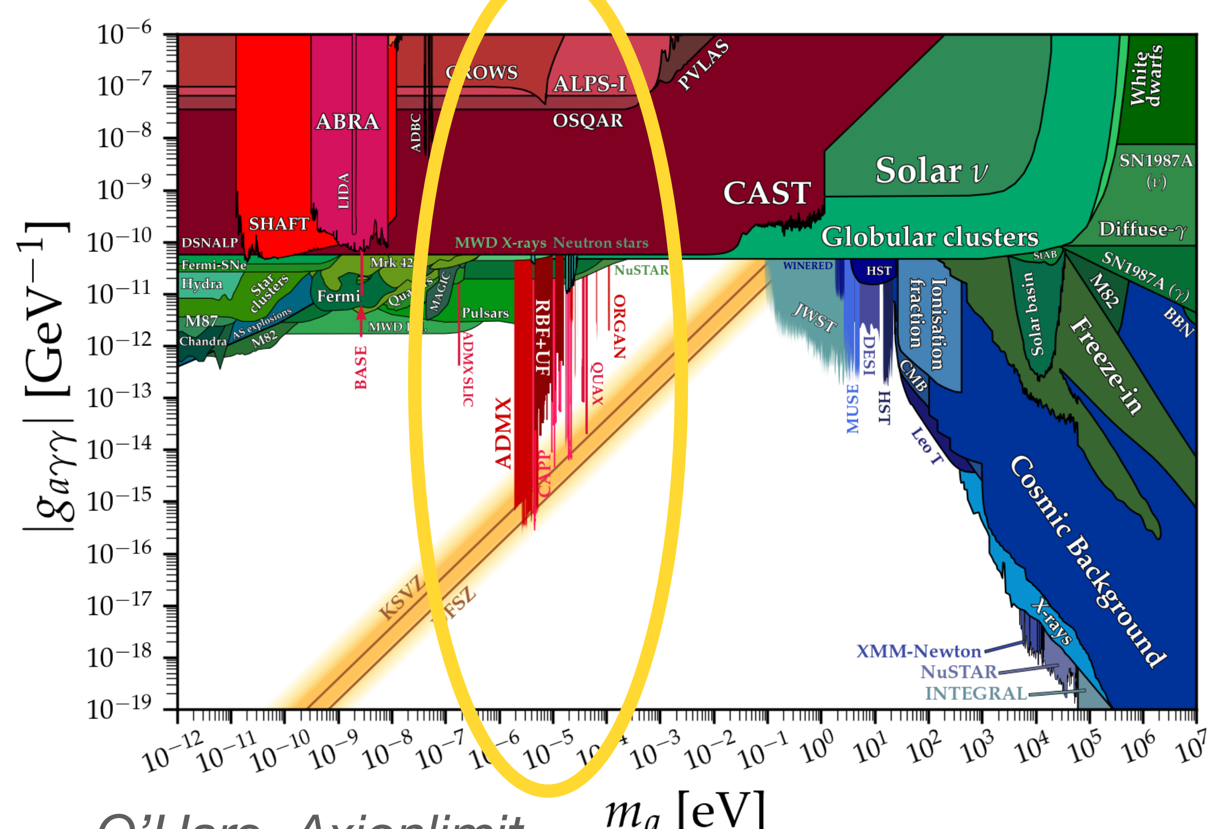
$$\nabla \times (\nabla \times \mathbf{E}) + \partial_t^2 \mathbf{E} = -\partial_t \mathbf{j}_{\mathrm{eff}}$$

DM induced effective current

$$\mathbf{J}_{\mathrm{eff}}(A') = \epsilon m_{A'}^2 \mathbf{A}'$$

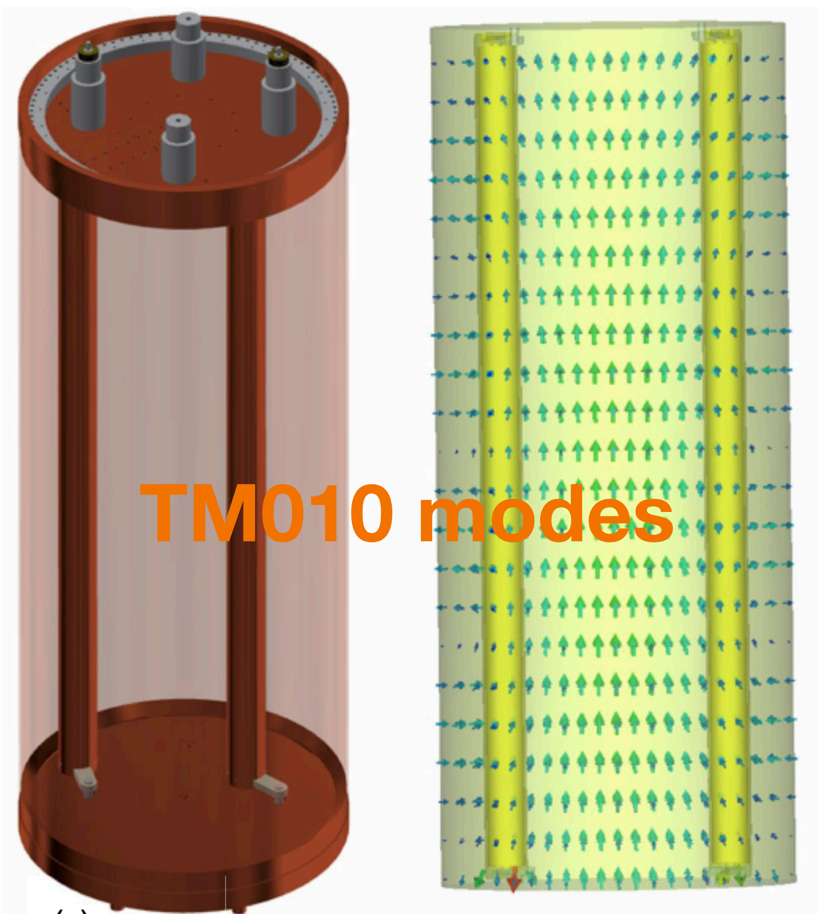
$$\mathbf{J}_{\mathrm{eff}}(a) = g_{a\gamma\gamma} \mathbf{B}_0 \dot{a}$$





O'Hare, Axionlimit

Cavity Searches For High-frequency DM



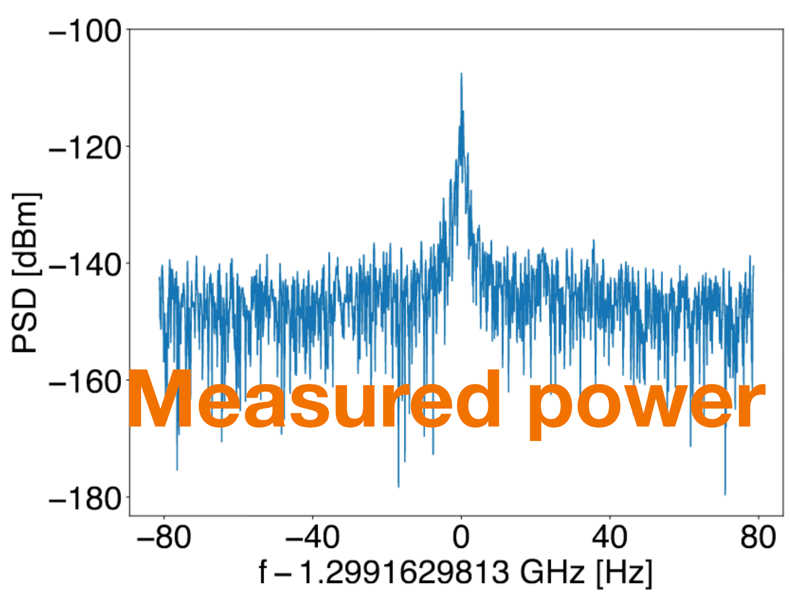
Source: ADMX

$$\mathbf{E}(\mathbf{x},t) = \sum_c e_c(t)\mathbf{E}_c(\mathbf{x})$$

Modes decomposition

$$\left(\partial_t^2 + \frac{\omega_c}{Q_c}\partial_t + \omega_c^2\right) e_c(t) = -\frac{\int_{V_{\rm Cav}} d^3\mathbf{x}\, \mathbf{E}_c^*(\mathbf{x})\cdot\partial_t \mathbf{j}_{\rm eff}}{\int_{V_{\rm Cav}} d^3\mathbf{x}\, |\mathbf{E}_c(\mathbf{x})|^2}$$

EOM for DM current driving cavity modes



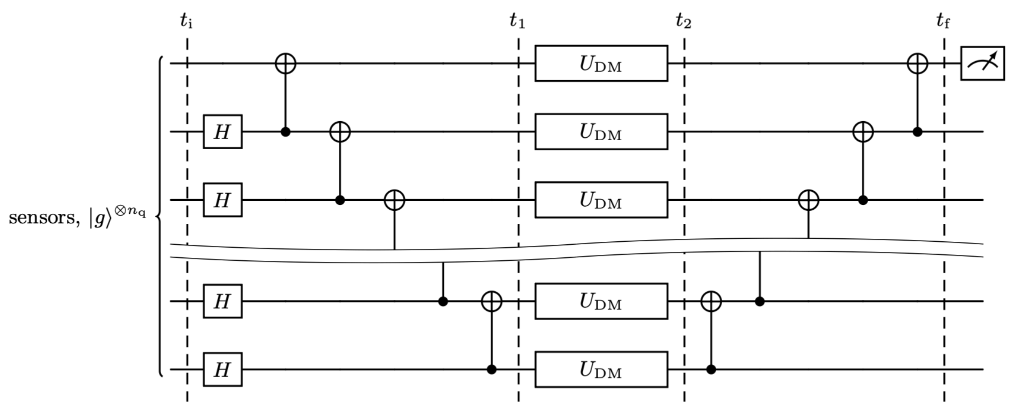
$$\epsilon \approx 10^{-16}\left(\frac{10^{10}}{Q_0}\right)^{\frac{1}{4}}\left(\frac{4\ \mathrm{L}}{V}\right)^{\frac{1}{2}}\left(\frac{0.5}{C}\right)^{\frac{1}{2}}\left(\frac{100\ \mathrm{s}}{t_{\rm int}}\right)^{\frac{1}{4}}\left(\frac{1.3\mathrm{GHz}}{f_0}\right)^{\frac{1}{4}}\left(\frac{T_{\rm amp}}{3\ \mathrm{K}}\right)^{\frac{1}{2}}$$

SHANHE Collabration, Phys.Rev.Lett. 133 (2024) 2, 021005

Into the realm of quantum sensing

Quantum resources-> quantum Fisher information

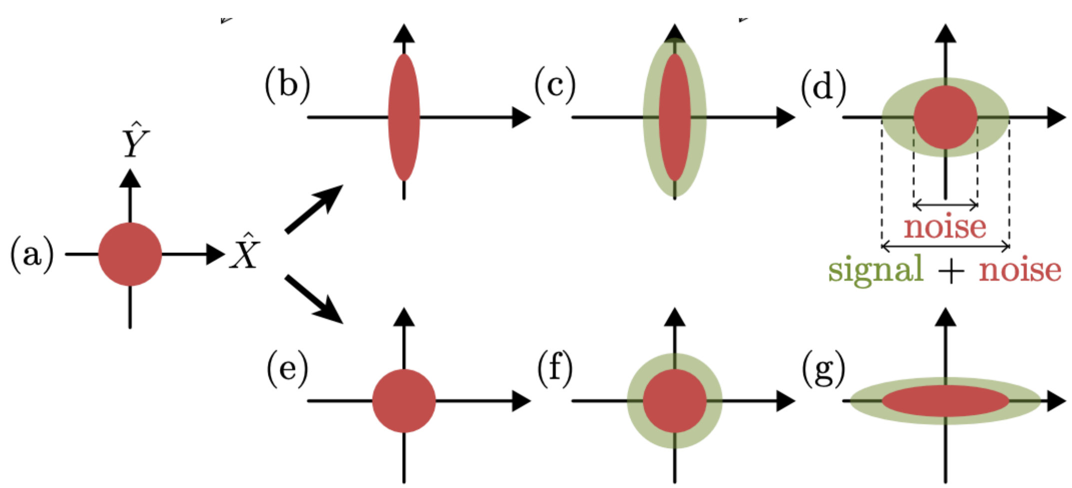
{Entanglement, squeezing, multi-levels etc} -> better parameter sensitivity



Entanglement

- Heisenberg sensitivity scaling of the sensors.

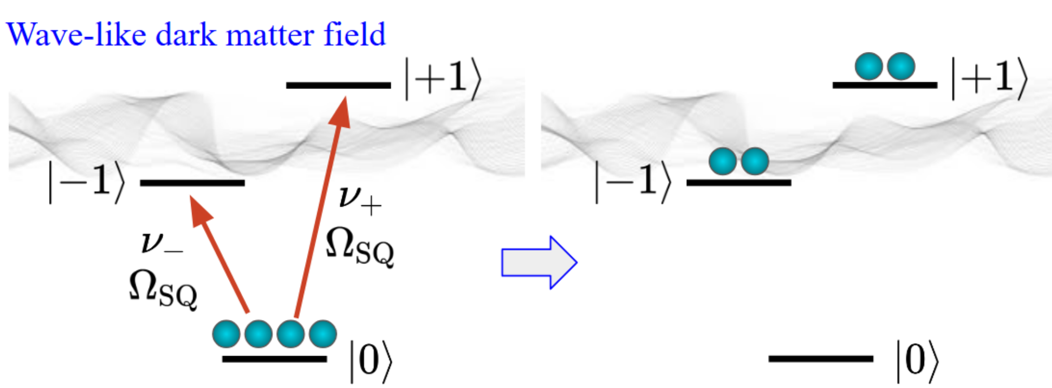
Chen et al, PRL.133.021801



Squeezing

- Injecting squeezed vacuum in cavity greatly enhances peak SNR and the scanning rate.

Malnou et al, PRX.9.021023



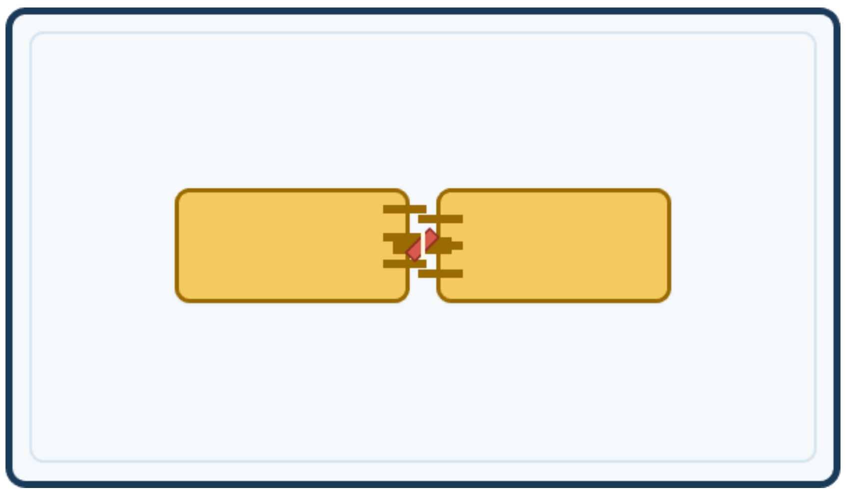
Multi-level

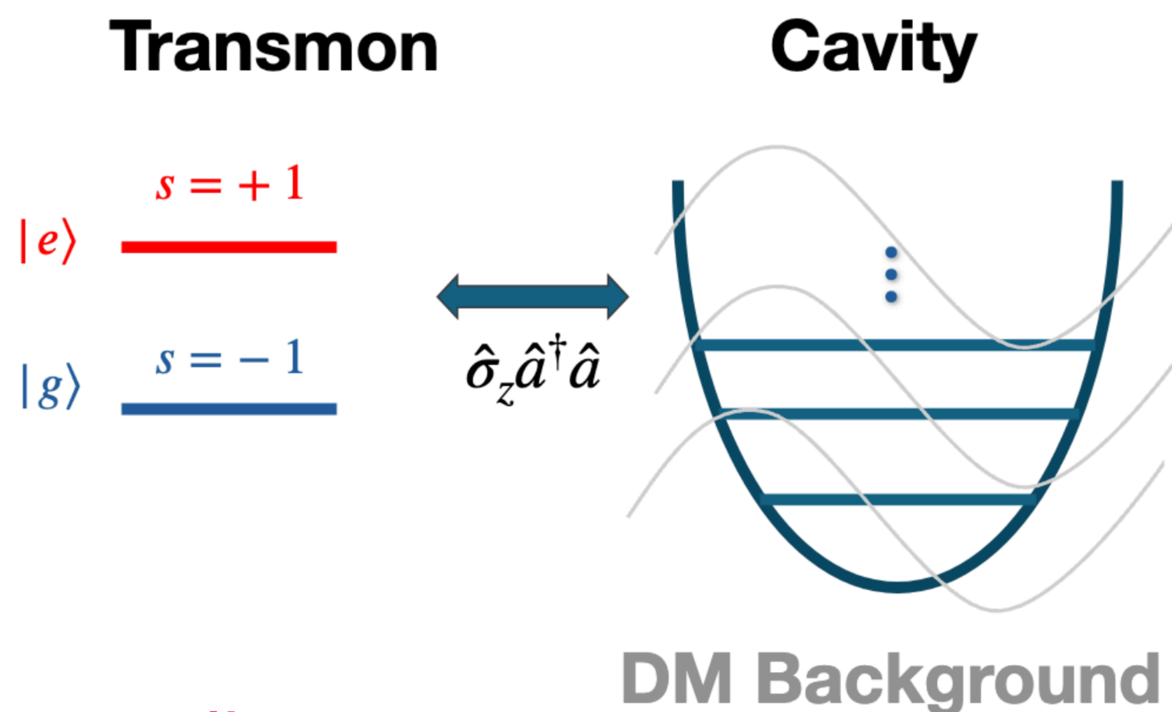
- Larger Hilbert space spectrum increase the quantum Fisher information.

Ma et al, PRA Lett 113 (2026)

Dispersive Qubit-Cavity Sensing of Dark Matter

Going quantum!





Dispersive coupling

$$\frac{H}{\hbar} = \omega_r\hat{a}^\dagger\hat{a} + \frac{\omega_q}{2}\hat{\sigma}_z + \frac{\chi}{2}\hat{a}^\dagger\hat{a}\,\hat{\sigma}_z + i\left(Ae^{-i(\omega_d t+\phi_1)}\hat{a}^\dagger - Ae^{i(\omega_d t+\phi_1)}\hat{a}\right).$$

Qubit

Cavity Modes

(TM010 mode considered)

DM drive (After RWA)

A: DM signal amplitude

$$\delta\omega_q = \langle a^\dagger a\rangle\chi/2$$

Qubit-Oscillator Platform for QND& freq tuning

$$\frac{H}{\hbar} = \underbrace{\boxed{\omega_r \hat{a}^\dagger \hat{a} + \frac{\omega_q}{2}\hat{\sigma}_z}}_{H_0} + \frac{\chi}{2}\hat{a}^\dagger \hat{a}\,\hat{\sigma}_z + i\left(A e^{-i(\omega_d t+\phi_1)}\hat{a}^\dagger - A e^{i(\omega_d t+\phi_1)}\hat{a}\right).$$

$$U(t_f, t_i) = e^{i\Phi_G(s)}\hat{R}(\Omega_s \tau)\hat{D}(\delta_s)\,.$$

DM amplitude&phases

- DM as effective current -> a tiny coherent drive into the cavity mode-> mode excitation.
- $\hat{D}(\alpha)\,|\,0\rangle \simeq |\,0\rangle + \alpha\,|\,1\rangle, \;\; P_1 \approx |\,\alpha\,|^2$
- Experiments with a high-Q cavity plus dispersively coupled transmon can prepare, control, and read out the cavity state.

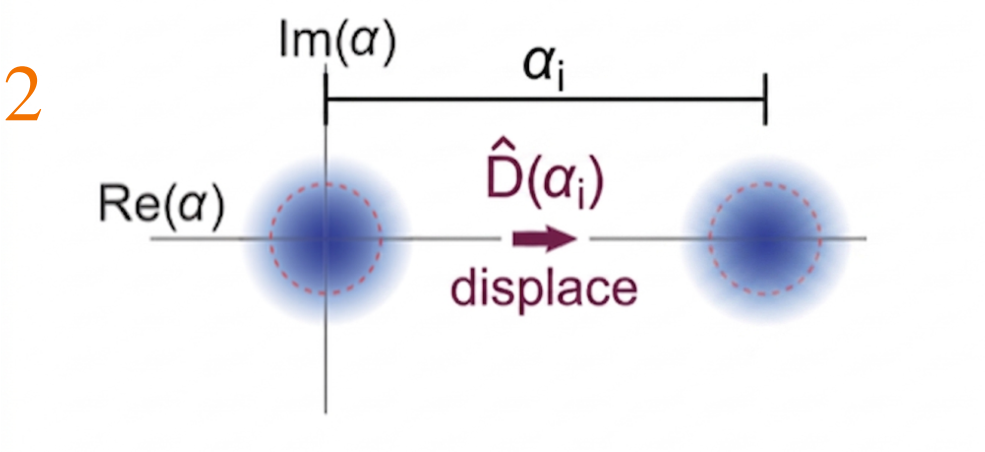
Pan Zheng et. al. 2507.23538

- Ideal platform: high-fidelity qubit readout, strong bosonic control, and long cavity coherence.

The displacement and squeezing operations

$$\Delta X_1 = \Delta X_2 = 1/2$$

Laser pulse



Coherent state $\Delta X_1 \Delta X_2 = 1/4$

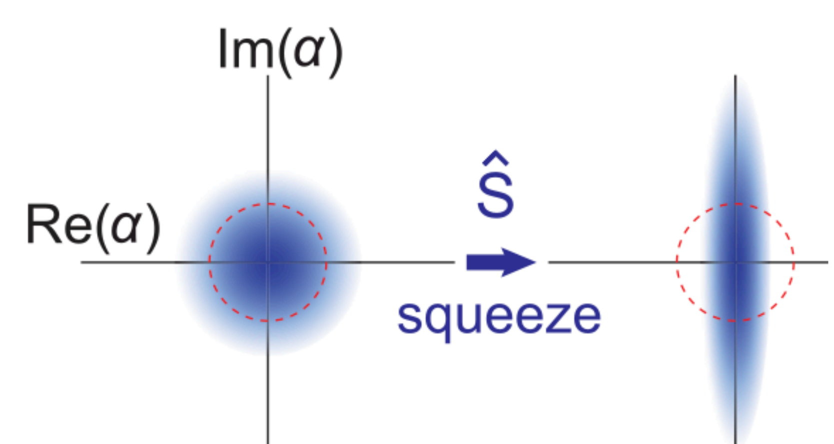
$$\hat{D}(\alpha) = \exp\left(\alpha \hat{a}^\dagger - \alpha^* \hat{a}\right)$$

$$\hat{a}\,|\,\alpha\rangle = \alpha\,|\,\alpha\rangle$$

$$|\,\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}\,|\,n\rangle$$

$$\Delta X_1 = e^{-r}/2$$

$$\Delta X_2 = e^{r}/2$$



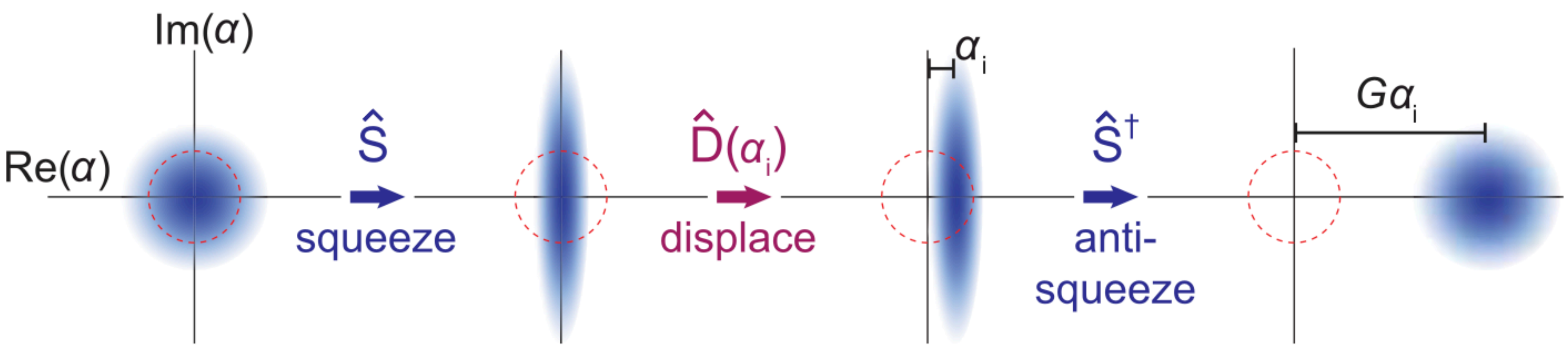
Beyond standard quantum limit

$$\hat{S}(\xi) = \exp\left(\frac{1}{2}\xi^* \hat{a}^2 - \frac{1}{2}\xi (\hat{a}^\dagger)^2\right)$$

Strong pump field at twice the resonance frequency

$$\xi = re^{i\theta}$$

Quantum Resource: Reversible Squeezing Amplifies Small Displacements

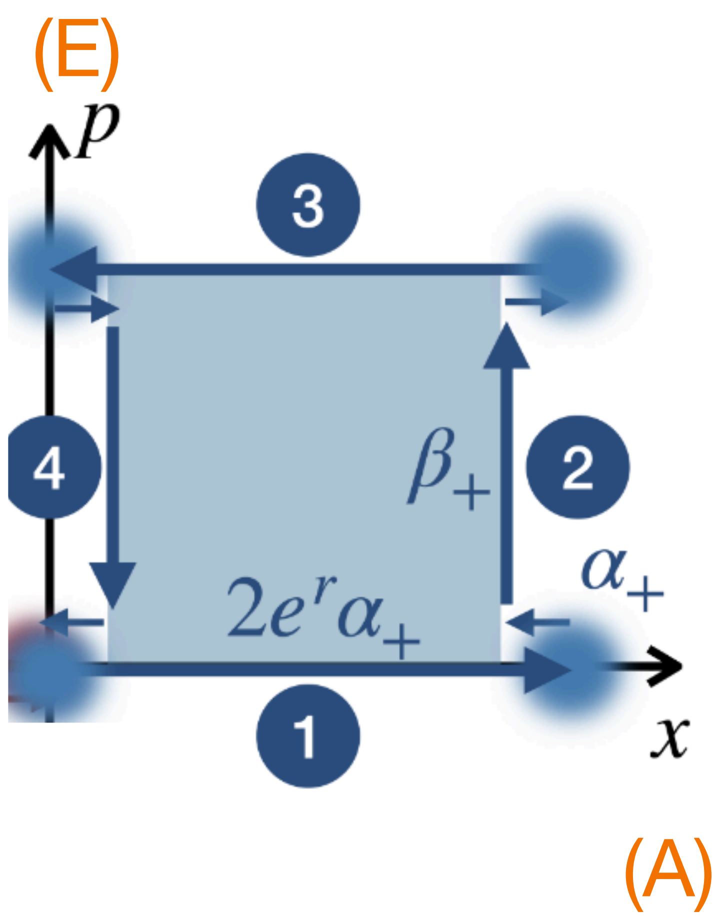


$$\hat{D}(\alpha_f) = \hat{S}^\dagger(\xi)\hat{D}(\alpha_i)\hat{S}(\xi), \qquad G = e^r$$

- Tiny displacement can be amplified by sandwiching it between squeezing and anti-squeezing operation. *Burd et. al. Science.aaw2884*
- For a displacement aligned with the squeezed quadrature, the amplification gain is exponential in the squeezing strength.
- The first key intuition for our protocol.

Applicable to any harmonic oscillator! (Faster reversible squeezing)

Geometric Readout for preserving the squeezing effect



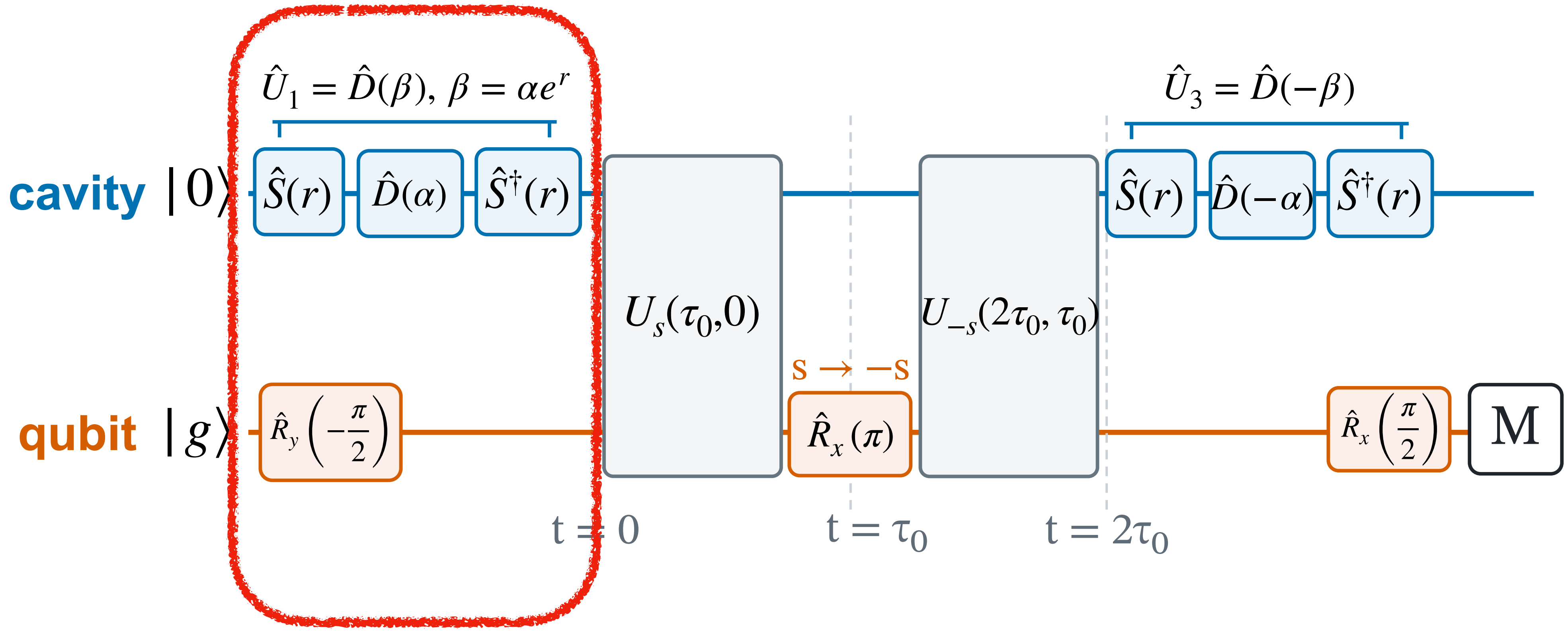
$$D(a)D(b) = e^{i\mathrm{Im}(a^*b)}D(a+b)$$
$$\neq D(b)D(a)$$

$$\mathrm{Area} = \mathrm{Im}\int_{\alpha(0)}^{\alpha(T)} \alpha^*\, d\alpha$$

- Subtlety: squeezing enlarges bosonic displacements, but the SNR not necessarily gain. *Suri, 2511.21983*
- Steer the oscillator phase trajectory around a loop: Signal->geometric phase.
- Conceptual bridge: “bigger displacement” to “bigger phase”.
- The second key intuition for our protocol

Geometric Protocol Designed for DM sensing

Preparation and squeezed displacement

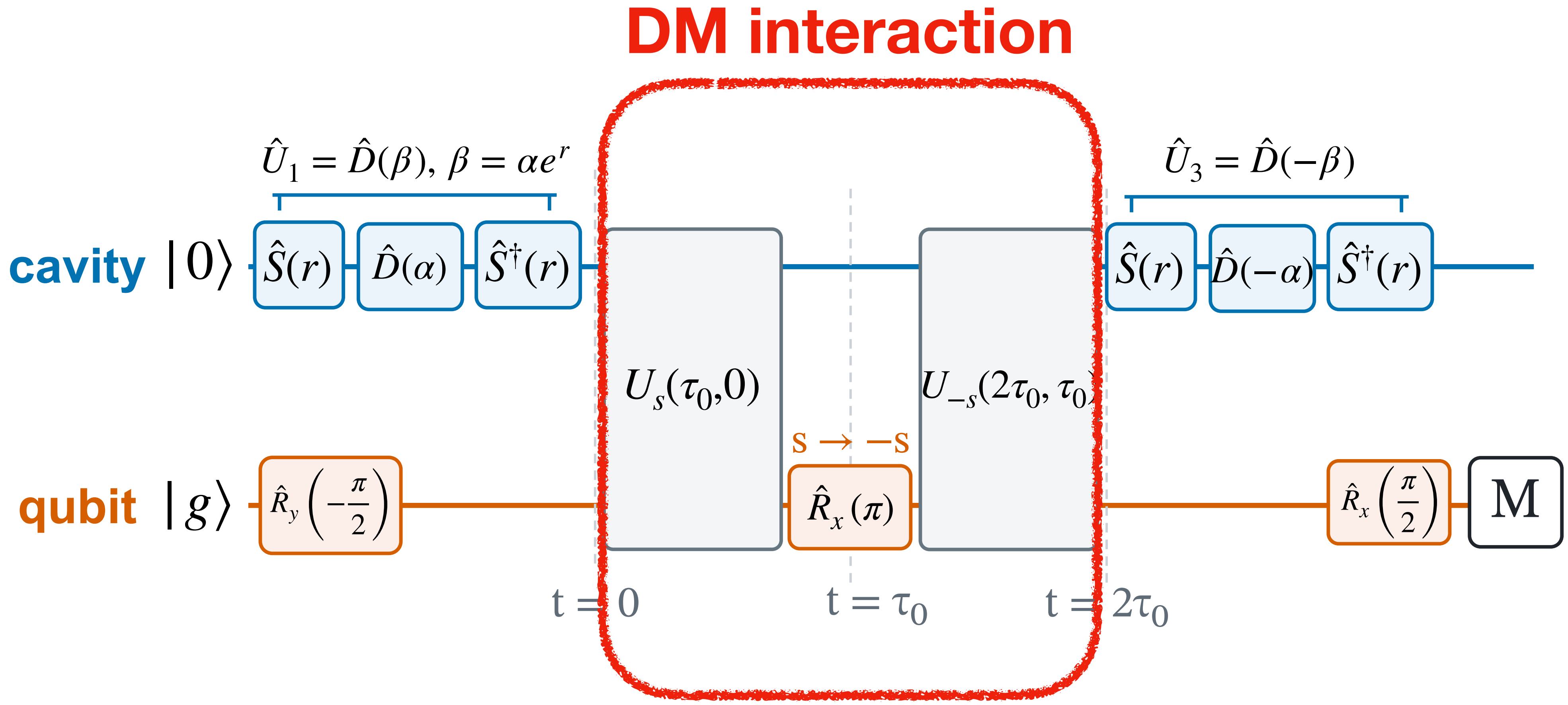


- Block 1: Prepare superposition of qubit, apply a large squeezed displacement on cavity, $U_1 = \hat{S}^\dagger(r)\hat{D}(\alpha)\hat{S}(r) = \hat{D}(\beta),\ \beta = \alpha e^r$.

Start from $1/\sqrt{2}(|e\rangle + |g\rangle) \otimes |0\rangle$

Qubit (Relative phase!)   Cavity

Geometric Protocol Designed for DM sensing

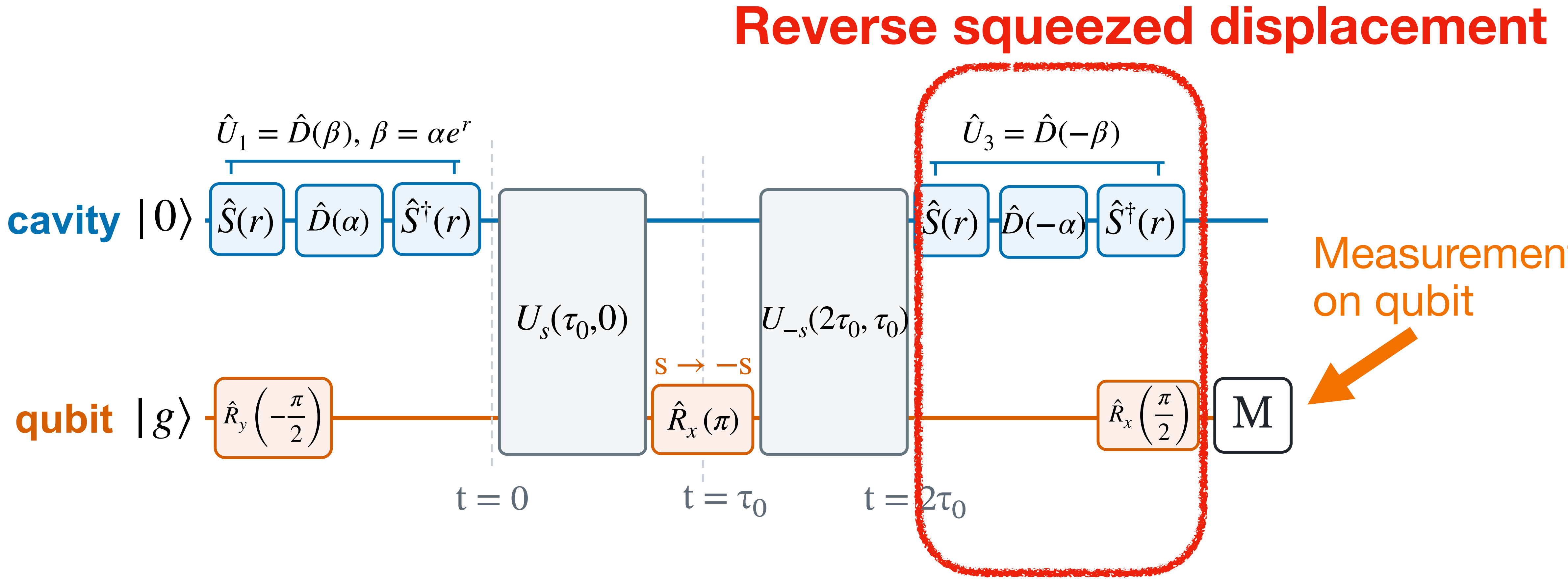


- Block 2: system evolve for $2\tau_0$ under the DM drive, with a qubit π -pulse at τ_0 to implement spin echo.

$2\tau_0 \lesssim T_2$

(necessary to cancel the spin dynamical phase)

Geometric Protocol Designed for DM sensing



- Block 3: apply the opposite squeezed displacement, $U_3 = \hat{D}^\dagger(\beta)$, to close the effective loop in phase space.

- $U_{\text{tot}} = U_3 U_2 U_1 = \hat{D}(\Sigma) e^{is\delta\Phi/2}$

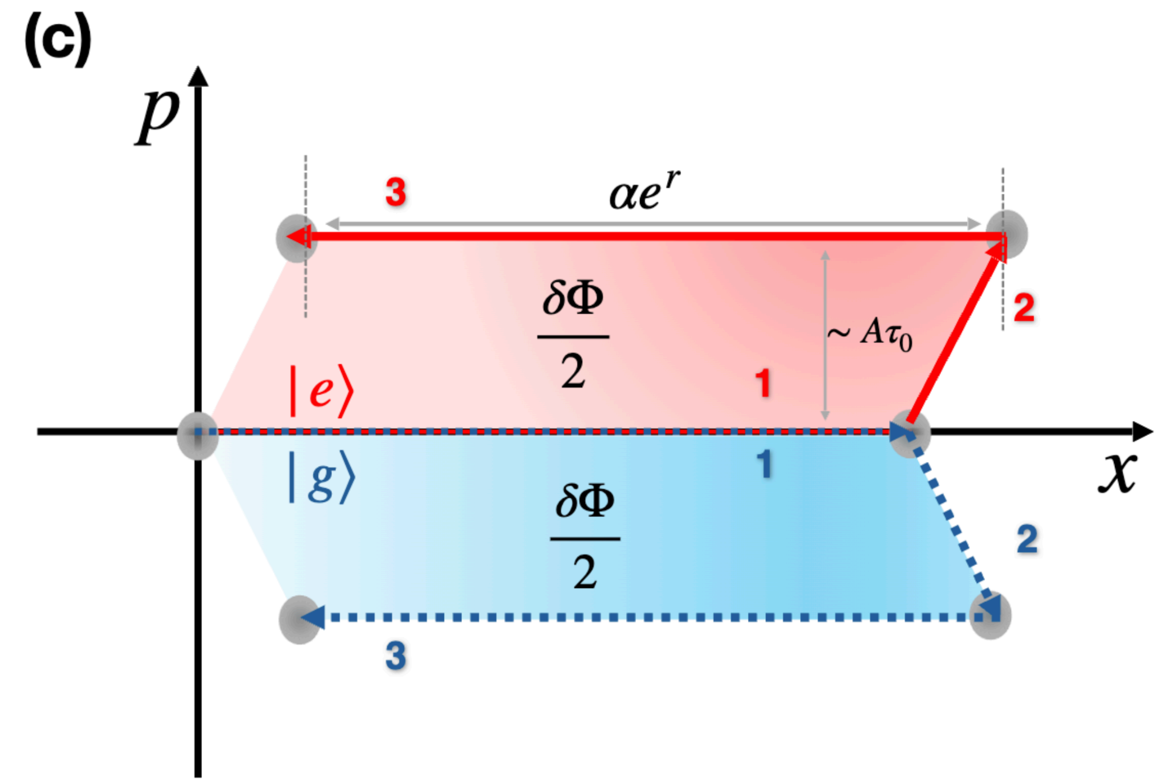
The Signal Becomes an Enhanced Geometric Phase of qubit

- Qubit picks up a large relative phase (Open-path geometric phase)
- The gain comes from noncommutativity of displacement operations and the squeezed pump amplitude $\beta = \alpha e^r$.
- $1/\sqrt{2}(|e\rangle + |g\rangle) \otimes |0\rangle$

$\rightarrow 1/\sqrt{2}(e^{i\delta\Phi/2}|e\rangle + e^{-i\delta\Phi/2}|g\rangle) \otimes |0\rangle$

+ small corrections

Cavity phase space with qubit spin branches (up/down)



$$\delta\Phi \propto A\beta\chi\tau_0^2 \mathrm{sinc}\left(\frac{(\Delta+\chi/2)\tau_0}{2}\right)\mathrm{sinc}\left(\frac{(\Delta-\chi/2)\tau_0}{2}\right)$$

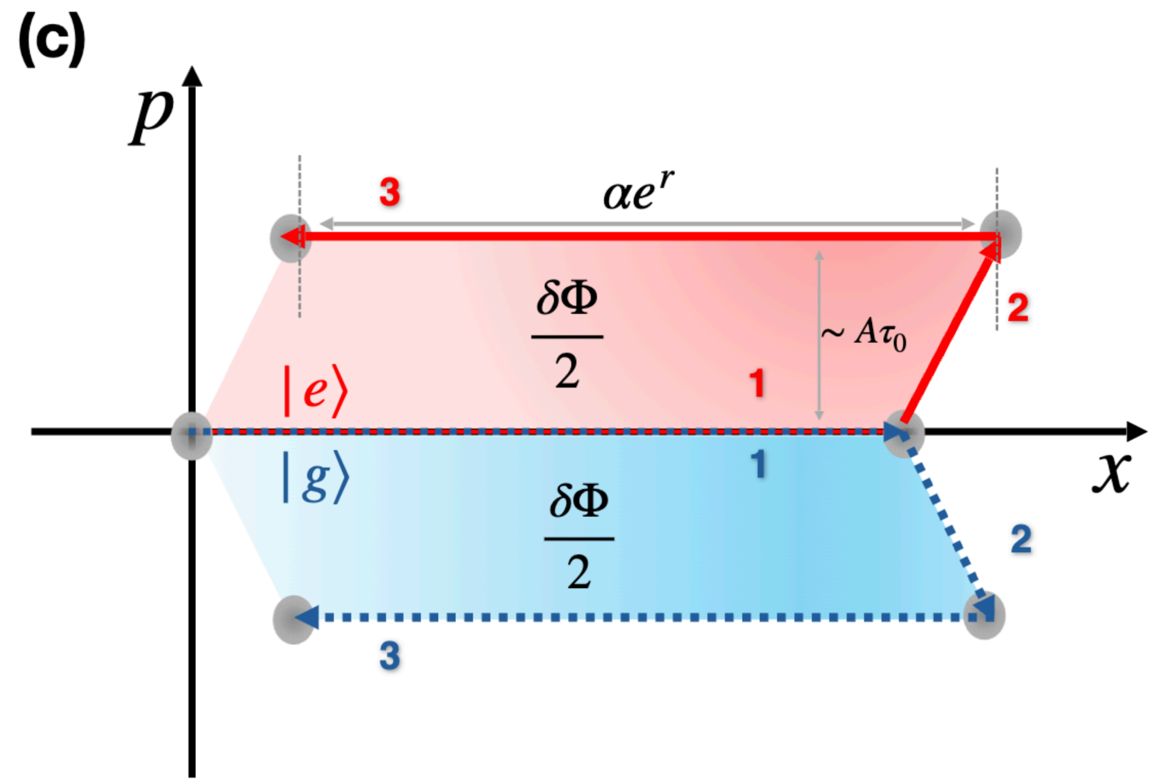
$\Delta \equiv \omega_C - \omega_D$

Detune

The Signal Becomes an Enhanced Geometric Phase

$$1/\sqrt{2}(e^{i\delta\Phi/2}|e\rangle + e^{-i\delta\Phi/2}|g\rangle) \otimes |0\rangle$$

- Ramsey readout turns this phase directly into an experimentally accessible qubit signal. (Expected non-vanishing variance)
- Noise effect to be discussed later.



$$\delta\Phi \propto A\beta\chi\tau_0^2 \mathrm{sinc}\left(\frac{(\Delta+\chi/2)\tau_0}{2}\right)\mathrm{sinc}\left(\frac{(\Delta-\chi/2)\tau_0}{2}\right)$$

$$\Delta \equiv \omega_C - \omega_D$$

Detune

QFI Enhancement and Spectral Profile

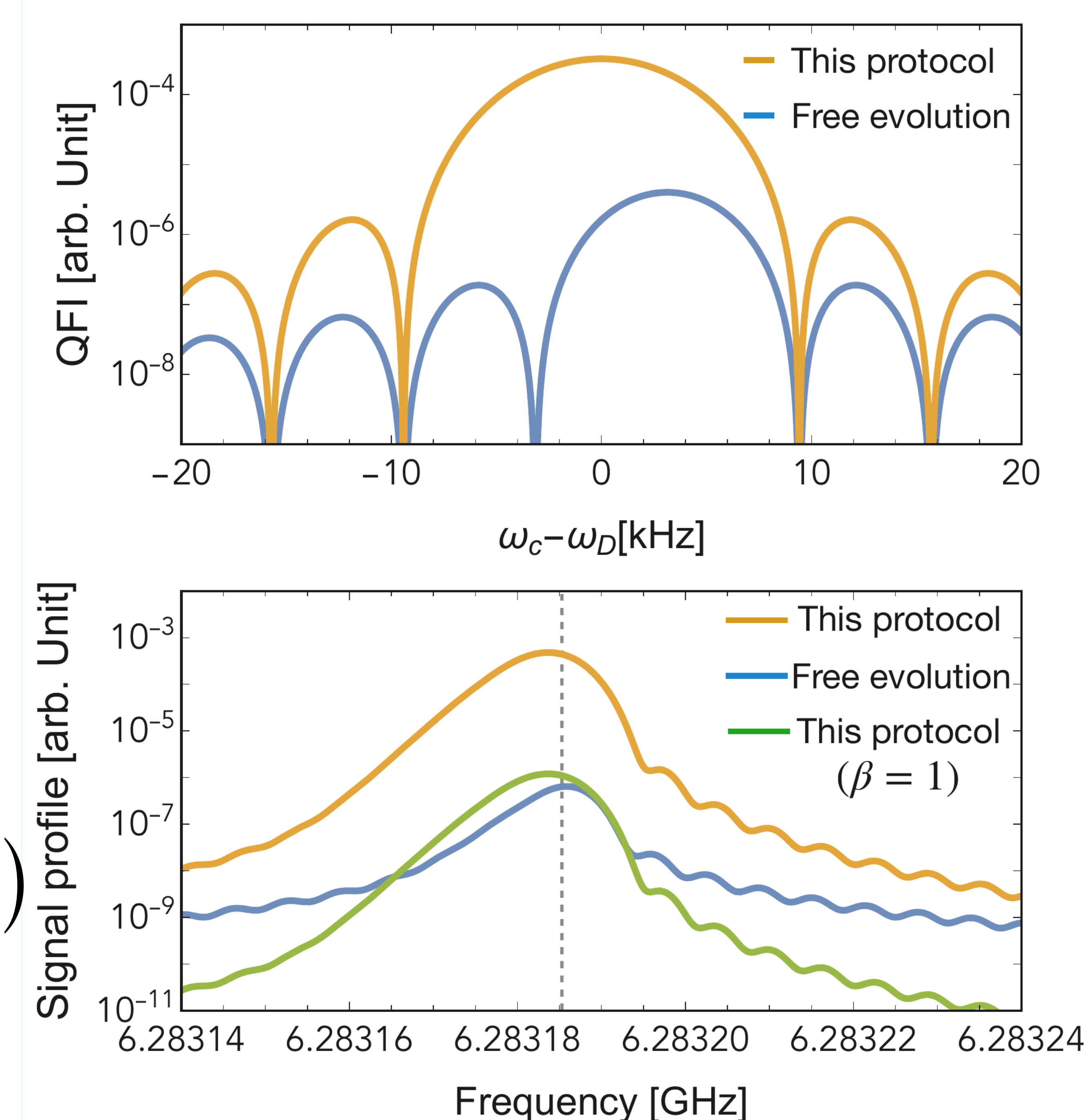
- Geometric protocol boosts the QFI by a factor set by
  $\beta^2 = (\alpha e^r)^2 \sim (3 \times 5.6)^2$

  Compatible with current experiments

- After convolution with the DM linewidth, the signal develops a characteristic double-sinc spectral profile with optimal choice $\underline{\chi\tau_0 = \pi}$.

- $\mathscr{F}_{Q,\mathrm{free}}(A) \simeq 16\tau_0^2\,\mathrm{sinc}^2\left(\Omega_{\mathrm{eff},s}\tau_0\right)$

  $\hat{U} = e^{-iA\hat{H}}$



$$\mathscr{F}_{Q,\mathrm{geo}}^{\mathrm{optimal}}(A) \approx \frac{1}{2}(\beta\chi\tau_0^2)^2 \mathrm{sinc}^2\left(\frac{(\Delta + \chi/2)\tau_0}{2}\right)\mathrm{sinc}^2\left(\frac{(\Delta - \chi/2)\tau_0}{2}\right)$$

New Scaling behavior from this protocol

$$P_{\rm DM} = \int d\omega f_{\rm DM}(\omega)\frac{1}{2}(\alpha e^r \chi \tau_0^2)^2 \text{sinc}^2\left(\frac{(\Delta+\chi/2)\tau_0}{2}\right)\text{sinc}^2\left(\frac{(\Delta-\chi/2)\tau_0}{2}\right).$$

GHz DM as an example

DM: random walk beyond coherent time

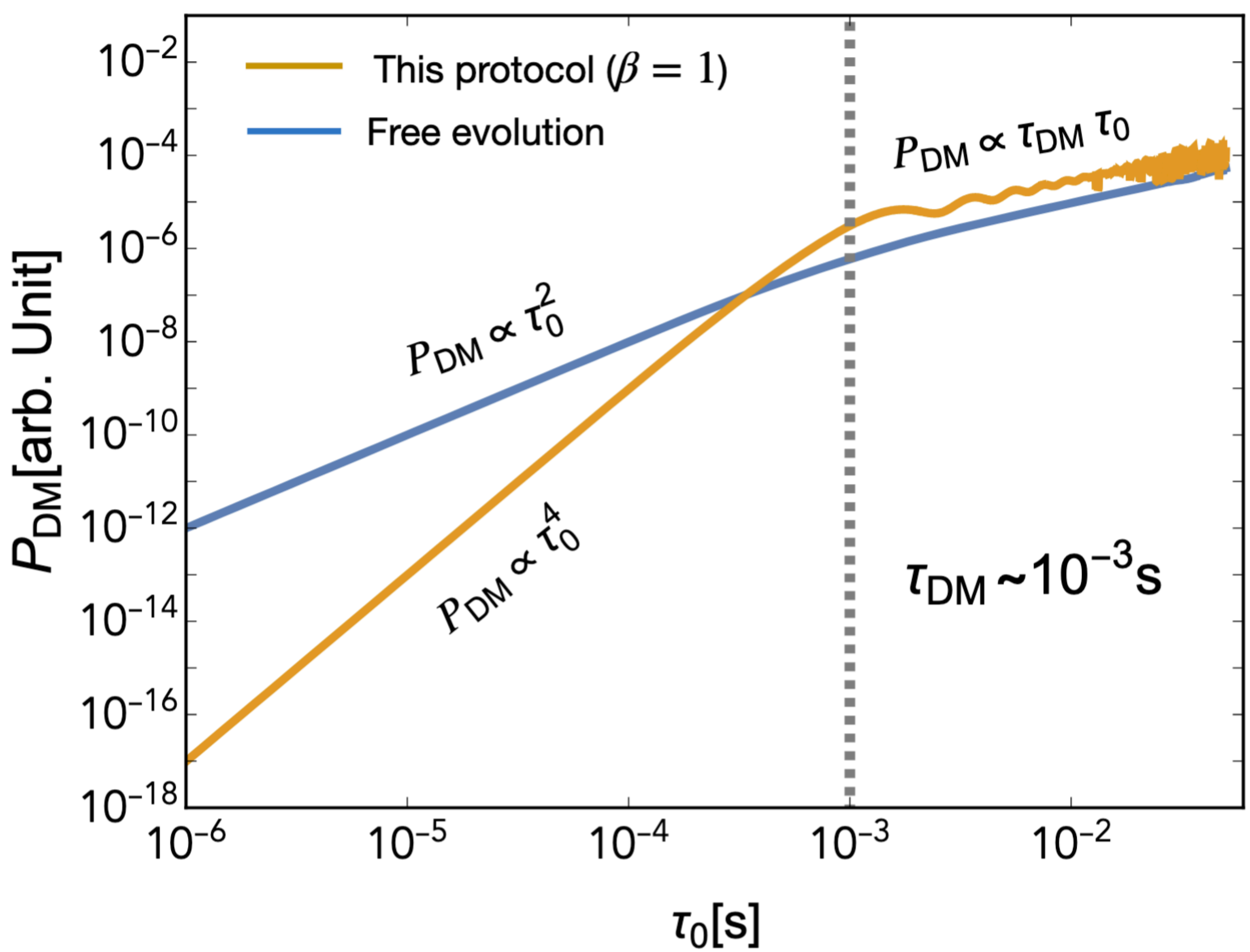
$$\frac{\sin^2(x\tau_0/2)}{(x/2)^2} \to 2\pi\tau_0\delta(x)$$

Standard:

$$P_{\rm DM} \propto \begin{cases} \tau_0^2, & \tau_0 < \tau_{\rm DM}, \\ \tau_{\rm DM}\tau_0, & \tau_0 \geq \tau_{\rm DM}. \end{cases}$$

New scaling behavior

$$P_{\rm DM} \propto \begin{cases} \chi^2\tau_0^4, & \tau_0 < \tau_{\rm DM}, \\ \tau_{\rm DM}\tau_0, & \tau_0 \geq \tau_{\rm DM}. \end{cases}$$

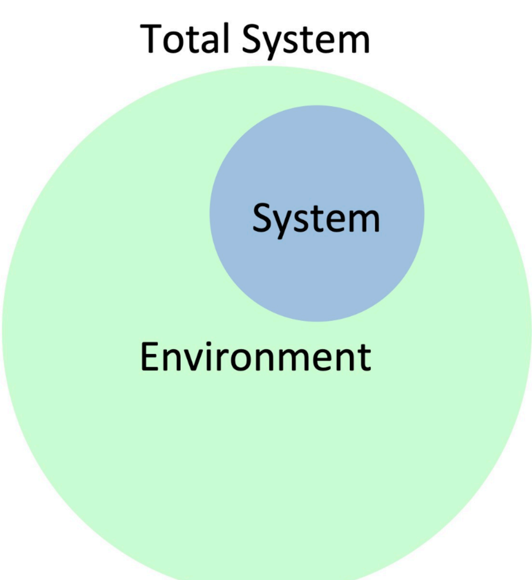


$$\tau_0 \sim \tau_{\rm DM} \sim \text{ms}$$

No free lunch: Faster decoherence and the Optimal Operating Point

Qubit dephasing

$$\dot{\rho}(t) = -\,i\left[\frac{\chi}{2}\hat{a}^{\dagger}\hat{a}\hat{\sigma}_z, \rho(t)\right] + \kappa(\bar{n}_{\mathrm{th}} + 1)\mathscr{D}[\hat{a}]\rho + \kappa\bar{n}_{\mathrm{th}}\mathscr{D}[\hat{a}^{\dagger}]\rho + \Gamma_1\mathscr{D}[\sigma_{-}]\rho + \frac{\Gamma_\phi}{2}\mathscr{D}[\sigma_z]\rho,$$



Cavity photon loss

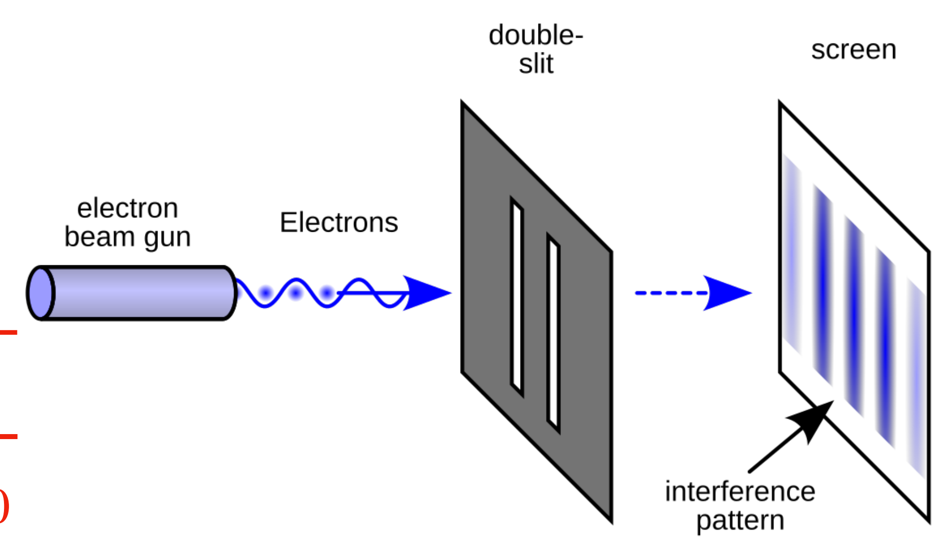
Spontaneous qubit flip

Cavity photon excitation

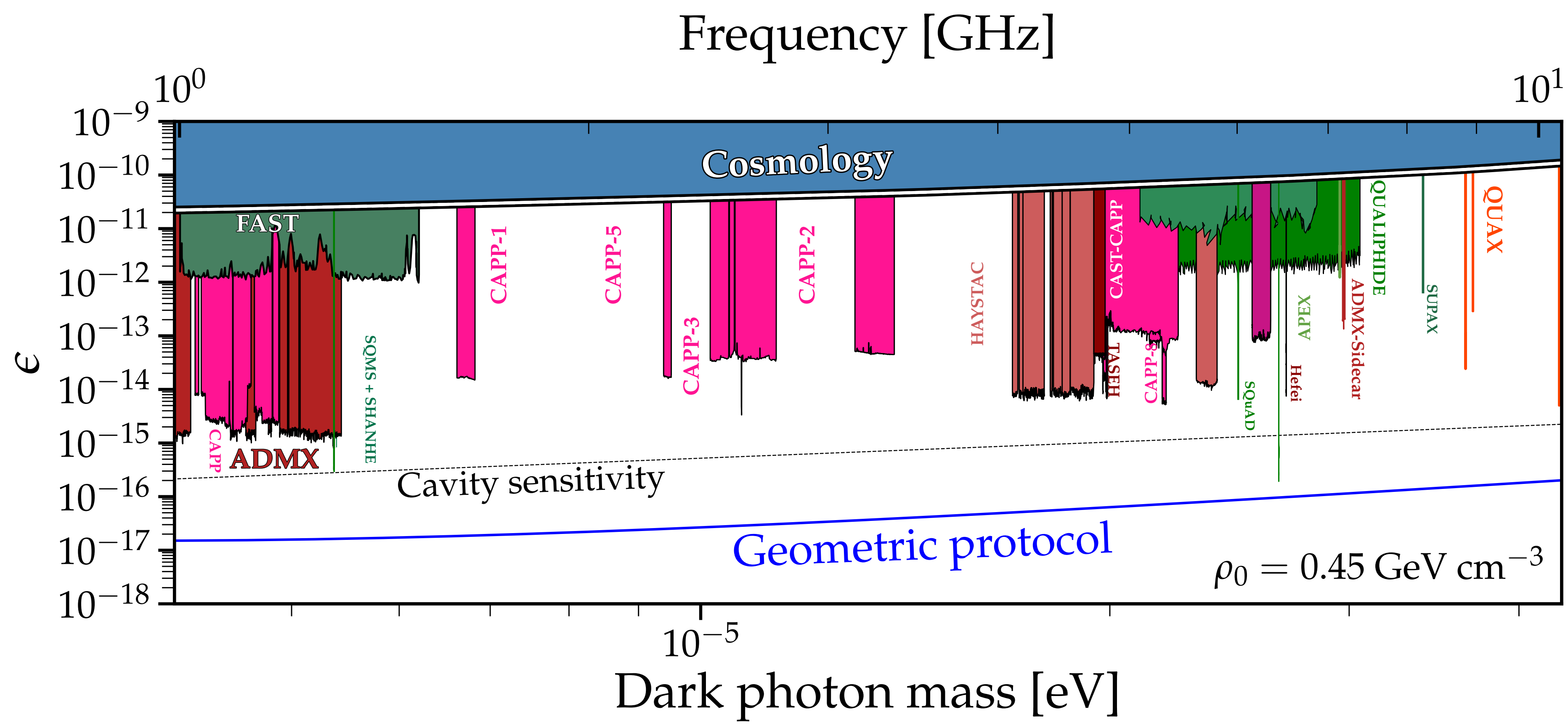
Lindblad equation for open quantum systems

$$\dot{\rho}(t) = -i\left[\frac{\chi}{2}\hat{a}^{\dagger}\hat{a}\hat{\sigma}_z, \rho(t)\right] + \kappa(\bar{n}_{\text{th}}+1)\mathscr{D}[\hat{a}]\rho + \kappa\bar{n}_{\text{th}}\mathscr{D}[\hat{a}^{\dagger}]\rho + \Gamma_1\mathscr{D}[\sigma_-]\rho + \frac{\Gamma_\phi}{2}\mathscr{D}[\sigma_z]\rho,$$

- The same enlarged phase-space separation that amplifies the signal also makes the protocol more sensitive to cavity loss and which-path dephasing.
- The signal therefore carries a competition between geometric gain and decoherence suppression.
- This tradeoff defines an optimal β for a given cavity lifetime and protocol time.
- $\eta = \exp\left[-2\tau_0/T_{2,\text{echo}}^{(0)} - \Lambda_\kappa\right],$
- $\Lambda_\kappa = \kappa(1+2\bar{n}_{\text{th}})|\beta|^2\tau_0\left(1-\frac{\sin(\chi\tau_0)}{\chi\tau_0}\right),\quad \beta_{\text{opt}} = \sqrt{\frac{T_1^c}{2\tau_0}}$



Projected Dark-Photon Reach



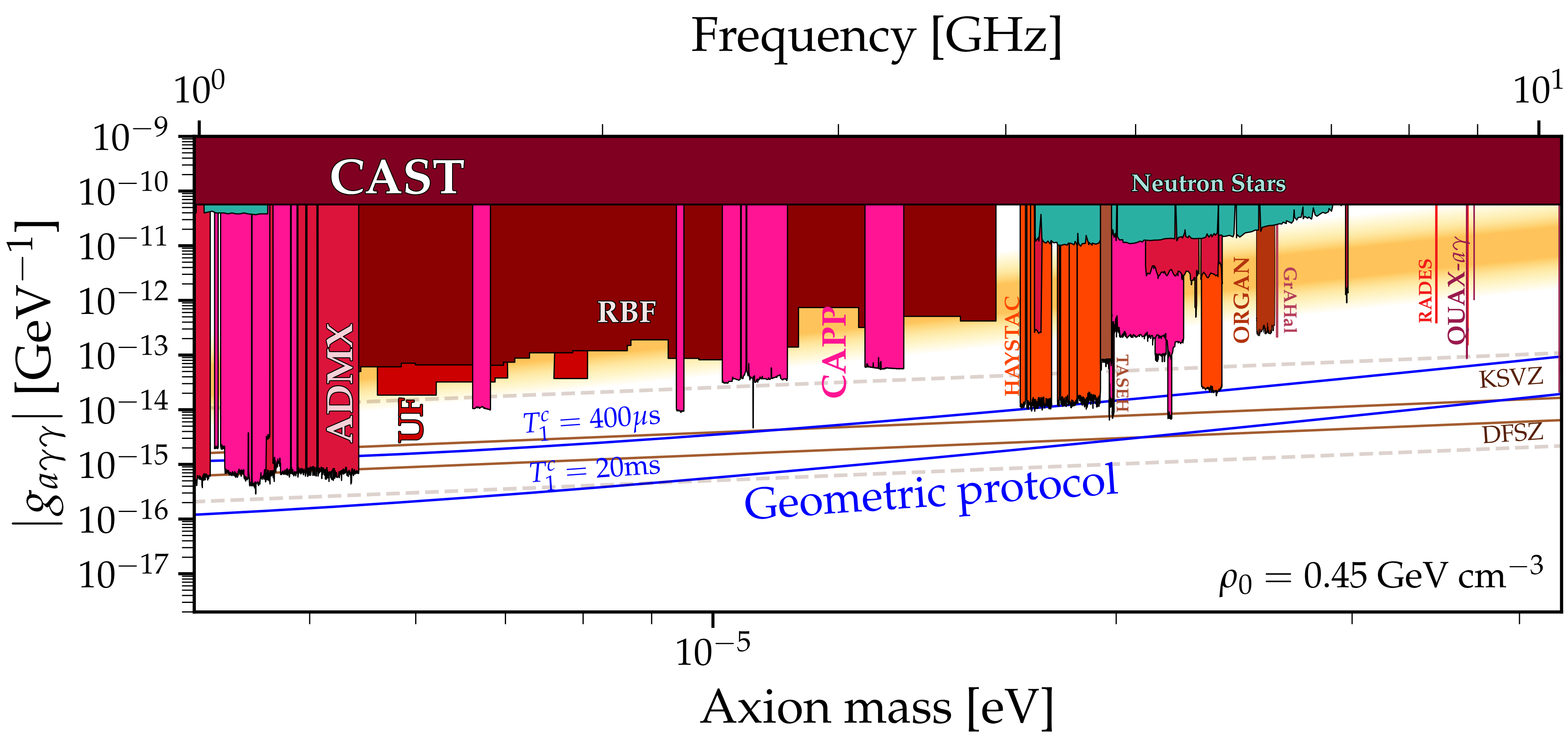
- Repeated Ramsey interferometry of the qubit readout, sensitivity derived from Asimov data analysis, $\mathcal{S}_k \propto 8\eta^2(\beta\tau_0)^2|A|^2/\pi^2$

Consider only spin projection noise

- For benchmark parameters $T_1^c \sim 300\text{ms},\ T_{2,\text{echo}}^{(0)} \sim 1\ \text{ms},\ t_{\text{obs}} = 60s$ per frequency bin, the reach improves substantially over cavity benchmarks.

- The projected improvement is up to one order of magnitude.

Projected Axion Reach



- For axions, the same framework applies once the cavity is driven through the axion-photon coupling in a strong magnetic field.
- The quality factor decreases a lot in strong magnetic field,

Strong magnetic environment caveat; taken with a grain of salt.

- Outlook: extend to multi-cavity architectures, more general control sequences, and other qubit-boson sensing platforms.

Summary

- μeV dark matter is well motivated, with cavity being the one of the most powerful detector in this region.
- Using squeezed displacement as the quantum sensing resource we propose a possible detection protocol utilizing the ultrahigh Q factor of modern cavities.
- With squeezing enhanced operations, the new protocol could have a potential to improve the sensitivity of the DM, but the experiment realization is highly challenging.

Thanks !

Backup

- Open geometric phase

$$\gamma_{\text{geo}}^{\text{open}} = \arg\langle \psi(0) \,|\, \psi(T) \rangle + i \int_0^T \langle \psi(t) \,|\, \dot{\psi}(t) \rangle dt \,.$$

For coherent state:

$$\gamma_{\text{geo}}^{\text{open}} = \mathrm{Im}\left[\alpha^*(0)\alpha(T)\right] - \mathrm{Im}\int_{\alpha(0)}^{\alpha(T)} \alpha^* \, d\alpha$$

Area