Black-Hole Ringdown Analysis with LIGO-Virgo-KAGRA O4 data

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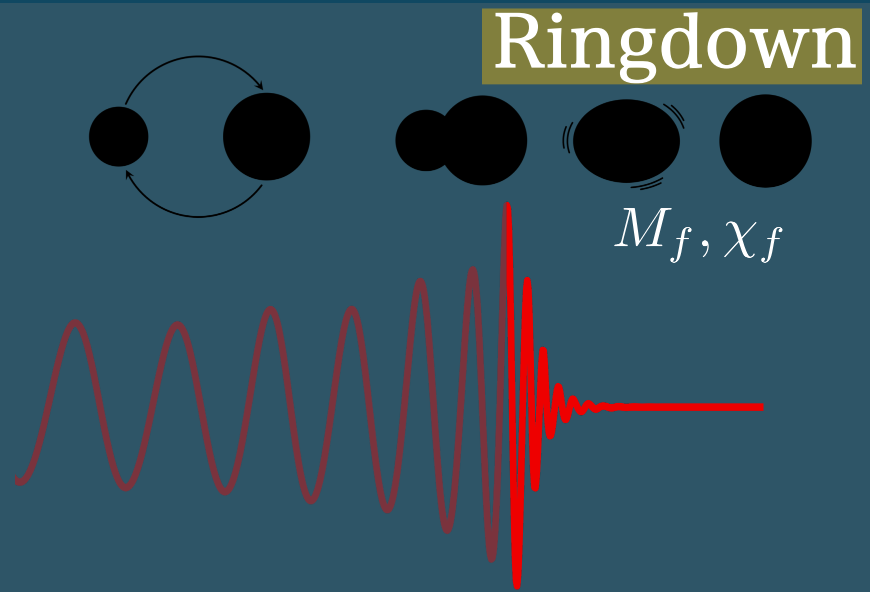
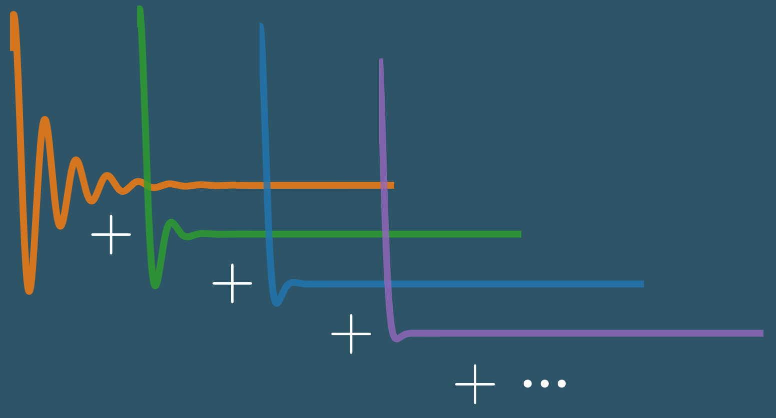
Introduction to Ringdown



$$h(t) = h_+(t) + ih_\times(t) = \sum_{\ell mn} \left[\mathcal{A}_{\ell mn} e^{-t/\tau_{\ell mn}} e^{-i2\pi f_{\ell mn} t} \quad \mathcal{A}'_{\ell mn} e^{-t/\tau_{\ell mn}} e^{i2\pi f_{\ell mn} t} \right]$$

Quasi-Normal Modes (QNMs)

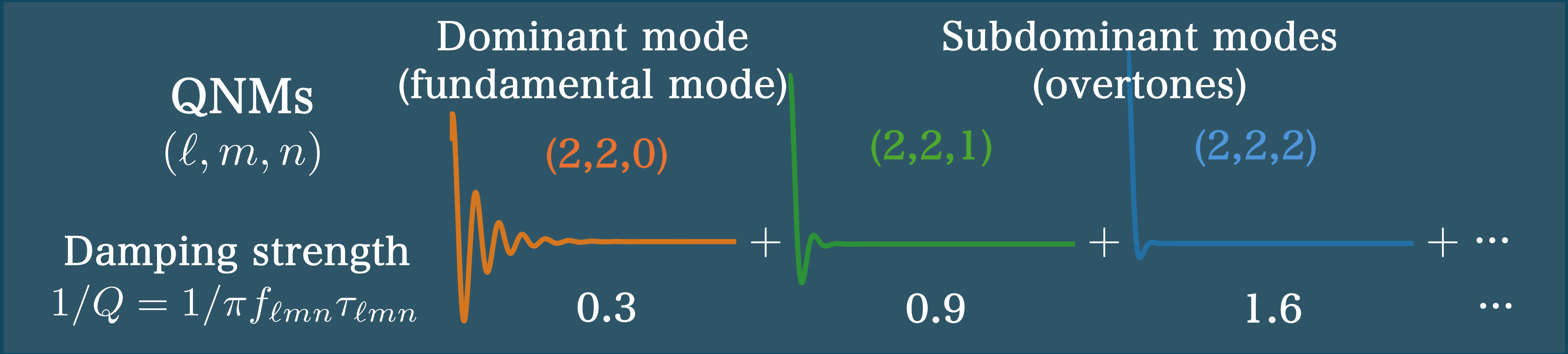


ℓ, m : Angular indices $\quad n$: Overtone number

- Ringdown signal can be modeled as a superposition of damped sinusoids.
- Frequency $f_{\ell mn}$ and damping time $\tau_{\ell mn}$ are determined solely by the remnant BH's mass M_f and spin χ_f (BH no-hair theorem).

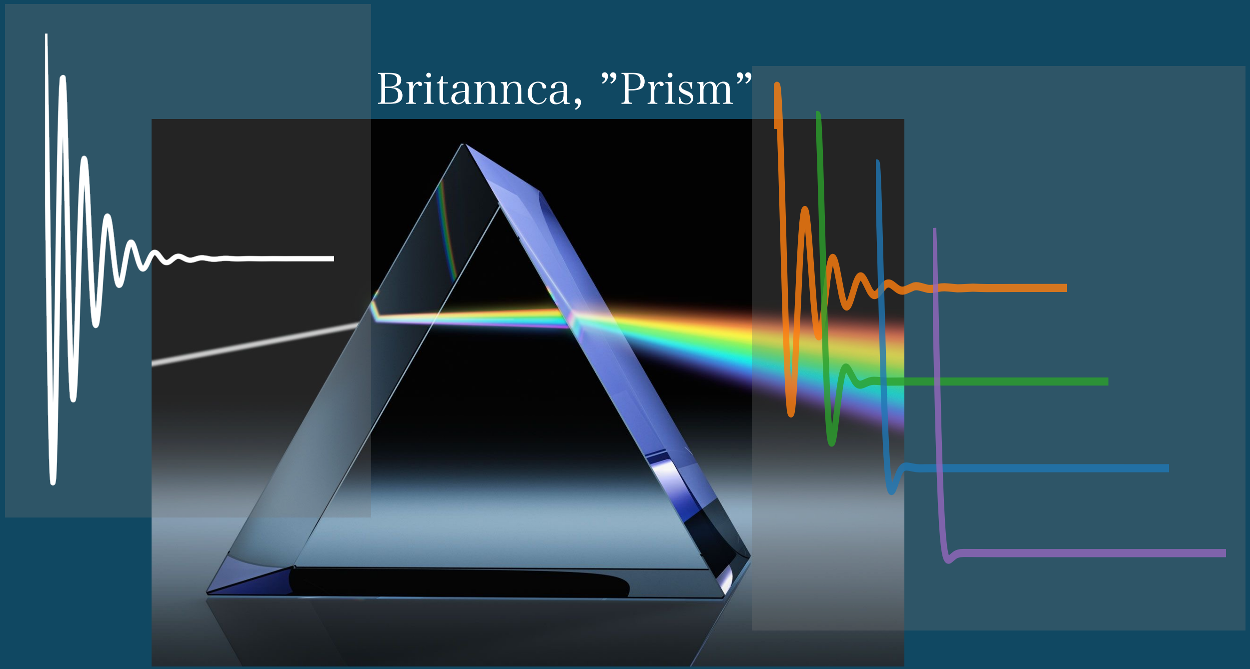
Challenges in Ringdown

- Typically, the (2,2,0) mode is the dominant one, and in recent years, overtones $(n \geq 1)$ have gained attention as subdominant modes.
  (M. Isi et al. 1905.00869, LVK Collaboration 2509.08099, and many others.)
- Overtones decay rapidly.



- It is unclear when the perturbation theory becomes valid.

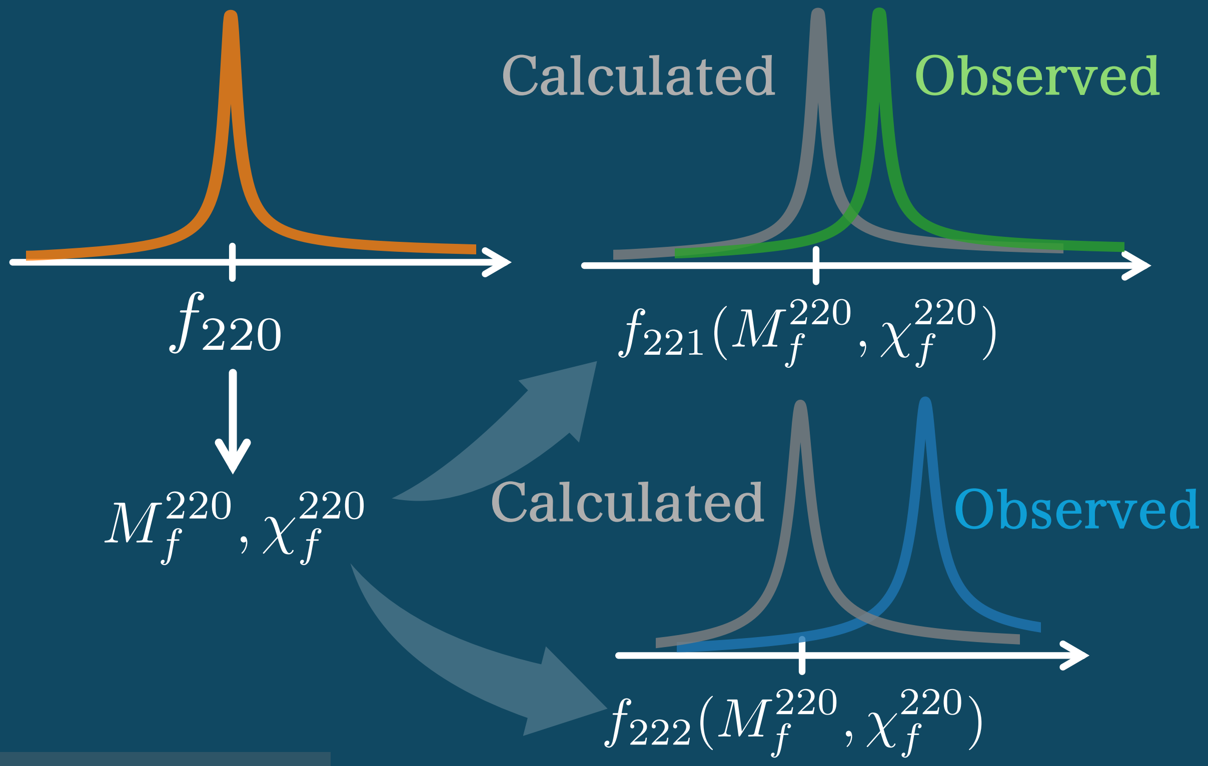
Test of General Relativity in Ringdown



Black-hole spectroscopy

Observed spectrum of the dominant mode

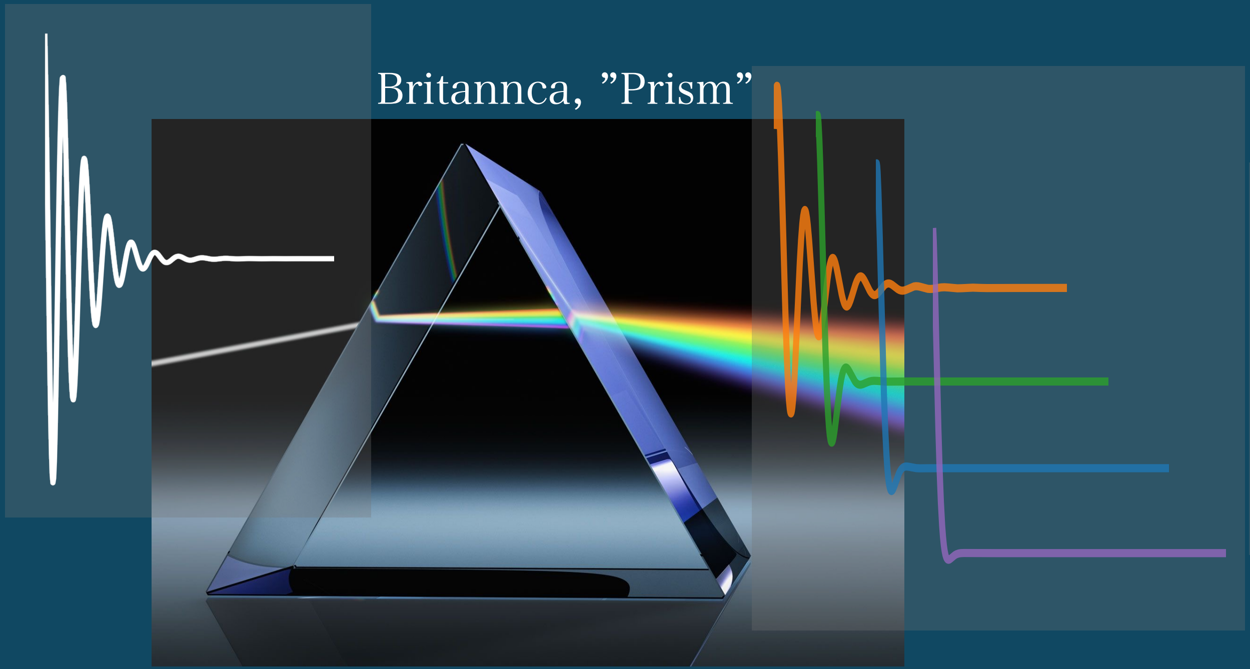
Spectrums of the subdominant modes



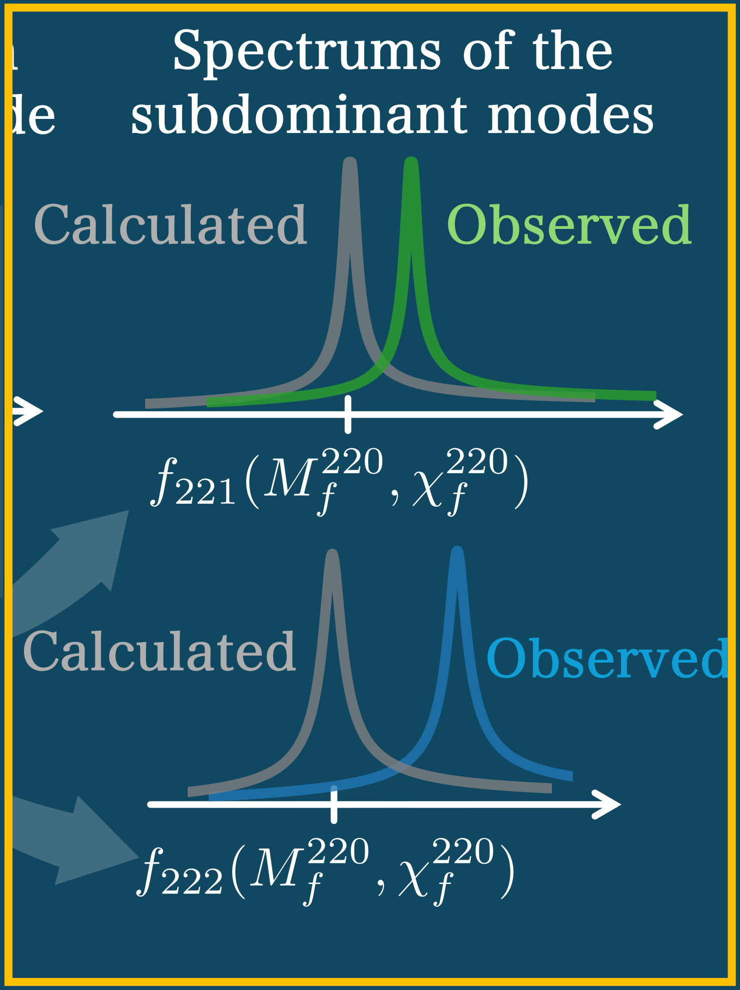
1. Estimate M_f and χ_f from the observed spectrum of the dominant mode.
2. Compare the calculated spectrums and observed spectrums of the subdominant modes.

→ **Test of General Relativity**

Test of General Relativity in Ringdown



The more QNMs can be observed, the more accurately we can test general relativity.



Black-hole spectroscopy

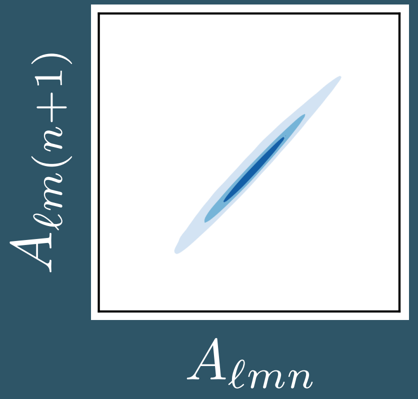
1. Estimate M_f and χ_f from the observed spectrum of the dominant mode.
2. Compare the calculated spectrums and obs-erved spectrums of the subdominant modes.

→ **Test of General Relativity**

Challenges in Ringdown Data Analysis

Challenges in ringdown analysis with multiple QNMs

1. QNMs are not orthogonal (they have similar oscillatory behaviors).



They cause **correlations between mode amplitudes $A_{\ell mn}$**.

2. Analysis with multiple modes increases the number of parameters.

Template waveform
$h(t; M_f, \chi_f, \mathcal{A}_{220}, \mathcal{A}'_{220}, \mathcal{A}_{221}, \mathcal{A}'_{221}, \ldots)$

It results in **a higher computational cost**.

↓

Semi-analytic method based on orthonormal QNMs

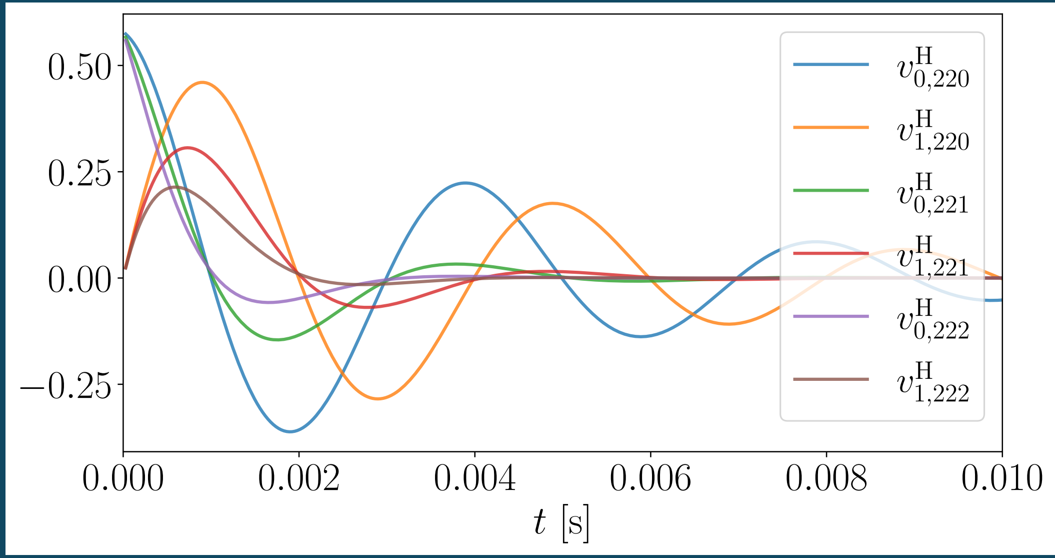
Semi-analytic Method

1. QNMs are not orthogonal (they have similar oscillatory behaviors).

⟶ **We use the numerically orthonormalized QNM templates.**

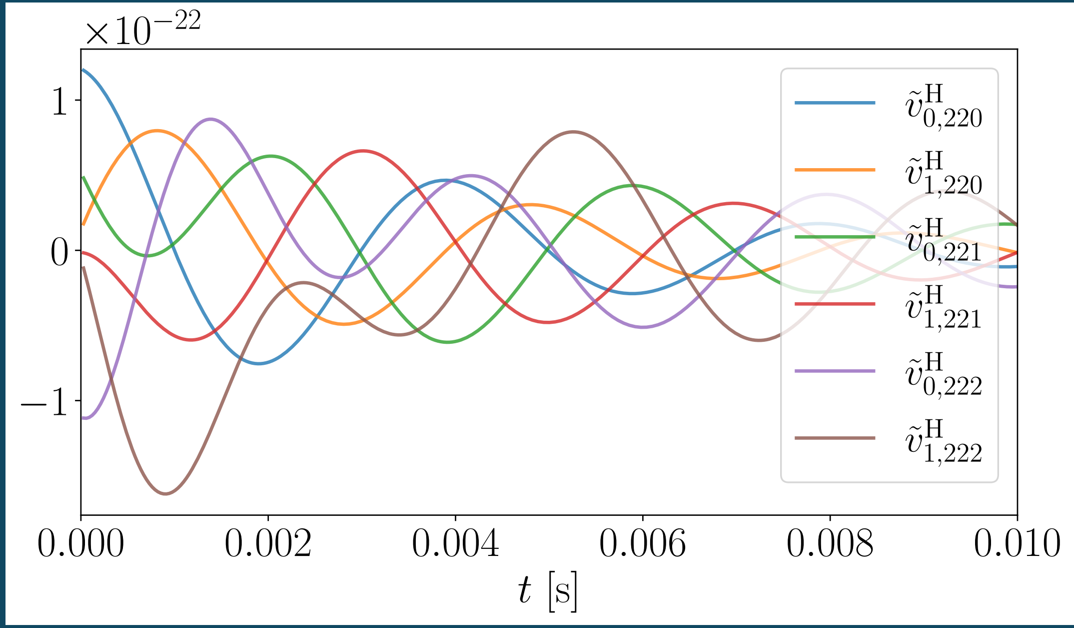
Template waveform

$$\boldsymbol{h}(t) = \sum_{\ell mn} \sum_{j=0}^{D-1} c_{j,\ell mn} \boldsymbol{v}_{j,\ell mn}$$



Orthonormalized template waveform

$$\boldsymbol{h}(t) = \sum_{\ell mn} \sum_{j=0}^{D-1} \tilde{c}_{j,\ell mn} \tilde{\boldsymbol{v}}_{j,\ell mn}$$



Correlations between mode amplitudes are reduced.

Semi-analytic Method

2. Analysis with multiple modes increases the number of parameters.

Orthonormalized template waveform

$$\boldsymbol{h}(t) = \sum_{\ell mn} \sum_{j=0}^{D-1} \tilde{c}_{j,\ell mn} \tilde{\boldsymbol{v}}_{j,\ell mn}$$

➢ Likelihood can be analytically marginalized over coefficients $\tilde{c}_{j,\ell mn}$:

$$\underbrace{P(M_f, \chi_f)}_{\text{Posterior}} \propto \int d\tilde{\boldsymbol{c}}\, \underbrace{\pi(M_f, \chi_f, \tilde{\boldsymbol{c}})}_{\text{Prior}} \underbrace{\mathcal{L}(M_f, \chi_f, \tilde{\boldsymbol{c}})}_{\text{Likelihood}}$$

Analytically integrate w.r.t. $\tilde{c}_{j,\ell mn}$

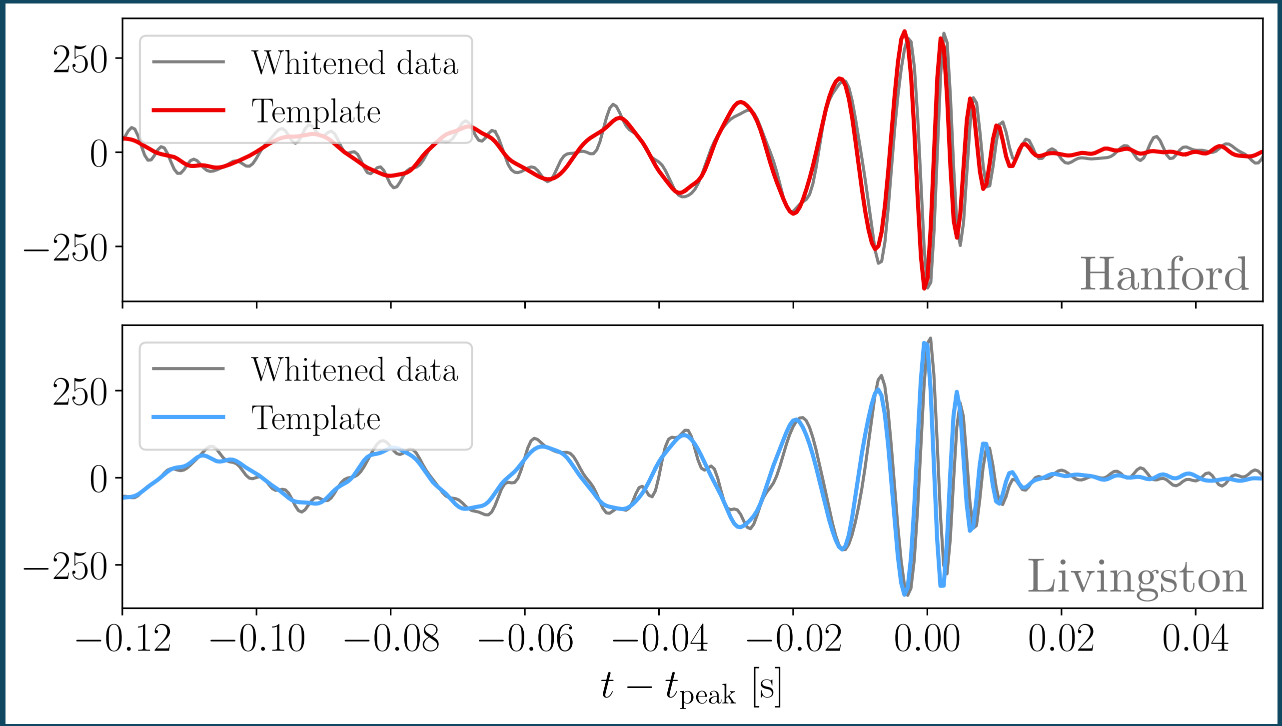
$$P(M_f, \chi_f) \propto \prod_{\ell mn} \frac{\pi}{4\sqrt{2}} \Gamma\left(\frac{D-1}{2}\right) \left[2D_1 F_1^{\mathrm{R}}\left(\frac{1}{2}, \frac{D}{2}+1, \frac{\tilde{d}_{\ell mn}^2}{2}\right) + \tilde{d}_{\ell mn}^2\, {}_1F_1^{\mathrm{R}}\left(\frac{3}{2}, \frac{D}{2}+2, \frac{\tilde{d}_{\ell mn}^2}{2}\right)\right]$$

S. Morisaki, H. Motohashi, M. Suzuki, and D. Watarai, arXiv:2507.12376

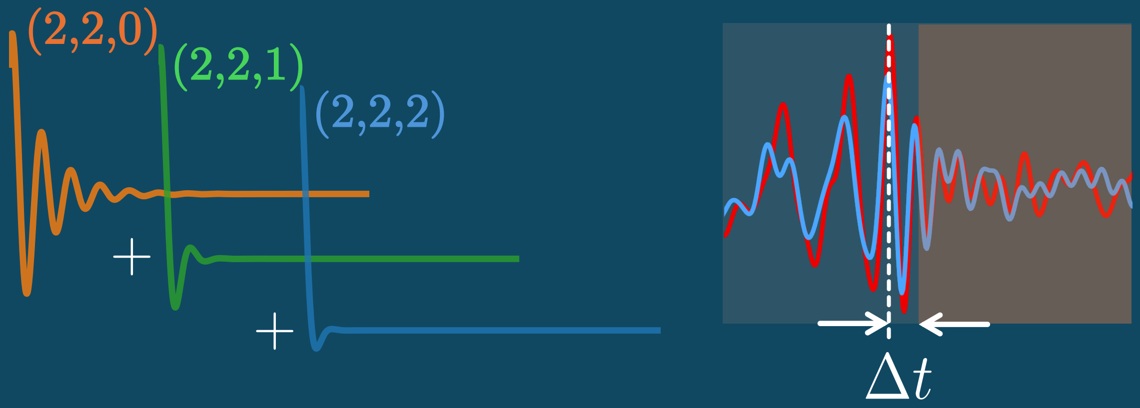
⟶ We do not need to sample parameters from likelihood $\mathcal{L}(M_f, \chi_f, \tilde{\boldsymbol{c}})$ using random sampling methods.

GW250114

- Detected on January 14, 2025, during the 4th observing run (O4) of the LIGO-Virgo-KAGRA collaboration.
- The loudest event so far (SNR∼80).



- Analyze using a template waveform including $(\ell, m, n) = (2,2,0), (2,2,1)$, and $(2,2,2)$ modes.
- Perform analyses on data segments starting at different times Δt.

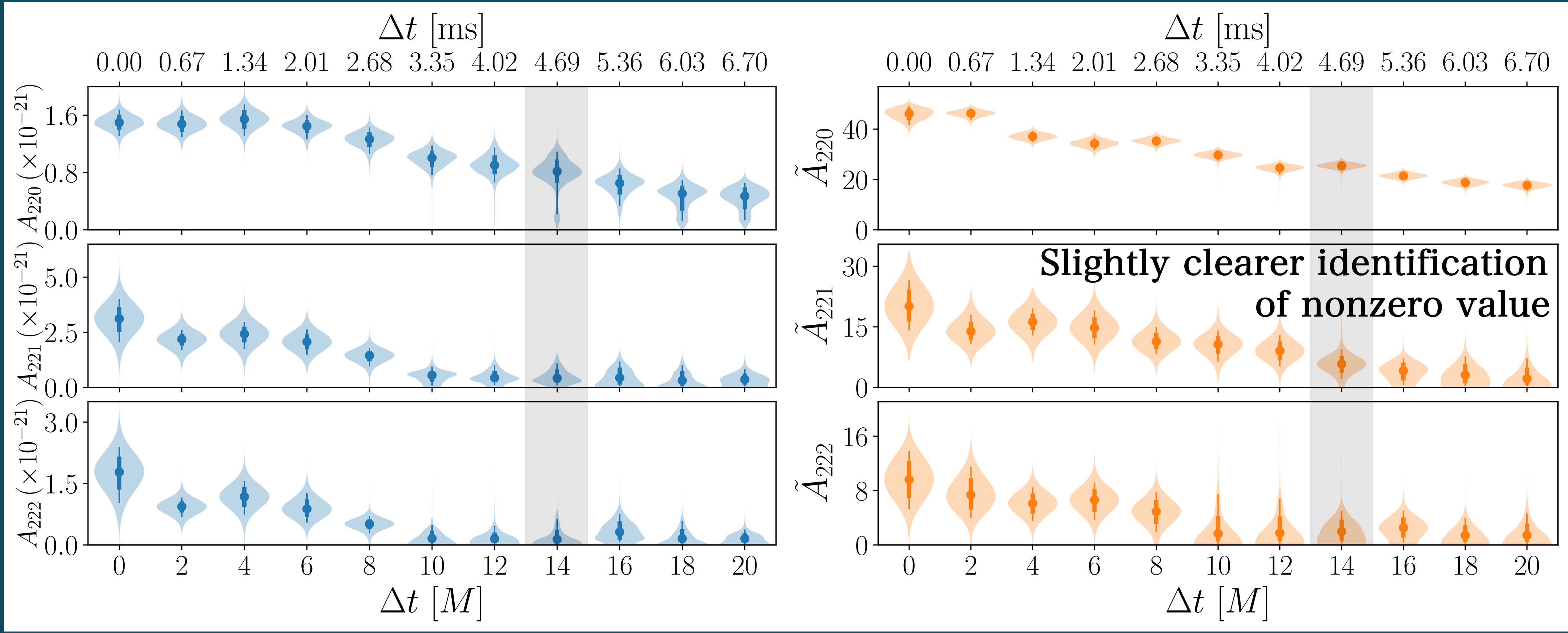


Application to the O4 event GW250114

Posterior distributions for different data segments

Random sampling method (original QNMs)

Semi-analytic method (orthonormal QNMs)

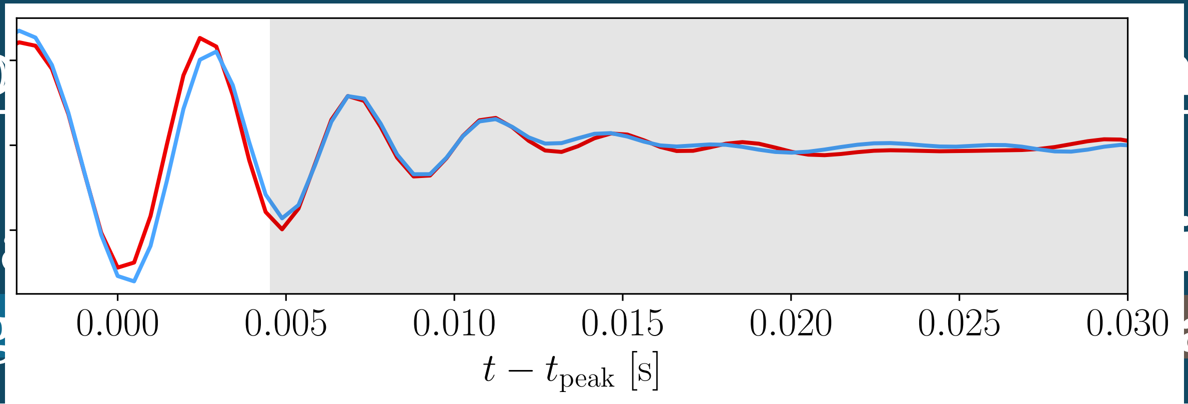


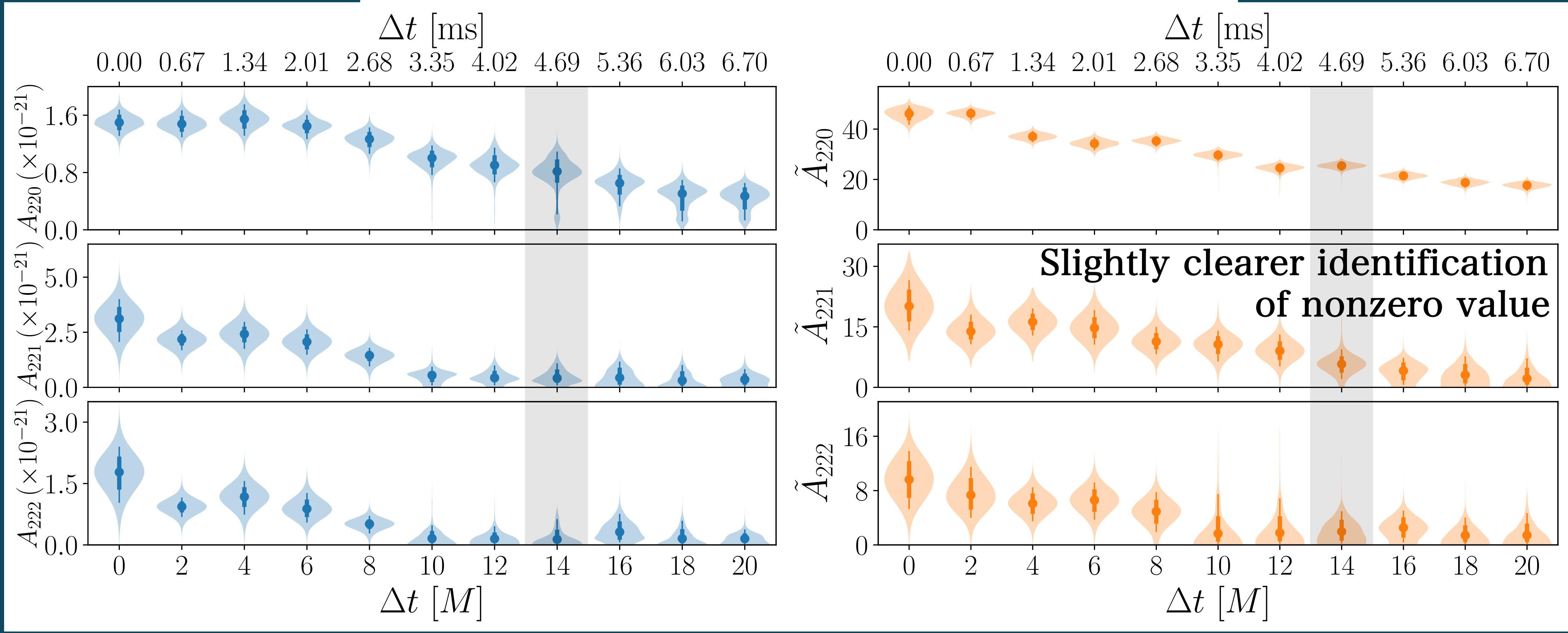
Application to the O4 event GW250114

Posterior distrib… …nents

Random s… …ic method

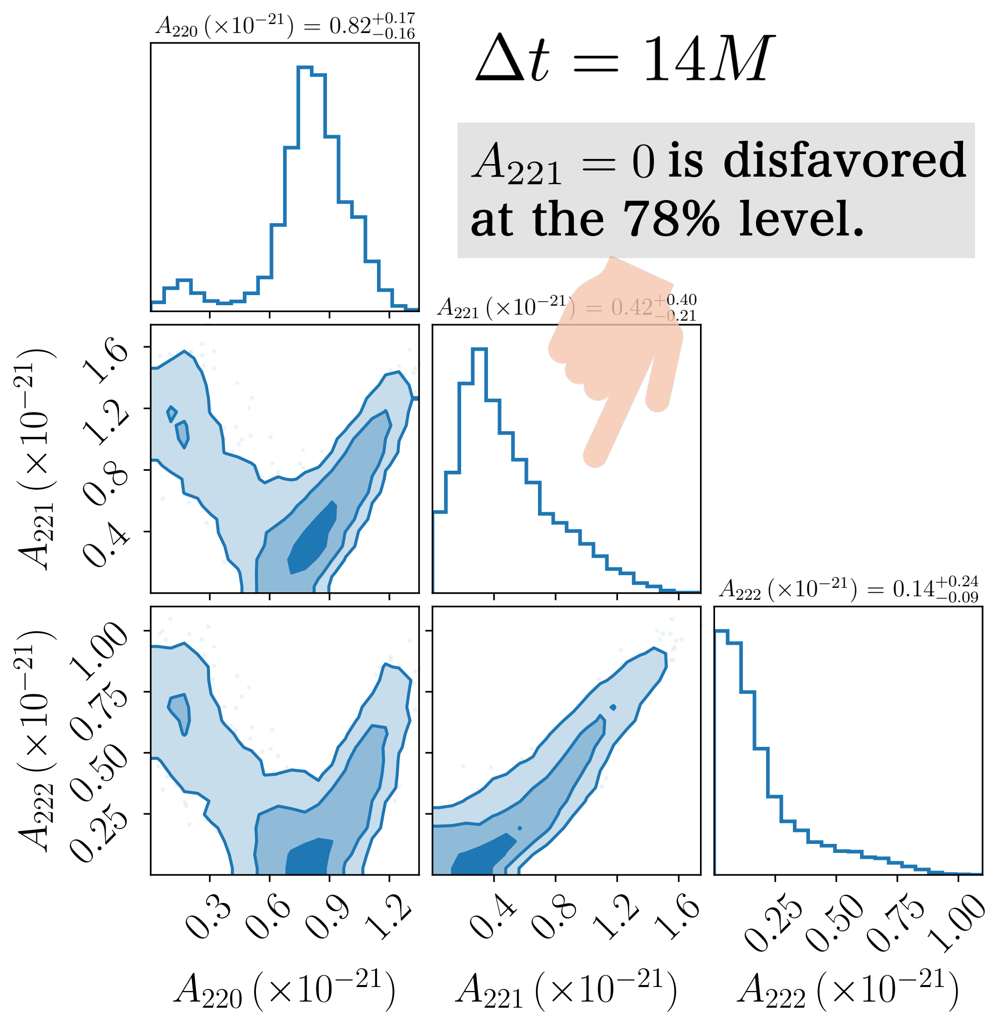
(orig… …al QNMs)



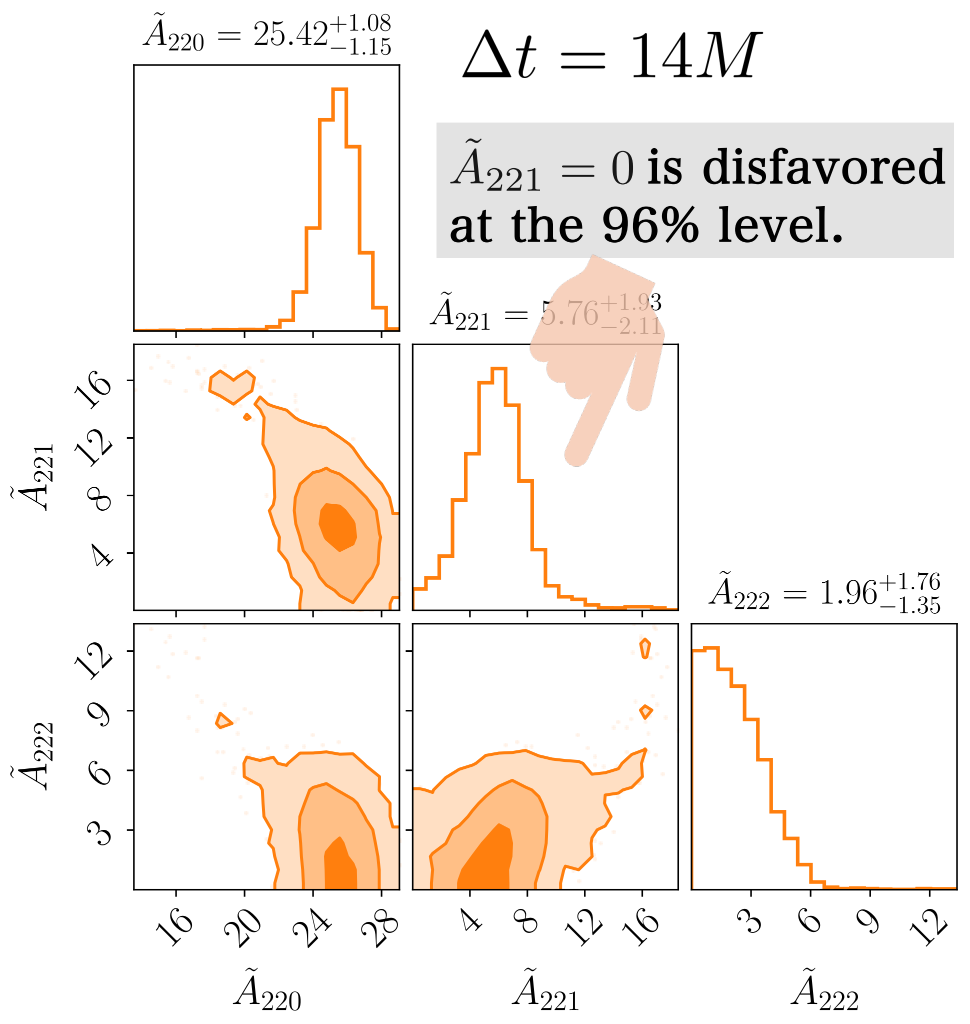


Application to the O4 event GW250114

Random sampling method (original QNMs)



Semi-analytic method (orthonormal QNMs)



- Semi-analytic method reduces correlations (e.g., (2,2,1) vs (2,2,2)).

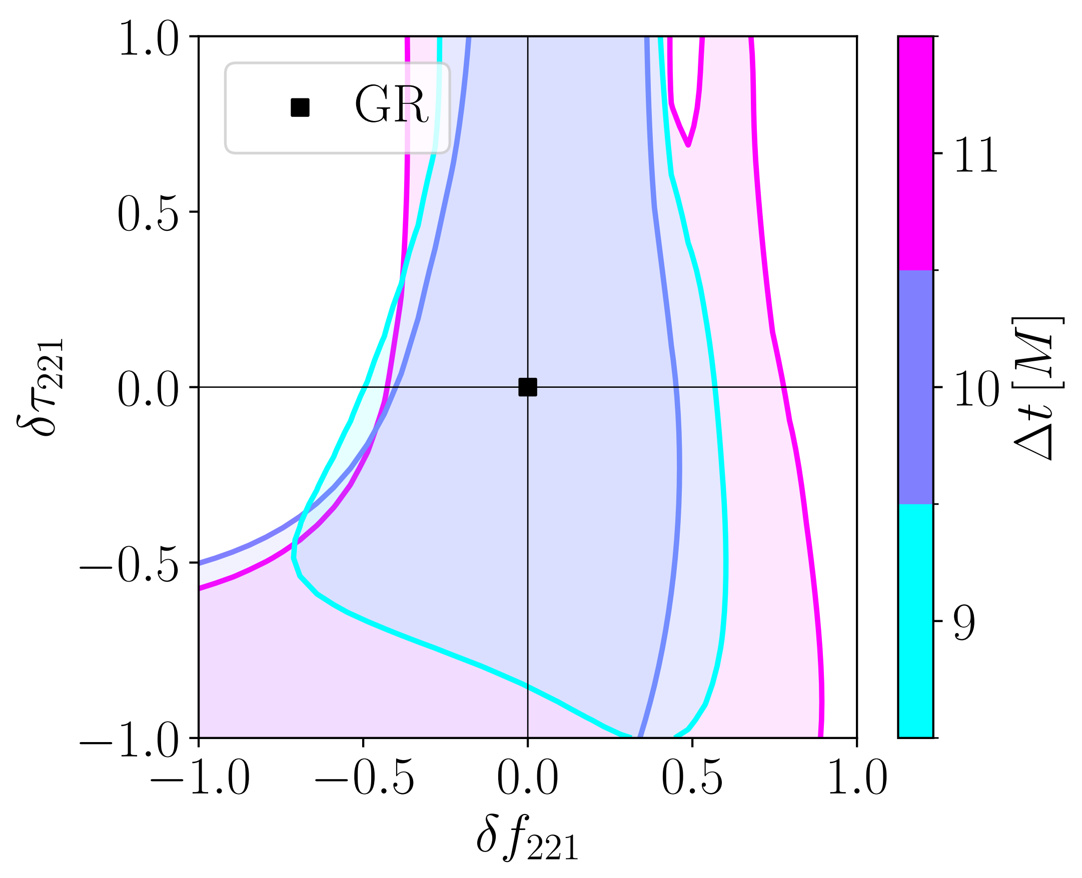
Semi-analytic method gives stronger support for the presence of the (2,2,1) mode.

Testing GR with GW250114

- Analyze using a template waveform including the (2,2,0) and (2,2,1) modes.
- Introduce the deviation parameters $\delta f_{221}, \delta\tau_{221}$ in the (2,2,1) mode, where $\delta f_{221} = \delta\tau_{221} = 0$ corresponds to GR.

(2,2,0): $f_{220}(M_f, \chi_f),\ \tau_{220}(M_f, \chi_f)$

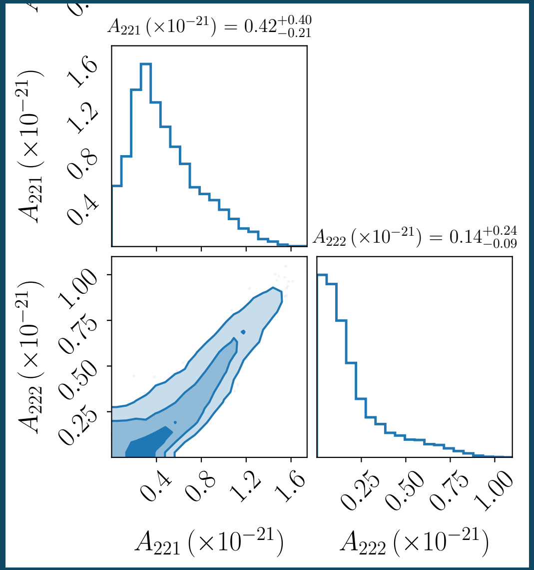
(2,2,1): $f_{221}(M_f, \chi_f)e^{\delta f_{221}},\ \tau_{221}(M_f, \chi_f)e^{\delta\tau_{221}}$

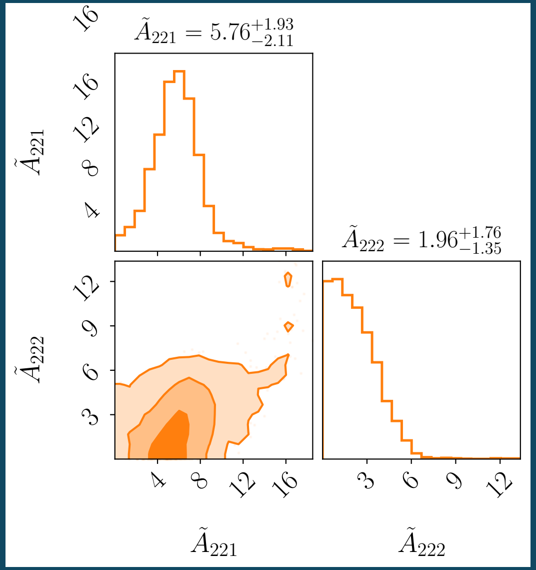


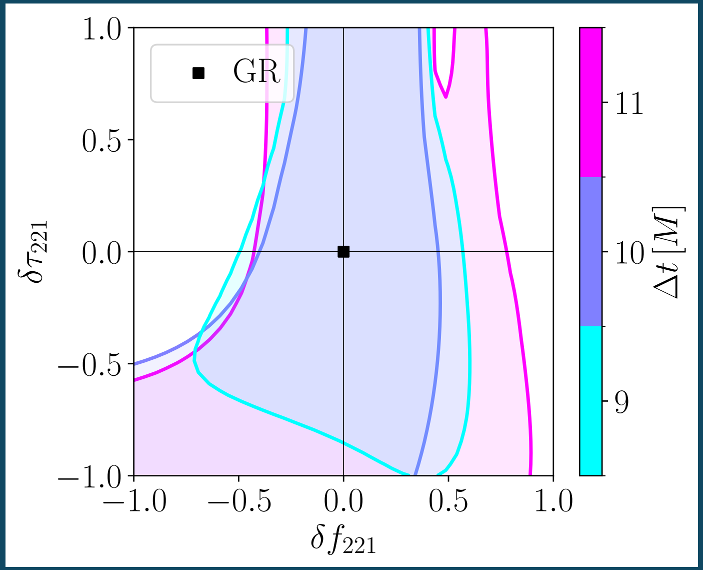
- With the current detector sensitivity, the deviation parameters connot be well constrained.
- No significant deviation from general relativity is found.

Summary

- By using orthonormalized QNM templates, the likelihood function can be analytically marginalized, enebling the semi-analytic method.
- It reduces not only the computational cost but also parameter correlations.
- It is valid for real data, and we found stronger evidence for the existence of the (2,2,1) mode in GW250114 compared to the conventional method.
- No significant deviation from general relativity is found in the ringdown regime of GW250114.



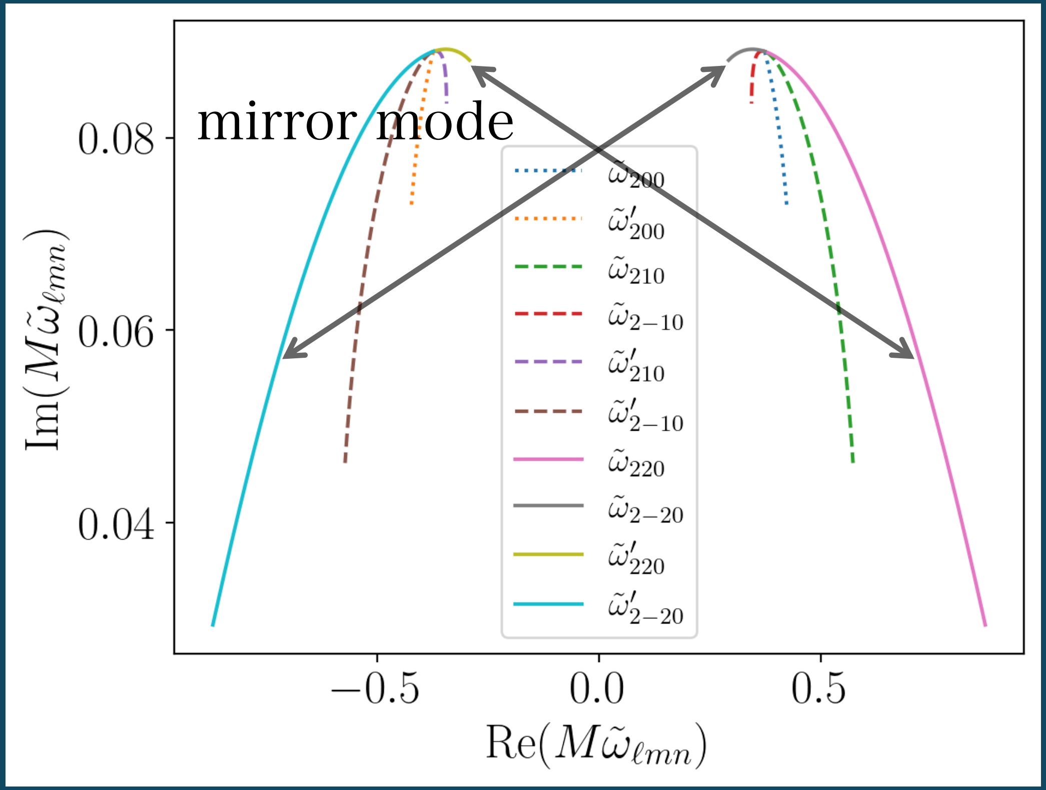




In general, a twin mode $\tilde{\omega}'_{\ell mn}$ appears as a pair with the original mode $\tilde{\omega}_{\ell mn}$, satisfying the relation

$$\tilde{\omega}'_{\ell mn} = -\tilde{\omega}^*_{\ell -mn}$$

(i.e. $\omega'_{\ell mn} = -\omega_{\ell -mn}$, $\tau'_{\ell mn} = \tau_{\ell -mn}$)



$$\begin{aligned} h(t) &= \sum_{\ell mn} \left[\mathcal{C}_{\ell mn} e^{-i\tilde{\omega}_{\ell mn} t} {}_{-2}S_{\ell mn}(\iota,\phi) + \mathcal{C}'_{\ell mn} e^{-i\tilde{\omega}'_{\ell mn} t} {}_{-2}S'_{\ell mn}(\iota,\phi)\right] \\ &= \sum_{\ell mn} \left[\mathcal{C}_{\ell mn} e^{-i\tilde{\omega}_{\ell mn} t} {}_{-2}S_{\ell mn}(\iota,\phi) + \mathcal{C}'_{\ell mn} e^{i\tilde{\omega}^*_{\ell -mn} t} {}_{-2}S_{\ell mn}(\iota,\phi)\right] \\ &= \sum_{\ell mn} \left[\mathcal{C}_{\ell mn} e^{-i\tilde{\omega}_{\ell mn} t} {}_{-2}S_{\ell mn}(\iota,\phi) + \mathcal{C}'_{\ell -mn} e^{i\tilde{\omega}^*_{\ell mn} t} {}_{-2}S_{\ell -mn}(\iota,\phi)\right] \\ &= \sum_{\ell mn} \left[\mathcal{C}_{\ell mn} e^{-i\tilde{\omega}_{\ell mn} t} {}_{-2}S_{\ell mn}(\iota,\phi) + \mathcal{C}'_{\ell mn} e^{i\tilde{\omega}^*_{\ell mn} t} {}_{-2}S_{\ell -mn}(\iota,\phi)\right] \end{aligned}$$

$({}_{-2}S'_{\ell mn}(\iota,\phi) = {}_{-2}S_{\ell mn}(\iota,\phi))$

$(m : -\ell \to \ell \Rightarrow \ell \to -\ell)$

$(\mathcal{C}'_{\ell -mn}(-1)^\ell \to \mathcal{C}'_{\ell mn})$

$m > 0$: Prograde modes
$m < 0$: Retrograde modes

The gravitational wave strain recorded by I-th detector:

$$h^I(t) = \mathrm{Re}[F^I h(t - t_{\mathrm{S}}^I)] = F_+^I h_+(t) + F_\times^I h_\times(t)$$

- $F^I = F_+^I - iF_\times^I$: Complex antenna pattern function of I-th detector
- t_{S}^I : Analysis start time at I-th detector

$$h^I(t) = \sum_{\boldsymbol{\alpha}} \sum_{j=0}^{3} c_{j,\boldsymbol{\alpha}} v_{j,\boldsymbol{\alpha}}^I(t)$$

Coefficients

$$\begin{aligned}
c_{0,\boldsymbol{\alpha}} &= \mathrm{Re}\left[\mathcal{C}_{\boldsymbol{\alpha}} S_{\boldsymbol{\alpha}}(\iota,\phi) + \mathcal{C}'_{\boldsymbol{\alpha}} S_{\boldsymbol{\alpha}'}(\iota,\phi)\right], \\
c_{1,\boldsymbol{\alpha}} &= \mathrm{Im}\left[\mathcal{C}_{\boldsymbol{\alpha}} S_{\boldsymbol{\alpha}}(\iota,\phi) - \mathcal{C}'_{\boldsymbol{\alpha}} S_{\boldsymbol{\alpha}'}(\iota,\phi)\right], \\
c_{2,\boldsymbol{\alpha}} &= \mathrm{Im}\left[\mathcal{C}_{\boldsymbol{\alpha}} S_{\boldsymbol{\alpha}}(\iota,\phi) + \mathcal{C}'_{\boldsymbol{\alpha}} S_{\boldsymbol{\alpha}'}(\iota,\phi)\right], \\
c_{3,\boldsymbol{\alpha}} &= \mathrm{Re}\left[-\mathcal{C}_{\boldsymbol{\alpha}} S_{\boldsymbol{\alpha}}(\iota,\phi) + \mathcal{C}'_{\boldsymbol{\alpha}} S_{\boldsymbol{\alpha}'}(\iota,\phi)\right],
\end{aligned}$$

Basis vectors

$$\begin{aligned}
v_{0,\boldsymbol{\alpha}}^I(t) &= F_+^I e^{-\frac{t-t_{\mathrm{S}}^I}{\tau_{\boldsymbol{\alpha}}}} \cos\left(\omega_{\boldsymbol{\alpha}}(t - t_{\mathrm{S}}^I)\right), \\
v_{1,\boldsymbol{\alpha}}^I(t) &= F_+^I e^{-\frac{t-t_{\mathrm{S}}^I}{\tau_{\boldsymbol{\alpha}}}} \sin\left(\omega_{\boldsymbol{\alpha}}(t - t_{\mathrm{S}}^I)\right), \\
v_{2,\boldsymbol{\alpha}}^I(t) &= F_\times^I e^{-\frac{t-t_{\mathrm{S}}^I}{\tau_{\boldsymbol{\alpha}}}} \cos\left(\omega_{\boldsymbol{\alpha}}(t - t_{\mathrm{S}}^I)\right), \\
v_{3,\boldsymbol{\alpha}}^I(t) &= F_\times^I e^{-\frac{t-t_{\mathrm{S}}^I}{\tau_{\boldsymbol{\alpha}}}} \sin\left(\omega_{\boldsymbol{\alpha}}(t - t_{\mathrm{S}}^I)\right).
\end{aligned}$$

- Theoretically, the ringdown waveform can be represented as a superposition of an infinite set of QNMs; however, in practical analyses, it is modeled using only a finite number of modes.

$$\{\boldsymbol{\alpha}_0, \boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_{K-1}\}$$

  - K : The number of modes included in the template

- For single-detector events, or when the detectors are nearly co-aligned, $v^I_{2,\boldsymbol{\alpha}}(t)$ and $v^I_{3,\boldsymbol{\alpha}}(t)$ become degenerate with $v^I_{0,\boldsymbol{\alpha}}(t)$ and $v^I_{1,\boldsymbol{\alpha}}(t)$. In such case, we set $c_{2,\boldsymbol{\alpha}} = c_{3,\boldsymbol{\alpha}} = 0$.

  - $D = 2$ or 4 : The number of coefficients used per mode

Template waveform $$h^I(t) = \sum_{k=0}^{K-1} \sum_{j=0}^{D-1} c_{j,\boldsymbol{\alpha}_k} v^I_{j,\boldsymbol{\alpha}_k}(t)$$

To reduce correlations between QNMs and to make the likelihood analytically marginalizable, we utilize orthonormalized basis:

$$\tilde{V} = \left(\tilde{\boldsymbol{v}}_{0,\boldsymbol{\alpha}_0}, \ldots, \tilde{\boldsymbol{v}}_{D-1,\boldsymbol{\alpha}_0}, \tilde{\boldsymbol{v}}_{0,\boldsymbol{\alpha}_1}, \ldots \tilde{\boldsymbol{v}}_{D-1,\boldsymbol{\alpha}_{K-1}}\right) = VU$$

- U : Upper triangular matrix that represents the Gram–Schmidt orthogonalization process w.r.t. the inner product

$$\left(\boldsymbol{v}_{j,\boldsymbol{\alpha}_k}, \boldsymbol{v}_{j',\boldsymbol{\alpha}_{k'}}\right) = \boldsymbol{v}_{j,\boldsymbol{\alpha}_k}^T R^{-1} \boldsymbol{v}_{j',\boldsymbol{\alpha}_{k'}}$$

- $\tilde{V}^T R^{-1} \tilde{V} = \mathbb{I}$

$$\begin{aligned}\ln \mathcal{L}(\boldsymbol{\theta}, \boldsymbol{c}) &= \boldsymbol{d}^T R^{-1} V \boldsymbol{c} - \frac{1}{2}\boldsymbol{c}^T V^T R^{-1} V \boldsymbol{c} + \text{const.} \\ &= \boldsymbol{d}^T R^{-1} \tilde{V} U^{-1} \boldsymbol{c} - \frac{1}{2}\boldsymbol{c}^T (U^{-1})^T \tilde{V}^T R^{-1} \tilde{V} U^{-1} \boldsymbol{c} + \text{const.} \\ &= \tilde{\boldsymbol{d}}^T \tilde{\boldsymbol{c}} - \frac{1}{2}\tilde{\boldsymbol{c}}^T \tilde{\boldsymbol{c}} + \text{const.} \\ &= -\frac{1}{2}(\tilde{\boldsymbol{c}} - \tilde{\boldsymbol{d}})^T (\tilde{\boldsymbol{c}} - \tilde{\boldsymbol{d}}) + \tilde{\boldsymbol{d}}^T \tilde{\boldsymbol{d}} + \text{const.}\end{aligned}$$

- $\tilde{\boldsymbol{d}} \equiv \tilde{V}^T R^{-1} \boldsymbol{d}$
- $\tilde{\boldsymbol{c}} \equiv U^{-1} \boldsymbol{c}$

Sampling procedure of the semi-analytic method

1. Draw samples of the pair (M_f, χ_f) from the marginal posterior $p^{A,\beta}_{\boldsymbol{\alpha}_k}(\boldsymbol{\theta})$.
2. Draw samples of $\tilde{A}_{\boldsymbol{\alpha}_k}$ from the conditional posterior corresponding to each sampled pair (M_f, χ_f), $p^{\beta}_{\boldsymbol{\alpha}_k}(\tilde{A}_{\boldsymbol{\alpha}_k}|\boldsymbol{\theta})$.
3. Draw angular parameters from $\mathcal{L}_{\boldsymbol{\alpha}_k}$.

$\longrightarrow$ Sample sets $\{M_f^{(i)}, \chi_f^{(i)}, \tilde{c}^{(i)}_{0,\boldsymbol{\alpha}_0}, \ldots, \tilde{c}^{(i)}_{D-1,\boldsymbol{\alpha}_{K-1}}\}_{i=1,2,\ldots,N_{\mathrm{samples}}}$

We do not need to use random sampling techniques such as the Markov Chain Monte Carlo method.

Mode identification

To identify subdominant QNMs from observed data, we adopt the probability that the marginal posterior $p(\tilde{A}_{\boldsymbol{\alpha}_k})$ excludes $\tilde{A}_{\boldsymbol{\alpha}_k} = 0$.

The orthonormalization is performed in order of significance: $c_n \begin{cases} \neq 0 & \text{for } n \leq N \\ = 0 & \text{for } n > N \end{cases}$

And we observe $\tilde{c}_n \begin{cases} \neq 0 & \text{for } n \leq \tilde{N} \\ = 0 & \text{for } n > \tilde{N} \end{cases}$

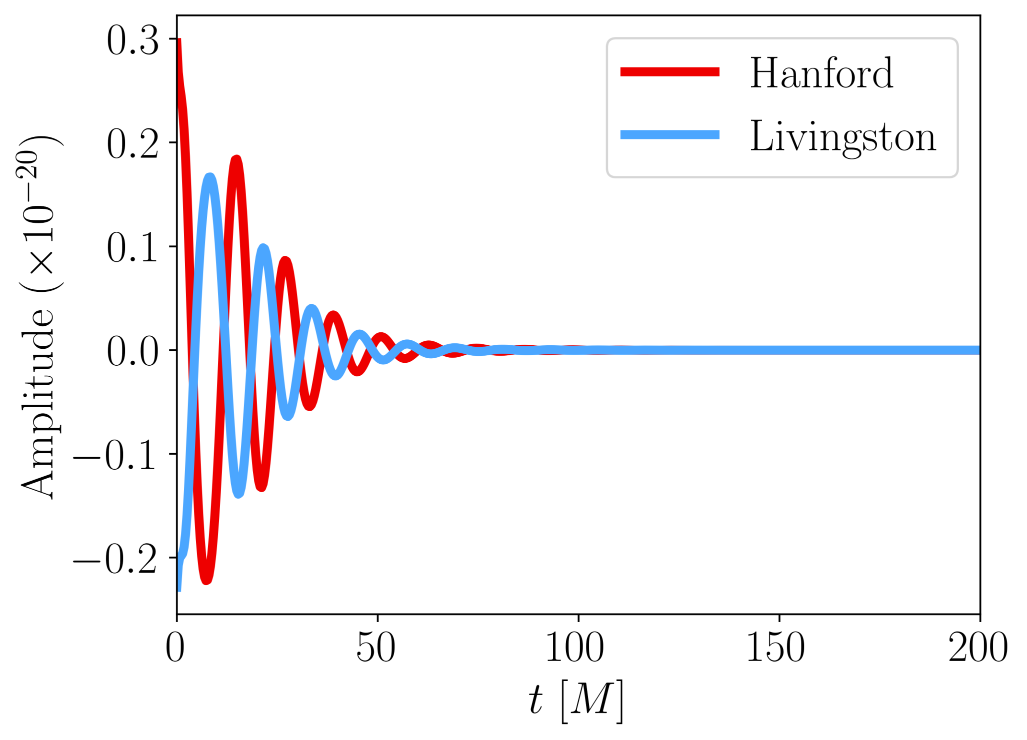
$$\Longrightarrow \quad c_n = \sum_{m=0}^{KD-1} U_{nm}\tilde{c}_m = \sum_{m=0}^{\tilde{N}-1} U_{nm}\tilde{c}_m = \sum_{m=n}^{\tilde{N}-1} U_{nm}\tilde{c}_m \qquad (U_{nm} = 0 \text{ for } n > m)$$

$$\therefore \; \tilde{c}_n \begin{cases} \neq 0 & \text{for } n \leq \tilde{N} \\ = 0 & \text{for } n > \tilde{N} \end{cases} \Longrightarrow \; c_n \begin{cases} \neq 0 & \text{for } n \leq \tilde{N} \\ = 0 & \text{for } n > \tilde{N} \end{cases}$$

e.g., for a given mode $\tilde{A}_{\boldsymbol{\alpha}_k} \begin{cases} \neq 0 & \text{for } k \leq k_{\max} \\ = 0 & \text{for } k > k_{\max} \end{cases} \Longrightarrow \; A_{\boldsymbol{\alpha}_k} \begin{cases} \neq 0 & \text{for } k \leq k_{\max} \\ = 0 & \text{for } k > k_{\max} \end{cases}$

We apply this semi-analytic method to mock data consisting of a superposition of damped sinusoids with GW150914-like parameters.

$$h^I(t) = \sum_{k=0}^{K-1} \sum_{j=0}^{D-1} c_{j,\boldsymbol{\alpha}_k} v^I_{j,\boldsymbol{\alpha}_k}(t)$$



Setup for mock data

- Modes: $(\ell, m, n) = (2,2,0), (2,2,1), (2,2,2), (2,2,3)$
- Final mass and spin: $M_f = 68.2 M_\odot$, $\chi_f = 0.692$
- Sky localization: $\alpha = 1.95\,\mathrm{rad}$, $\delta = -1.27\,\mathrm{rad}$, $\psi = 0.82\,\mathrm{rad}$
- Coefficients ($\times 10^{-20}$):

k	c_{0,α_k}	c_{1,α_k}	c_{2,α_k}	c_{3,α_k}	A_{α_k}
0	−0.4768	0.8502	−0.3376	−0.3609	1.0929
1	2.0116	−0.7439	−0.2217	0.7646	2.2877
2	−0.4013	−0.0456	0.2399	−0.0094	0.46989
3	−0.7062	−0.1697	0.1110	−0.0951	0.7409

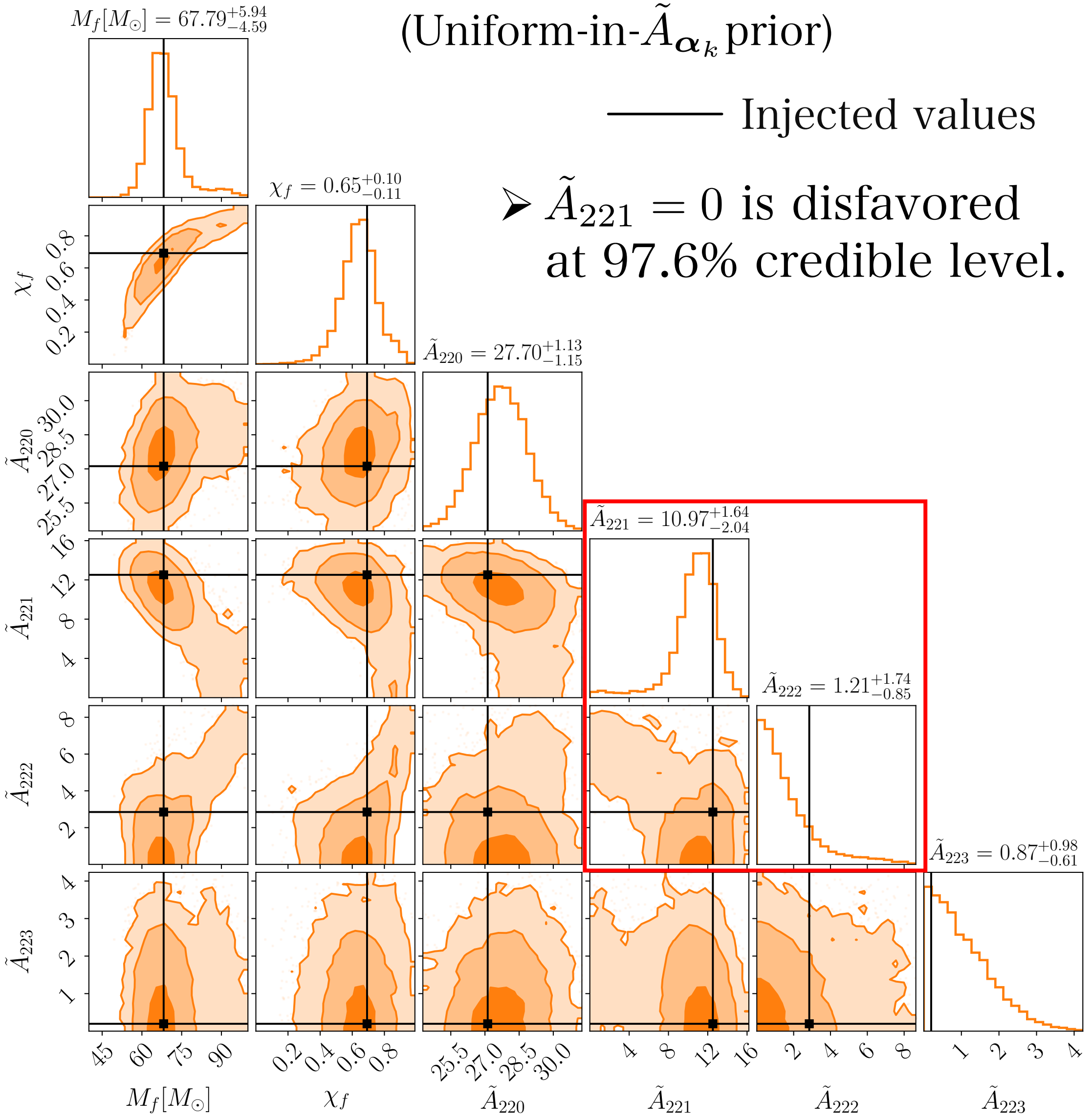
- Observed by LIGO Hanford and Livingston
- No random detector noise
- The resulting post-merger SNR is 30.

Analysis setup

- Sampling rate: 2048 Hz
- Analysis duration: $300M$ ($\approx 0.1\,\mathrm{s}$)

Semi-analytic method

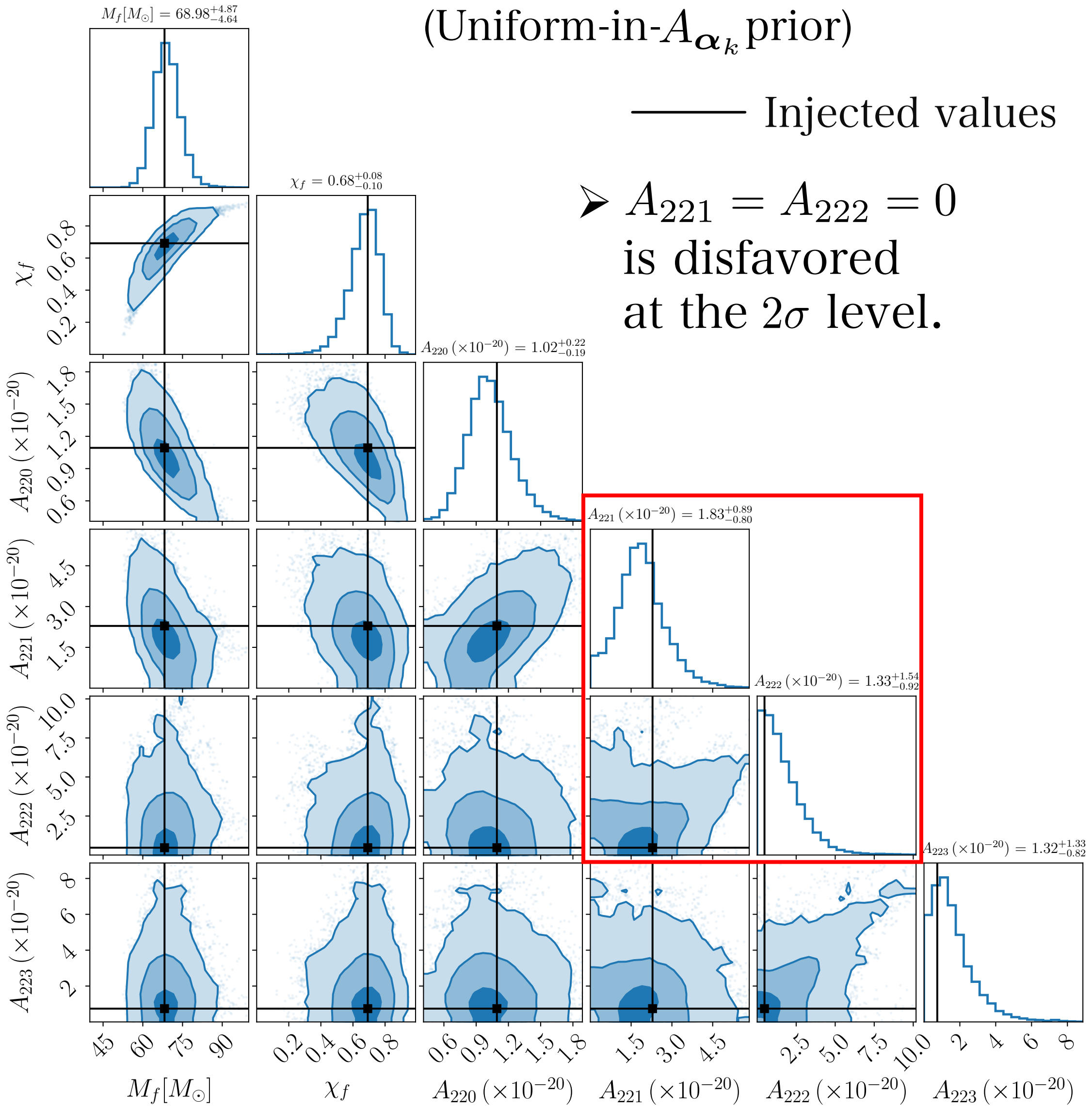
(Uniform-in-$\tilde{A}_{\boldsymbol{\alpha}_k}$ prior)



- ➢ $\tilde{A}_{221} = 0$ is disfavored at 97.6% credible level.

Random sampling method

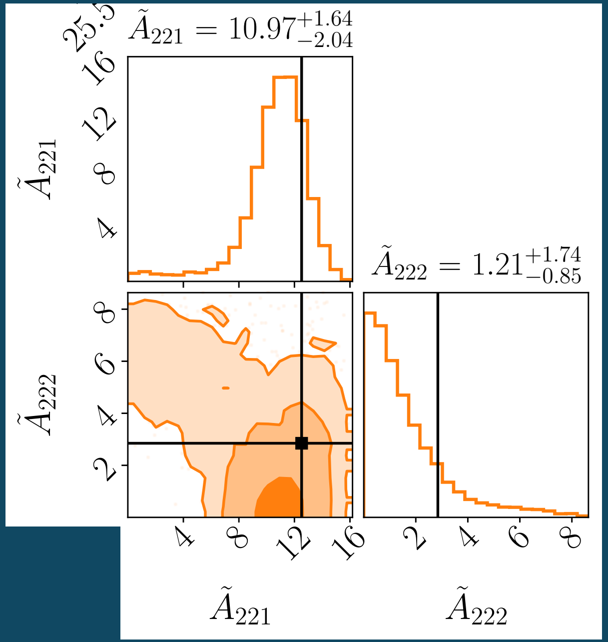
(Uniform-in-$A_{\boldsymbol{\alpha}_k}$ prior)



- ➢ $A_{221} = A_{222} = 0$ is disfavored at the 2σ level.

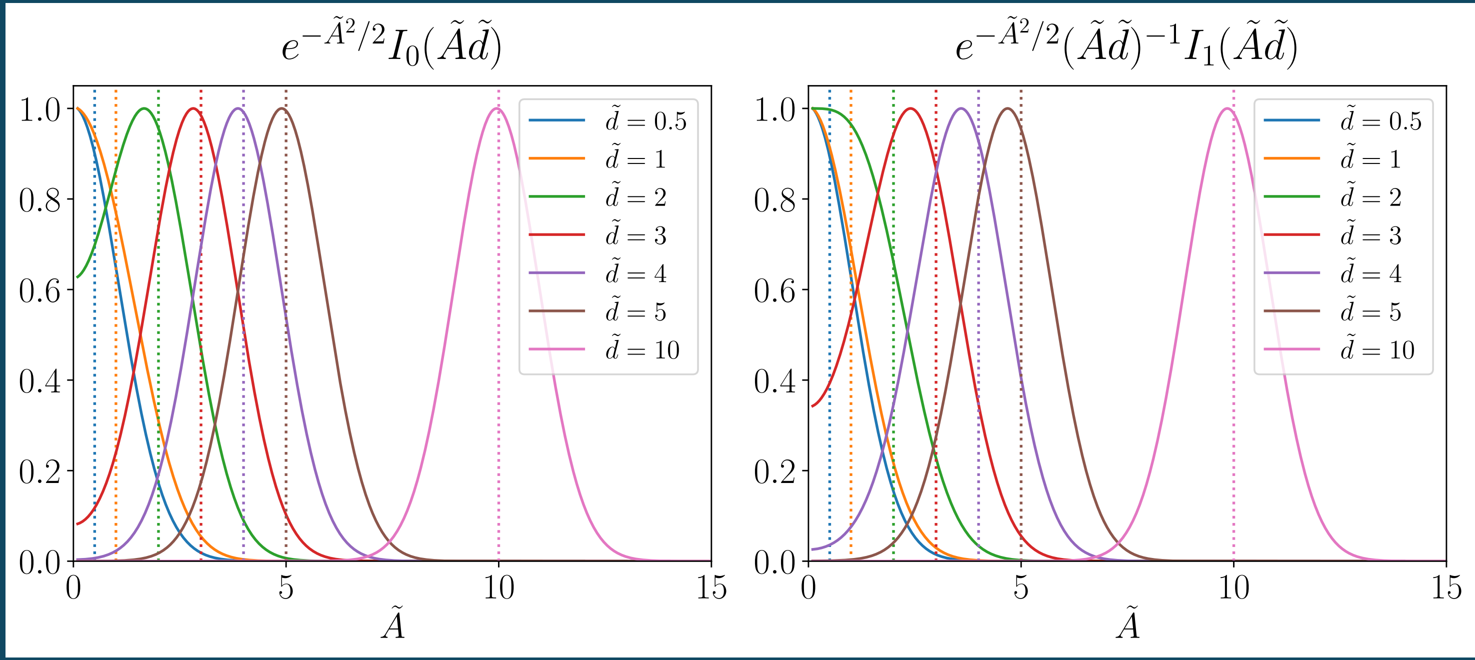
The peak of the distribution is shifted from the true values.

Semi-analytic method
(Uniform-in-$\tilde{A}_{\boldsymbol{\alpha}_k}$ prior)



Marginal posterior

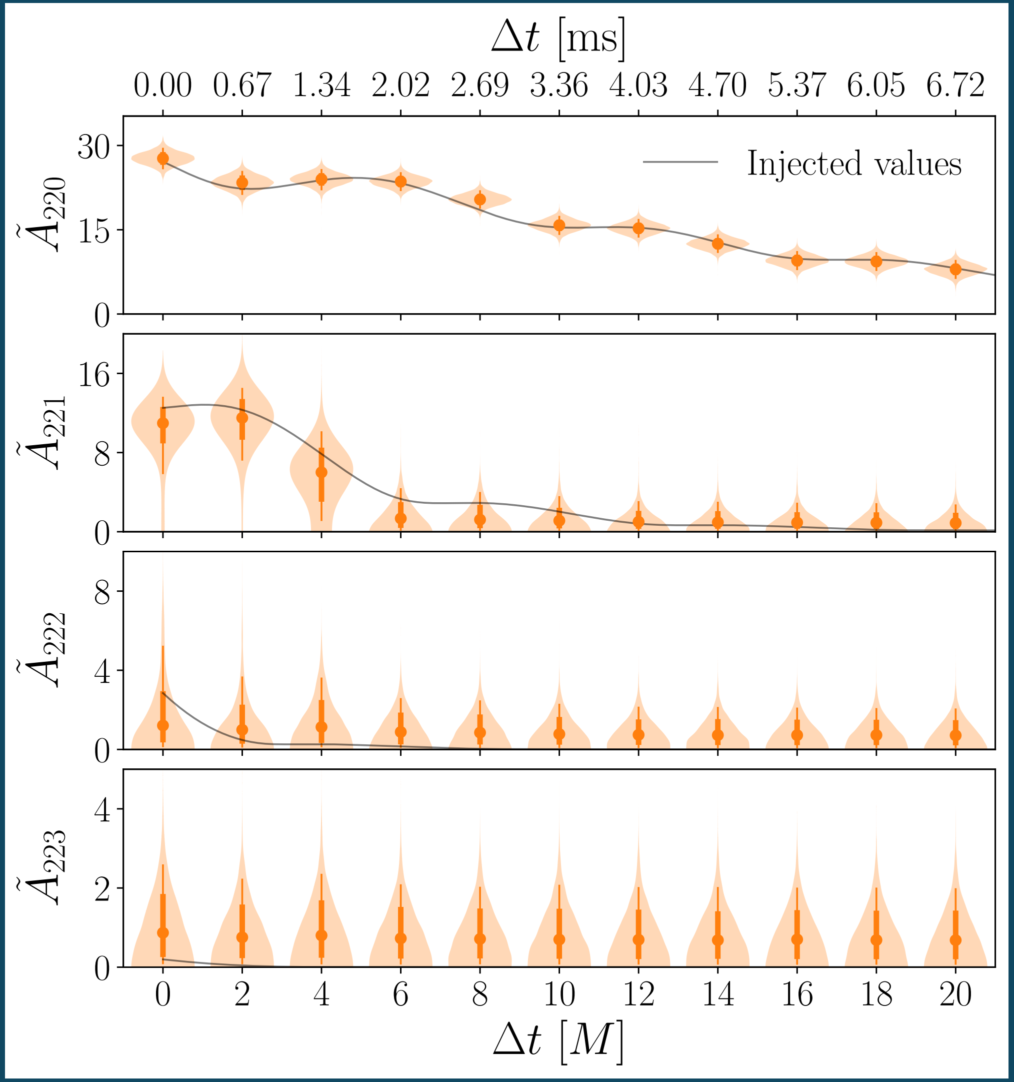
$$p^{\beta}_{\boldsymbol{\alpha}_k}(\boldsymbol{\theta}, \tilde{A}_{\boldsymbol{\alpha}_k}) \propto \begin{cases} e^{-\tilde{A}^2_{\boldsymbol{\alpha}_k}/2} I_0(\tilde{A}_{\boldsymbol{\alpha}_k} \tilde{d}_{\boldsymbol{\alpha}_k}) & (D=2), \\ e^{-\tilde{A}^2_{\boldsymbol{\alpha}_k}/2} (\tilde{A}_{\boldsymbol{\alpha}_k} \tilde{d}_{\boldsymbol{\alpha}_k})^{-1} I_1(\tilde{A}_{\boldsymbol{\alpha}_k} \tilde{d}_{\boldsymbol{\alpha}_k}) & (D=4). \end{cases}$$



The marginal posterior exhibits an offset toward supporting values smaller than the true ones, particularly when the true value is small.

It does not lead to false positives.

Results with shifted analysis start times



- Shift of the analysis start time from the peak time: Δt
- For all choices of the analysis start time, and in particular for late times when the template contains more modes than are actually present in the signal, we observe no indecation of false positives in the inf-rred mode amplitudes.

Reweighting samples

In principle, we can obtain the samples under the uniform-in-$A_{\boldsymbol{\alpha}_k}$ by reweighting samples generated under the uniform-in-$\tilde{A}_{\boldsymbol{\alpha}_k}$ prior.

$$\{M_f^{(i)}, \chi_f^{(i)}, c_{0,\boldsymbol{\alpha}_0}^{(i)}, \ldots, c_{D-1,\boldsymbol{\alpha}_{K-1}}^{(i)}\}_{i=1,2,\ldots,N_{\mathrm{samples}}}$$

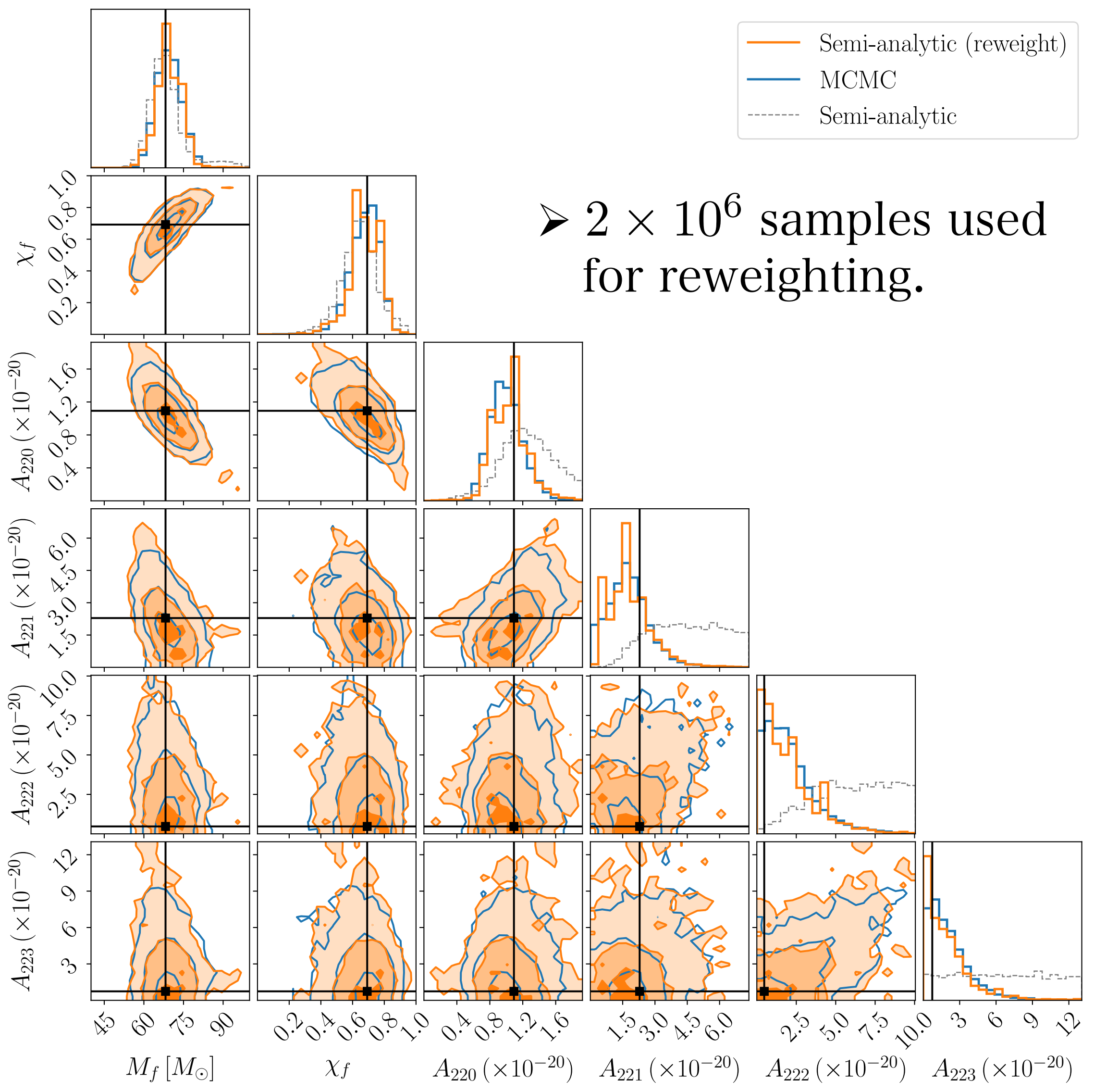
under the uniform-in-$\tilde{A}_{\boldsymbol{\alpha}_k}$ prior

$$w^i = \left|\det U(M_f^{(i)}, \chi_f^{(i)})\right| \prod_{k=0}^{K-1} \left(\frac{\tilde{A}_{\boldsymbol{\alpha}_k}^{(i)}}{A_{\boldsymbol{\alpha}_k}^{(i)}}\right)^{D-1}$$

$$\{M_f^{(i)}, \chi_f^{(i)}, c_{0,\boldsymbol{\alpha}_0}^{(i)}, \ldots, c_{D-1,\boldsymbol{\alpha}_{K-1}}^{(i)}\}_{i=1,2,\ldots,N_{\mathrm{samples}}}$$

under the uniform-in-$A_{\boldsymbol{\alpha}_k}$ prior

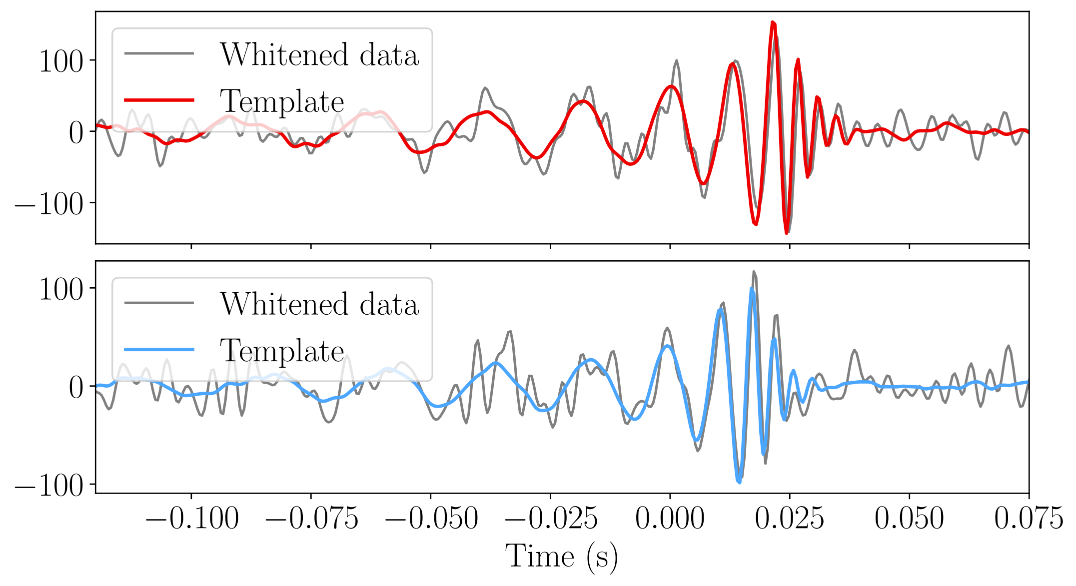
- Reweighted results (orange) are agree with the direct inference results (blue).
- Lack of smoothless in reweighted results (e.g. A_{221}) due to the prior difference.



- 2×10^6 samples used for reweighting.

We apply the semi-analytic method to real data.

GW150914



Data setup

- Analyze data from LIGO Hanford and Livingston
- Sky localization: $\alpha = 1.95\,\mathrm{rad}$, $\delta = -1.27\,\mathrm{rad}$, $\psi = 0.82\,\mathrm{rad}$
- Geocentric arrival time: $t_{\mathrm{geocent}} = 1126259462.4083147$ s

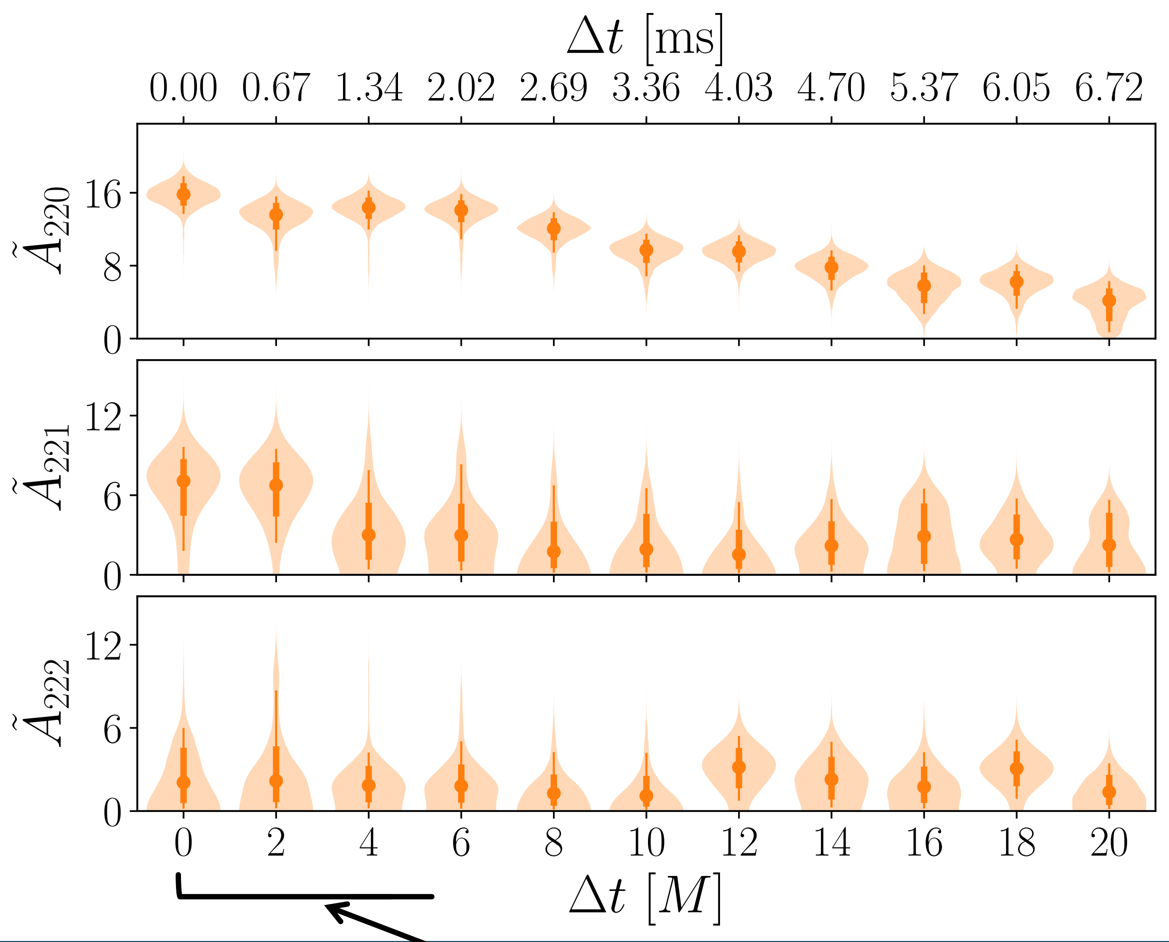
Analysis setup

- Sampling rate: 4096 Hz
- Apply a high-pass filter at 20 Hz
- Analysis duration: $300M$ ($\approx 0.1\,\mathrm{s}$)
- Modes included in template:

$$(\ell, m, n) = (2,2,0), (2,2,1), (2,2,2)$$

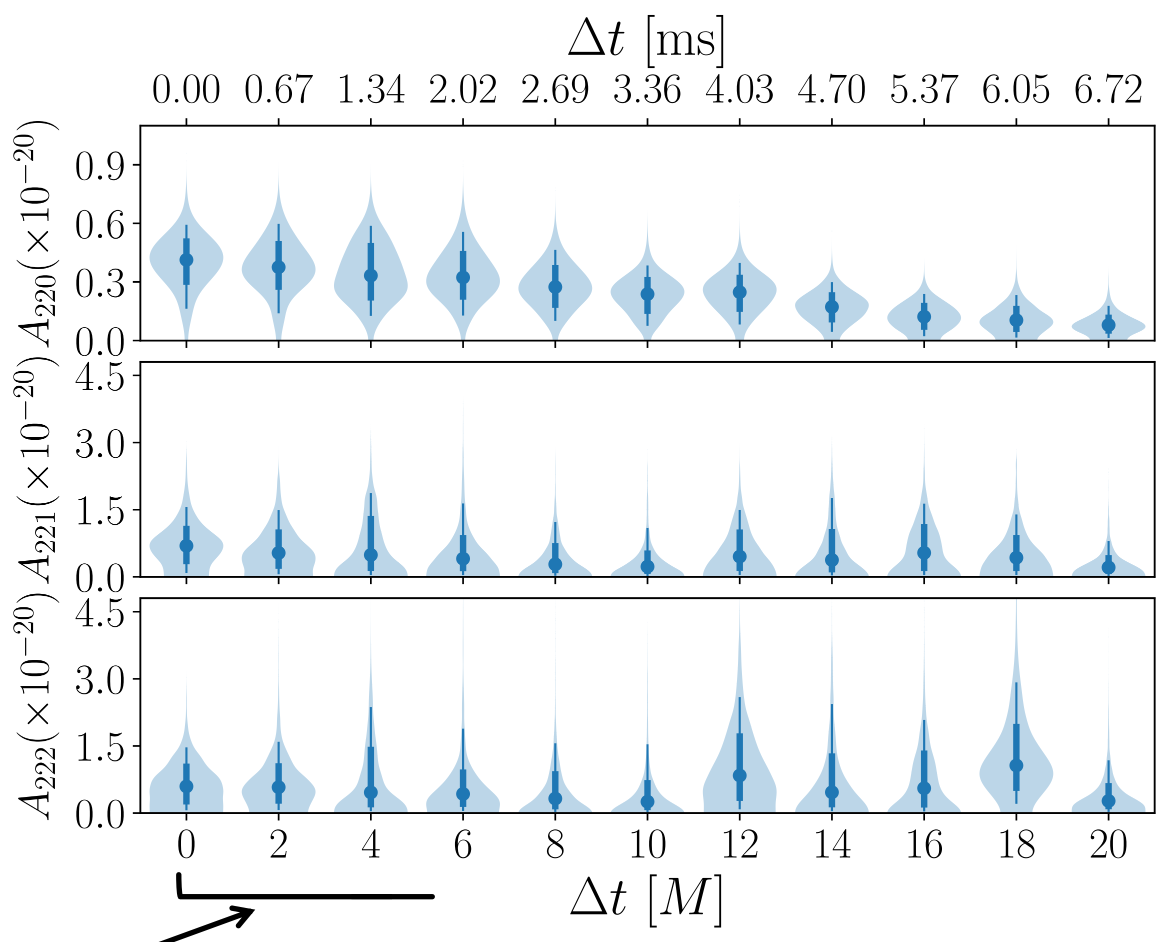
Semi-analytic method

(Uniform-in-$\tilde{A}_{\boldsymbol{\alpha}_k}$ prior)



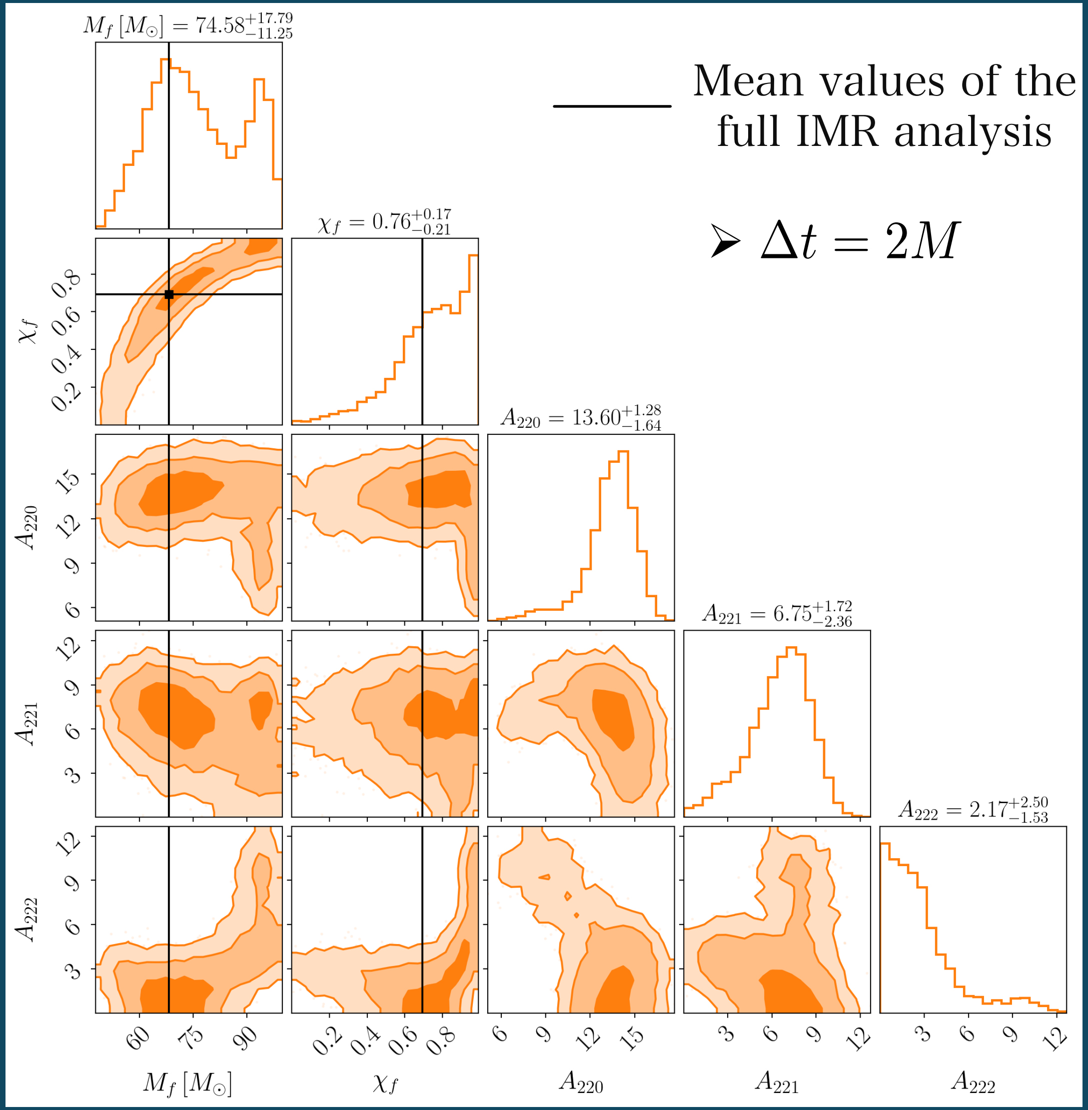
Random sampling method

(Uniform-in-$A_{\boldsymbol{\alpha}_k}$ prior)



Linear QNMs might not completely describe the early post-merger signal.
(Non-linearity, direct waves, ...)

Semi-analytic method
(Uniform-in-$\tilde{A}_{\boldsymbol{\alpha}_k}$ prior)



Random sampling method
(Uniform-in-$A_{\boldsymbol{\alpha}_k}$ prior)