Theoretical and numerical analysis for growth rate of relativistic tearing instability

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1. Introduction — What is “*tearing instability*” ?

- Tearing instability is a spontaneous mode that appears in a magnetized plasma with a current sheet.
- Be driven by small perturbation, such as pressure, density, and magnetic field.
- Change topology of magnetic field sandwiching the current.

- Trigger *magnetic reconnection* and create magnetic islands (so called “*plasmoids*”, see Fig.1).
- Drive vast energy release in relativistic magnetized plasma via reconnection.

(Here, “relativistic” means that magnetization parameter $\sigma \equiv \frac{B^2}{4\pi\rho c^2} > 1$.)

- It is important to understand the spatio-temporal scales of tearing instability.

- The spatial wavenumber dependence of the growth rate was derived (M. Hoshino, *ApJ*, 2020).

-While prediction for maximum growth rate agrees with simulations, not for maximum growth wave number.

-We have revised the conventional growth rate to more accurately estimate the maximum growth wave number.

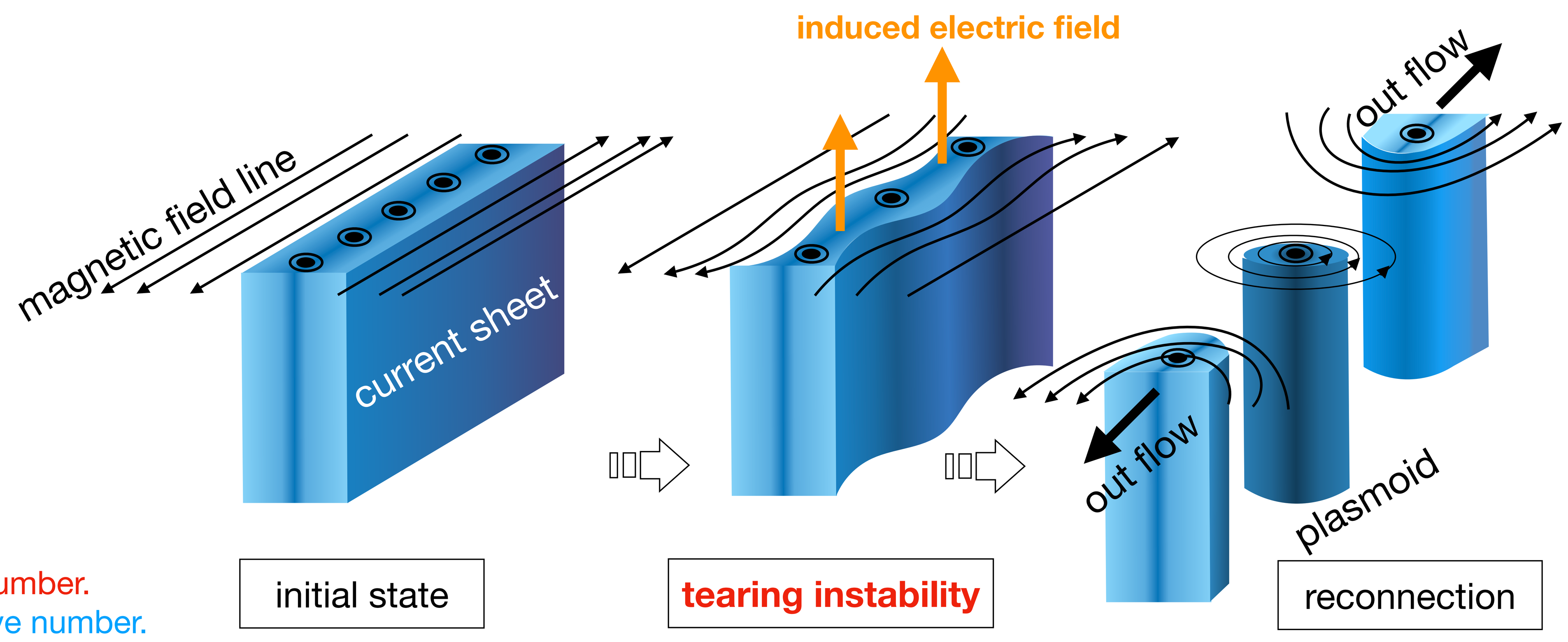


Fig.1 Simple diagram for the instability process

2. Theory — Dependence of the growth rate on wave number

- Consider “*Harris equilibrium*” in two-dimensional coordinates (x-y plane. $\partial/\partial z = 0$).

magnetic field: $\boldsymbol{B}(x) = B_0 \tanh(x/w)\mathbf{e}_y$

electron(ion) density: $n(x) = n_0 \cosh^{-2}(x/w)$

(See Fig.3-2 for more visual informations)

In the “inner region”, the magnetic field is relatively weak. → Meandering orbits (Fig.2 (b))

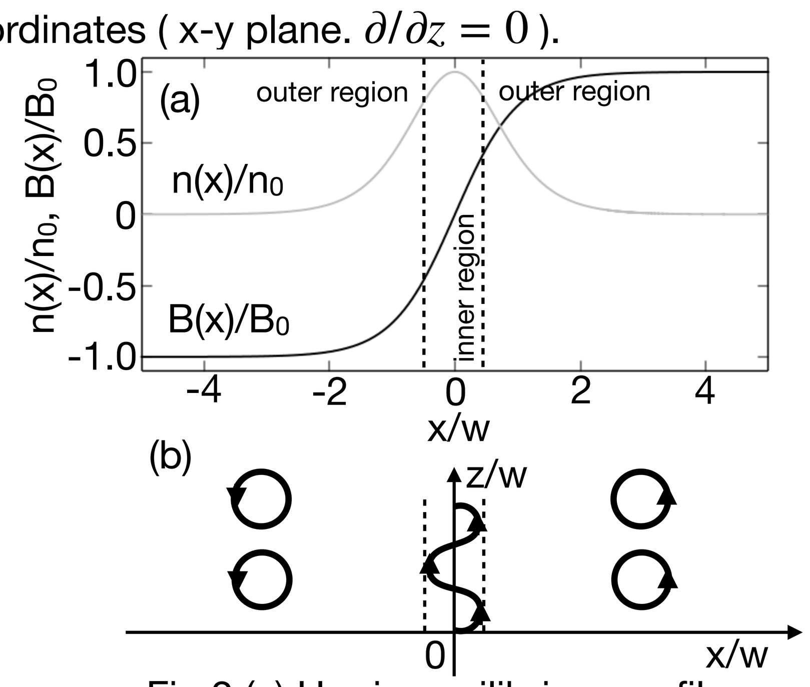


Fig.2 (a) Harris equilibrium profile.
(b) Positively charged particle orbits.

- Energy conversion equation in MHD scheme

$$\frac{\partial}{\partial t}\iint \frac{\boldsymbol{B}^2}{8\pi}dxdy = -\iint \boldsymbol{E}\cdot\boldsymbol{j}dxdy$$

- Introduce perturbation (e.g. $B_1(x,y) = \tilde{B}_1(x)\exp(iky - i\omega t)$).

$$\underbrace{\frac{\partial}{\partial t}\int_{y=-\frac{\pi}{k}}^{y=\frac{\pi}{k}}\int_{x=-\infty}^{x=\infty}\frac{|\boldsymbol{B}_1|^2}{8\pi}dxdy + \int_{y=-\frac{\pi}{k}}^{y=\frac{\pi}{k}}\int_{x=-\infty}^{x=\infty}\left(\boldsymbol{E}_1^*\cdot\boldsymbol{j}_1\right)_{\rm ideal}dxdy}_{\text{ideal MHD (dominate in outer region)}} = \underbrace{\int_{y=-\frac{\pi}{k}}^{y=\frac{\pi}{k}}\int_{x=-l_m}^{x=l_m}\left(\boldsymbol{E}_1^*\cdot\boldsymbol{j}_1\right)_{\rm non-ideal}dxdy}_{\text{non-ideal MHD (dominate in inner region)}}$$

- Assume the perturbation property $\tilde{\phi}(x) \propto \left(1+\tanh\left(|x|/w\right)/kw\right)\exp\left(-k|x|\right)$ in the inner region, and evaluate the time-scale for energy conversion (growth rate),

growth rate: $\gamma(k)\tau_c = \frac{2\sqrt{2}}{\pi}\frac{kw(1-k^2w^2)}{kw+\sqrt{r_g/w}}\frac{\beta^{3/2}}{\Gamma_\beta}$

- Previous research (M. Hoshino, ApJ, 2020) in which $\tilde{\phi}(x) = const$. gives

growth rate: $\gamma(k)\tau_c = \frac{2\sqrt{2}}{\pi}kw(1-k^2w^2)\frac{\beta^{3/2}}{\Gamma_\beta}$

$\tau_c = w/c$, β and r_g are average velocity (normalized by c) and gyro-radius of particle, $\Gamma_\beta = 1/\sqrt{1-\beta^2}$, and $l_m \sim \sqrt{r_g w}$.

3. Numerical approach — Particle In Cell (PIC) simulation

- Plasma is represented by clusters of *macro particles*.
- *Grids* are set, and electromagnetic field and current (charged) density are defined on them.

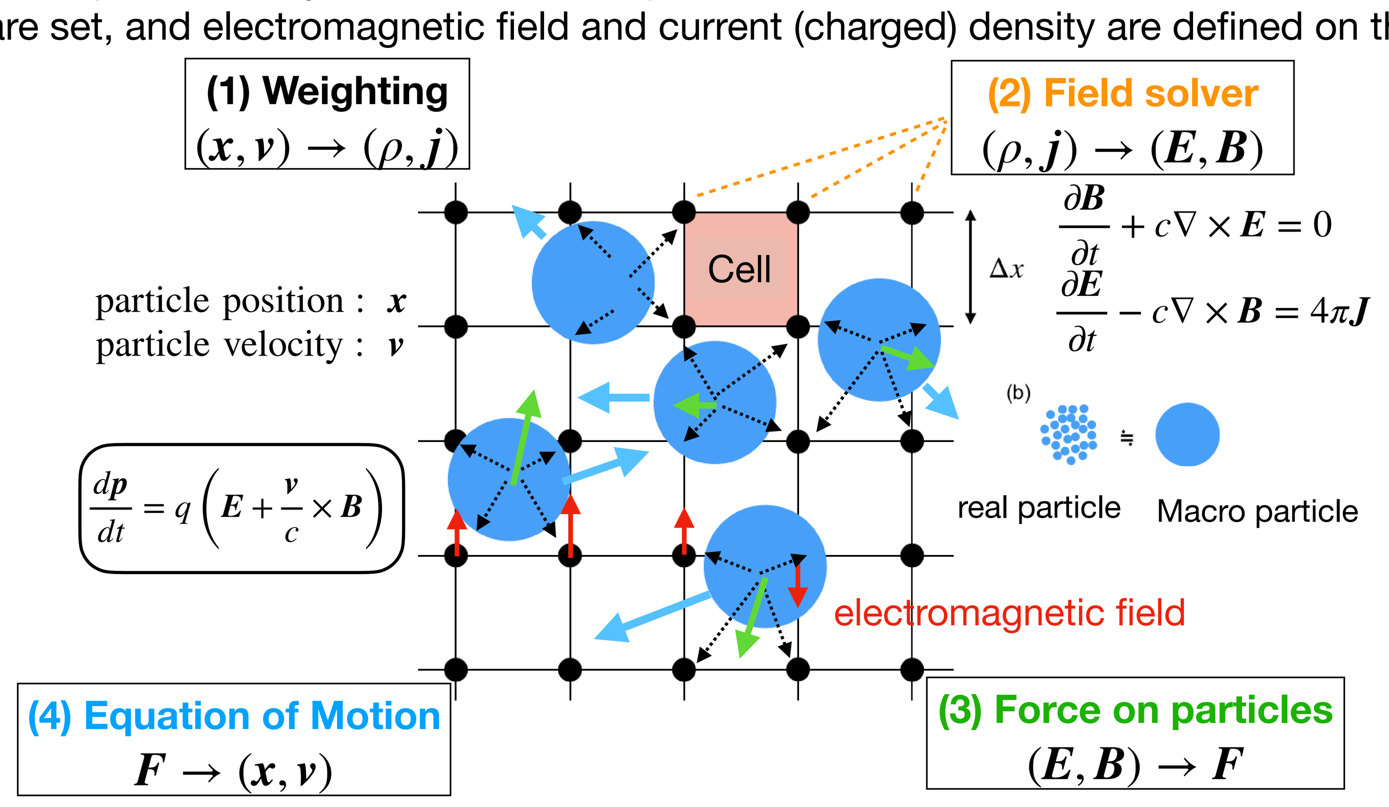


Fig.3-1 Calculation flow of PIC simulation

- 2D-PIC simulation setup

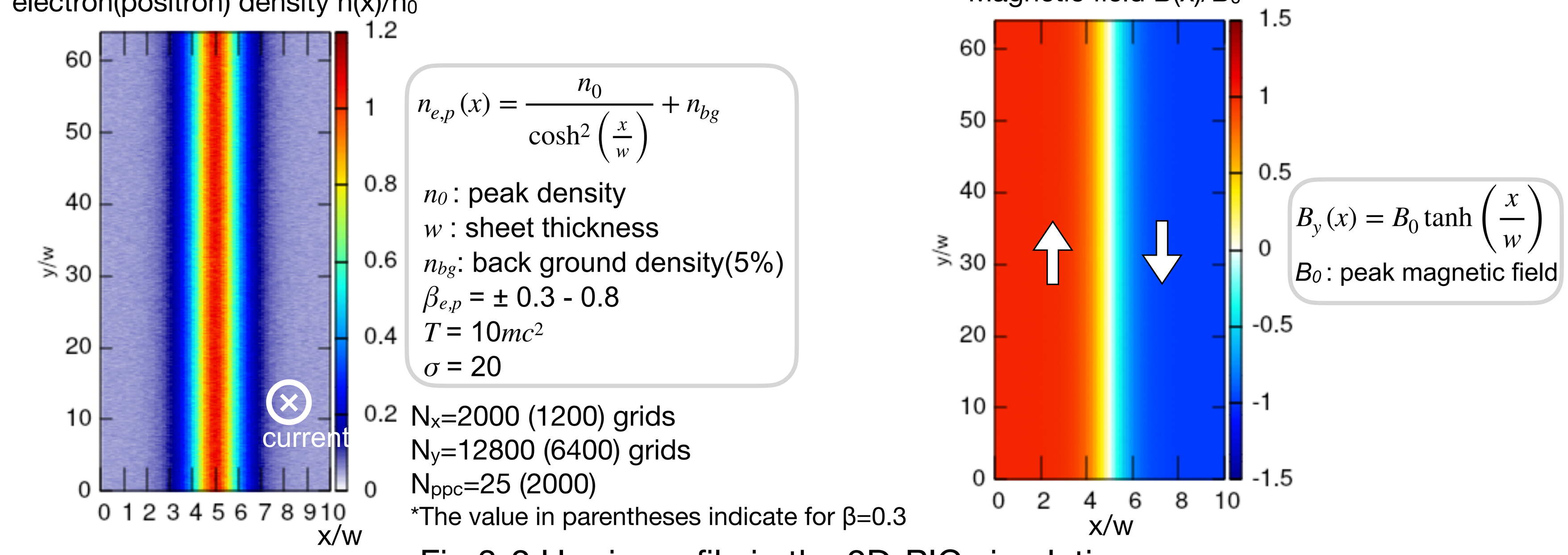
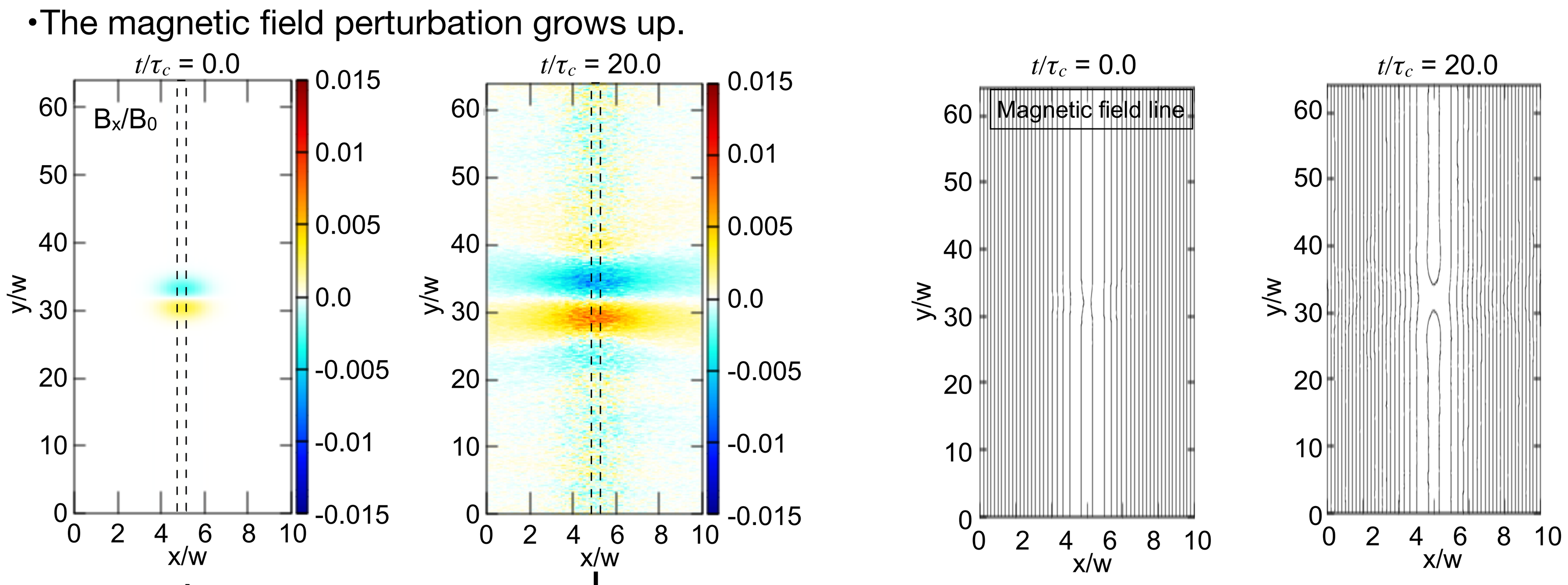


Fig.3-2 Harris profile in the 2D-PIC simulation

4. Results — Comparison of the theory and simulation

- The magnetic field perturbation grows up.

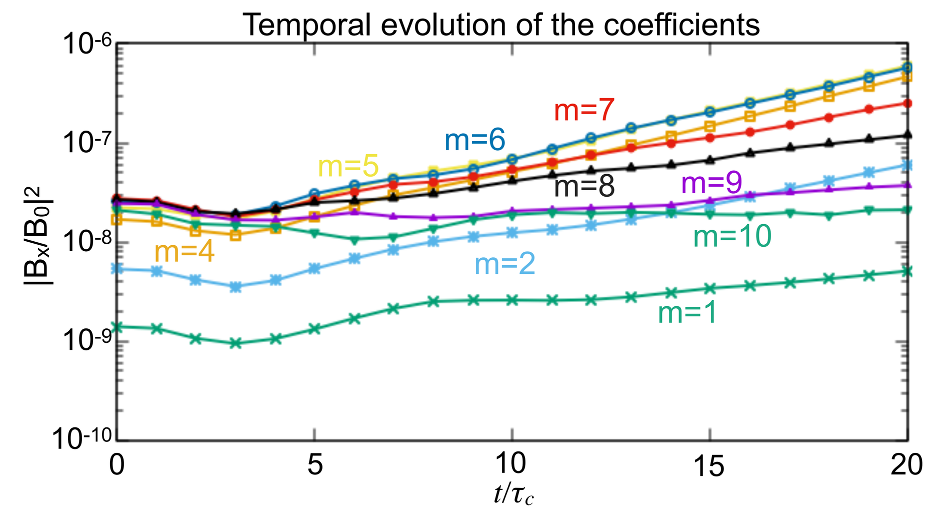


- Take averaged value of the magnetic field (B_x)
- → $\overline{B}_x(y)$
- Fourier transformation of $\overline{B}_x(y)$, and obtain the coefficients of wavenumber k_m.

wavenumber : $k_m = \frac{2\pi m}{L_y}$

m: mode number

L_y: system length in y-direction



- Comparison the growth rates. The points represent simulation results.

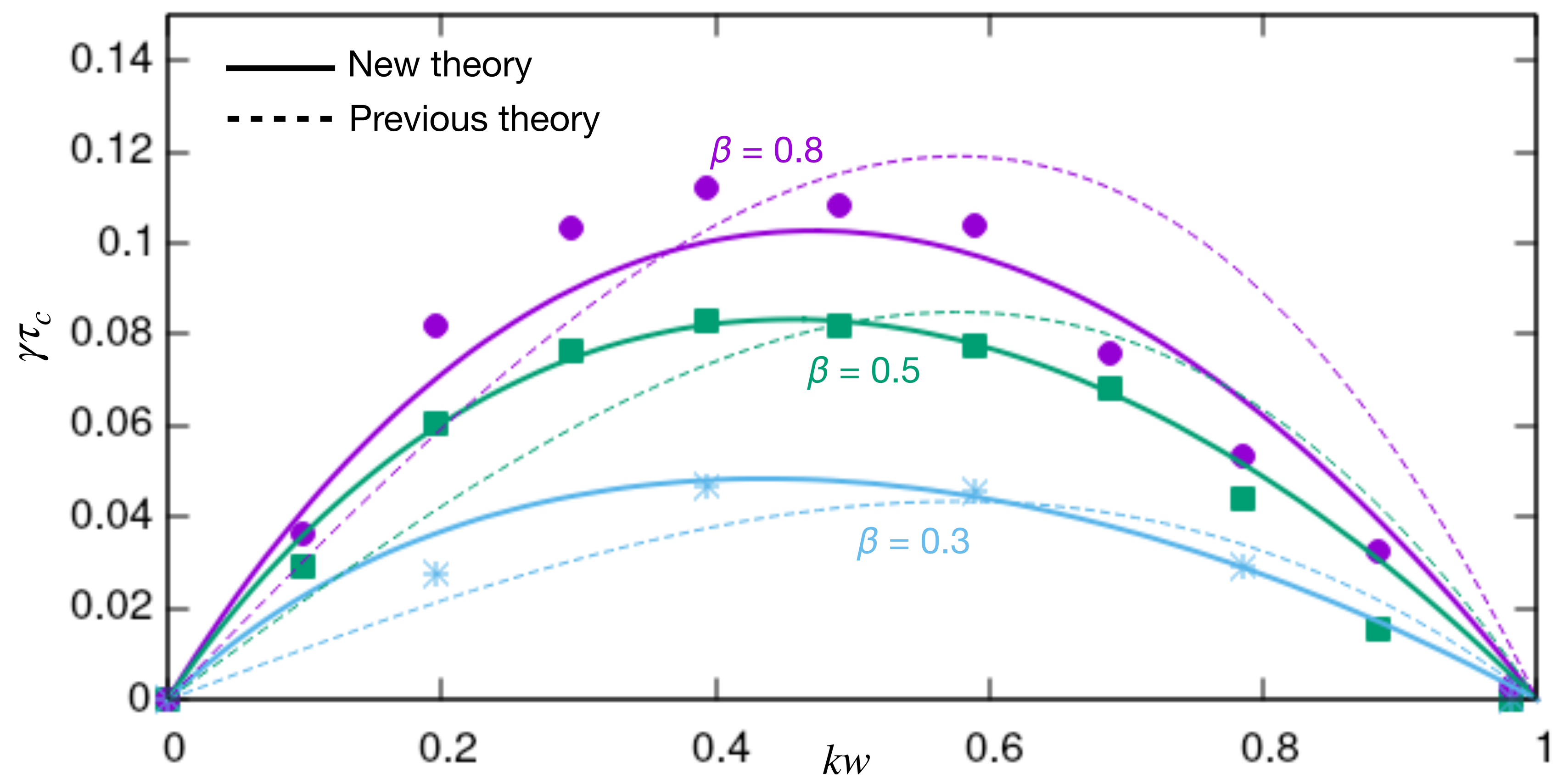


Fig.4 2D-PIC simulation results

5. Discussion & Future work

- By modifying the form of the perturbation in the “inner region” of the current sheet, it has become possible to predict the maximum growth wave number more accurately.
- In particular, the correction becomes significant as the average particle velocity degreases.
- Estimates of the size and number of plasmoids generated from the current sheet become more accurate.

- Investigate the effects on electromagnetic radiation processes associated with plasmoid coalescence.

➡ This may be related to the current sheet around the light cylinder and the precursor in BNSM, etc.

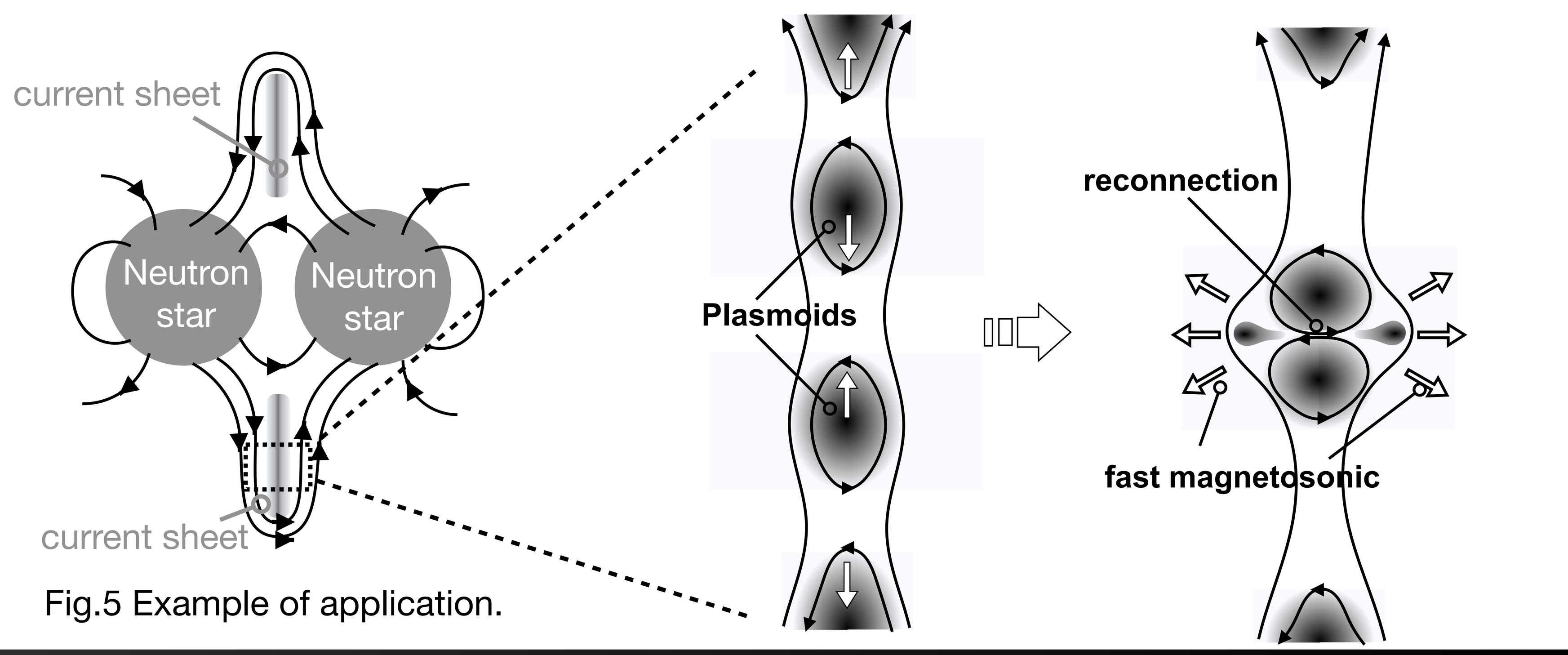


Fig.5 Example of application.

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